Talbot effect in anti-PT symmetric synthetic photonic lattices

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We investigated the Talbot effect in an anti-parity-time (PT) symmetric synthetic photonic lattice composed of two coupled fiber loops. We calculated the band structures and found that with an increase in the gain-loss parameter, the band transitions from a real spectrum to a complex spectrum. We study the influence of phase in the Hermitian operator on the Talbot effect, and the Talbot effect disappears when the period of the input field is \( N > 8 \). Further study shows that the variation of Talbot distance can also be modulated by non-Hermitian coefficients of gain and loss. This work may find significant applications in pulse repetition-rate multiplication, temporal invisibility, and tunable intensity amplifiers.

Keywords: Talbot effect; synthetic dimensions; anti-PT symmetry; photonic lattices.
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1. Introduction

The Talbot effect, initially discovered by H. F. Talbot in 1836¹, refers to the self-replication phenomenon of periodic structures under quasi-monochromatic coherent light illumination, which directly arises from near-field Fresnel diffraction. The Talbot distance \( Z_T \) is defined as the position at which waves periodically reproduce themselves along the propagation direction. In addition to optics, this effect has also been widely studied and applied in condensed matter, quantum systems, and two-dimensions. Unlike in continuous media, the occurrence of the Talbot effect in discrete media requires the pattern period to meet specific conditions, whether in one-dimension or two-dimensions.

The non-Hermitian Hamiltonians have completely real spectra if they obey parity-time (PT) symmetry, which was proved by Bender and their colleagues. The anti-PT symmetry demands that the refractive index \( n(x) = n_R(x) + i n_I(x) \) should satisfy \( n(-x) = -n^*(x) \). This means that the real part \( n_R(x) \), the phase distribution, should be an odd function, while the imaginary part \( n_I(x) \), the gain/loss, should be an even function. PT-symmetry in optics was initially observed in the context of two coupled waveguides. However, experimental achievement on an anti-PT symmetric large-scale lattice is highly limited. To overcome limitations, recently, there has been a generalization of the lattice concept from real space to synthetic dimensions, including temporal and spectral lattices.

By representing a temporal grid lattice, the evolution of pulses in two coupled fiber rings can be analogous to the dynamics of light beams in a spatial grid lattice. Researchers have, to the best of their knowledge, successfully achieved experimental realization of PT-symmetry in large-scale lattices for the first time. In this study, we employ an anti-PT symmetric synthetic photonic lattice composed of two coupled fiber loops. Electro-optic and acousto-optic modulators are utilized to adjust the phase distribution as well as gain and loss coefficients of the system. First, the band structure is calculated, followed by a theoretical analysis of the conditions for generating the Talbot effect and the calculation method of the Talbot distance. Additionally, simulations are conducted under two scenarios, Hermitian phase modulation and non-Hermitian modulation, to investigate the Talbot self-imaging phenomenon. The goal is to explore the
influence of these different modulation methods on the Talbot effect and the characteristics of self-imaging.

2. Model and Theory

The anti-PT symmetric synthetic photonic lattice is formed by two coupled fiber loops, which are shown in Fig. 1(a). A 50:50 directional coupler connects two fiber rings with a length difference $\Delta L$.[19,29] Once the coupler receives the initial light pulse injected into the long loop, it generates two new pulses that flow into each loop. After completing one round trip, the pulses in the short loop return to the coupler first, and subsequently propagate in the long loop. As a result, the two pulse trains interfere with each other through the coupler.[20]. Like the dynamics of beams in spatial grids, the pulse train undergoes incremental evolution as the number of round trips increases. The equivalent lattice in the time domain is shown in Fig. 1(b), where $m$ is the number of pulses entering the coupler, and $n$ is the position of the pulse in the lattice.

As mentioned in the introduction section, to guarantee the odd symmetry of the real part, the phase modulator needs to be adjusted so that $\varphi_n$ satisfies

$$
\varphi_n = \begin{cases} 
-\varphi, & n = 2k \\
+\varphi, & n = 2k + 1 
\end{cases}
$$

where $\varphi$ is the amplitude of phase modulation, $k \in \mathbb{Z}$. The distributions of $\varphi_n$, $\gamma$, and $\beta$ are shown in Fig. 1(c).

Similarly, to ensure that the imaginary part is an even function, the acousto-optic modulator needs to be adjusted, and the gain ($G > 0$) and loss ($\gamma < 0$) are set as

$$
\begin{align*}
\gamma_p &= \gamma, \gamma_q = G, & \text{for } \text{mod}(n + 3; 4) = 0 \\
\gamma_p &= \gamma, \gamma_q = \gamma, & \text{for } \text{mod}(n + 3; 4) = 1 \\
\gamma_p &= G, \gamma_q = \gamma, & \text{for } \text{mod}(n + 3; 4) = 2 \\
\gamma_p &= G, \gamma_q = G, & \text{otherwise}
\end{align*}
$$

Assuming that the amplitudes of the pulses in the short and long loops are $U_m$ and $D_m$, respectively, the pulse iteration equation governing the dynamics in the anti-PT symmetric synthetic photonic lattice can be expressed as

$$
\begin{align*}
U_{m+1} &= \frac{1}{2} e^{i\varphi} (U_m + iD_m) e^{-i\varphi} \\
D_{m+1} &= \frac{1}{2} e^{-i\varphi} (D_m + iU_m)
\end{align*}
$$

To better study the optical pulse transmission characteristics in the synthesized photonic lattice, we use $e^{i\varphi} e^{i\beta}$ to solve the dispersion relation of the system. The Bloch mode is $(U_n^m, D_n^m)^T = (U, D)^T e^{i\varphi} e^{i\beta}$, where $(U, D)^T$ is the periodic Bloch function, $k$ and $\beta$ represent the transverse Bloch momentum and longitudinal propagation constant, respectively.[36,37]. By inserting Eqs. (1) and (2) and the Bloch mode $(U_n^m, D_n^m)^T = (U, D)^T e^{i\varphi} e^{i\beta}$ into Eq. (3), we can obtain the dispersion relation

$$
\cos(4k) = 2 + 4 \cos(2\beta) + \cos(2\varphi) + 8 \cos(2\beta) \cos(\varphi) \cosh(2G) + 2 \cosh(4G).
$$

The band structure can be manipulated by adjusting the wave vector $k$, the phase $\varphi$, and the gain $G$ (loss $\gamma$). For $G = 0$, Eq. (4) becomes

$$
\cos(4k) = 1 + 4 \cos(2\beta) + 2 \cosh(4G).
$$

Figure 2 illustrates the band structures with various gain $G$. In Fig. 2(a), the band structures become entirely real and exhibit anomalies. In Figs. 2(b)–2(e), the bands exhibit both real and imaginary parts. However, in Fig. 2(f), the bands become entirely imaginary.

Considering the essential prerequisites for the occurrence of the Talbot effect in the synthetic photonic lattice, we assume that the input field has the period $N$ in the time domain grid. Due to transverse discreteness, $N$ is an integer.[19]. Therefore, the period of the incident pulse train in the coupling loop is $N\Delta T$. To satisfy the periodic boundary condition, the Bloch momentum is $k_l = 2\pi l/N$, where $M = N/4$ and $l = 0, 1, 2, \ldots, M - 1$. The propagation constant corresponding to the Bloch momentum $k_l$ is expressed as $\beta_{lj}$, where $j$ is the band index and is equal
3. Results and Discussion

3.1. Talbot effect of Hermitian phase modulation

First, we study the influence of the phase of the Hermitian operator on the Talbot effect. For $N = 4$, the gain factor takes a value of $G = 0$. Considering the periodic boundary conditions with $k_0 = 0$, the propagation constant is $\beta_{4,0} = \frac{1}{4} \arccos \left( \frac{1}{2} \left( \frac{1}{2} - \cos \varphi \right) \right)$, and the Talbot effect occurs when the mode of $\beta_{4,0}$ can be replicated during propagation. Assuming efficient recovery of the $\beta_{4,0}$ mode, we can express $\beta_{4,0}$ as $p \pi/q$, where $p$ and $q$ are relatively prime integers.

For odd $p$ and $q$, the modal recovery distance for $\beta_{4,0}$ is $\lambda_{ij} = 4q$. For $G = 0$, $\varphi = 0$, and $\beta_{4,0} = \pi/4$, according to the symmetry of the bands, the propagation constants are $\beta_{1,0} = 3\pi/4$, $\beta_{2,0} = \pi/2$, and $\beta_{3,0} = \pi/2$. The recovery distances are $\lambda_{1,0} = 16$, $\lambda_{2,0} = 8$, $\lambda_{3,0} = 8$, and $\lambda_{4,0} = 16$. Thus, the Talbot distance is $Z_T = 16$. In Fig. 3(a1), the evolution of the pulse intensity is the Talbot effect plot, where the pulse train achieves complete recovery every 16 steps. At $m = 8$, the intensity distribution is the same as the initial distribution at $m = 0$, which only accumulates a $\pi$ phase delay. For $\varphi = \pi/2$, according to the symmetry of the bands, the corresponding propagation constants of the bands are $\beta_{1,0} = 5\pi/6$, $\beta_{2,0} = 2\pi/3$, $\beta_{3,0} = \pi/3$, and $\beta_{4,0} = \pi/6$. The recovery distances are $\lambda_{1,0} = 24$, $\lambda_{2,0} = 24$, $\lambda_{3,0} = 24$, and $\lambda_{4,0} = 24$. Thus, the Talbot distance is $Z_T = 24$. The corresponding Talbot diagram is presented in Fig. 3(b1). For $N = 8$ and $\varphi = \pi/2$, the Bloch momentums are $k_0 = 0$ and $k_1 = \pi$, and the propagation constants of the fourth band are

$$\beta_{4,0} = \frac{1}{4} \arccos \left( -\frac{1}{2} \cosh(4G) \right),$$

$$\beta_{4,1} = \frac{1}{4} \arccos \left( -\frac{1}{2} \cosh(4G) \right).$$

To support recovery, $\beta_{4,0}$ and $\beta_{4,1}$ are both in the form of $p\pi/q$. The Talbot effect occurs only when $G$ equals zero, and the Talbot distance is $Z_T = 24$, as shown in Fig. 3(c1). Due to the periodic incidence conditions and transverse discreteness, the set of $N$ should be restricted to $\{4, 8, 12, \ldots, 2M_L\}$, where $M_L$ is the longest transmission distance. For $N > 8$, the Talbot effect does not occur. We use the case of $N = 12$ as an example for illustration. For $N = 12$, the corresponding Bloch momentum is $k_1 = 2\pi/3$, and the propagation constant is $\beta_{4,1} = \frac{1}{4} \arccos(-\frac{1}{2})$ and is no longer rational multiples of $\pi$. Consequently, the condition required for the Talbot effect, as illustrated in Fig. 3(d1), is not satisfied, the propagation image along the propagation direction does not exhibit periodic recovery, and the energy shows irregular oscillation.

On the other hand, the Talbot distance can be manipulated by applying linear phase modulation to the incident pulse train. We denote the interval between adjacent pulse phase shifts as $\varphi_0$. In Fig. 4, the input field of the temporal grid also has an initial momentum $\varphi_0$, which causes the Bloch momentum to become $k_0 = 2\pi t/M + \varphi_0$. For $N = 4$, the fourth band with the corresponding transmission constant is $\beta_{4,0} = \frac{1}{4} \arccos \left( \frac{1}{2} \left( -1 + \cos(4\varphi_0) - 2 \cosh(4G) \right) \right)$. The Talbot effect exists only when $\beta_{4,0}$ is a rational multiple of $\pi$, and we express $\beta_{4,0}$ as $\alpha\pi/b$, where $\alpha$ and $b$ are positive integers of relative prime numbers. Since $-\pi < \varphi_0 < \pi$ and $0 \leq G < 0.33$, the range of values for $\beta_{4,0}$ is...


\[(\pi/4, \pi/3]\), \(b\) should be at least 3, and the corresponding Talbot distance is \(Z_T = 12\). The set of Talbot distances is denoted by \(\{12, 16, 20, \ldots, 4b, \ldots, M_1\}\). For \(G = 0\), when \(\phi_0\) takes the values of 0.36\(\pi\) and 0.46\(\pi\), it corresponds to \(Z_T = 28\) and \(Z_T = 16\), respectively. For \(G = 0.21\), when \(\phi_0\) takes the values of 0.15\(\pi\) and 0.5\(\pi\), it corresponds to \(Z_T = 28\) and \(Z_T = 36\), respectively. The corresponding Talbot images are depicted in Figs. 4(a1)–4(d1).

3.2. Non-Hermitian modulated Talbot effect

The Talbot effect can also be observed by modifying the non-Hermitian coefficients of gain and loss. For \(N = 4\) and \(\varphi = 0\), the fourth band propagation constant is \(\beta_{40} = \arccos \sqrt{1 - \frac{\cosh (G)^2}{4} - \frac{\sinh (G)^2}{4} + \frac{1}{4} \times (1 - 4 \cosh (G) \sinh (G))^2}\).

When the gain coefficients are \(G = 0.29\) and \(G = 0.35\), respectively, the propagation constants of the fourth band are \(\beta_{40} = \frac{3\pi}{8}\) and \(\beta_{40} = \frac{3\pi}{10}\). The corresponding Talbot distance is \(Z_T = 32\) and \(Z_T = 40\), and the Talbot images are shown in Figs. 5(a1) and 5(b1).

Then, we investigated the case where \(N\) equals 4 and \(\varphi\) equals \(\pi\). We found that when the gain-loss coefficient and the input pulse period are identical, the Talbot effect and the amplitude of energy remain consistent for the two initial phases of \(\varphi = 0\) and \(\varphi = \pi\).

For \(N = 4\) and \(\varphi = \pi/2\), according to Fig. 2, the first and second bands are symmetrical with respect to \(\beta = 3\pi/4\), the third band is symmetrical with the fourth band \(\beta = \pi/4\), and the first (second) band is symmetrical with the fourth (third) band about \(\beta = \pi/2\). The fourth band propagation coefficient is \(\beta_{40} = \frac{1}{4} \arccos(-4\cosh(4G))\). The Talbot effect occurs when \(\beta_{40}\) is a rational multiple of \(\pi\), expressed as \(\pi n/q\). In this case, the Talbot distance can be calculated as \(4q\). Considering that the gain-loss factor falls within the range of \(0 \leq G < 0.33\), the propagation constant \(\beta_{40}\) is in the range of \(\pi/8\) to \(\pi/4\), with \(q\) being at least 5. Thus, the Talbot distance falls within the range \(\{20, 24, 28, \ldots, 4q, \ldots, M_1\}\). Accompanying the energy oscillations during Talbot imaging, for \(G = 0.25\), the propagation constant of the fourth band is \(\beta_{40} = \pi/5\). Consequently, the corresponding Talbot distance is \(Z_T = 20\), which is illustrated in Fig. 5(c1). For \(G = 0.31\), the propagation constant of the fourth band is \(\beta_{40} = 2\pi/9\), corresponding to the Talbot distance \(Z_T = 36\), as shown in Fig. 5(d1). From Figs. 5(c1) and 5(d1), the amplitude of energy oscillation increases as the gain-loss factor augments. For \(G \geq 0.33\), the band structure becomes entirely imaginary, as illustrated in Fig. 2(f), the input field distribution cannot be recovered due to varying degrees of amplification or attenuation experienced by these modes. Therefore, when the included modes are excited in the purely imaginary region of the band structure, the Talbot effect does not occur. We attempted to utilize the enumeration method to detect the presence of the Talbot effect for \(N > 4\). However, the Talbot effect does not occur.

The influence of parameters \(\varphi\) and \(\gamma\) on the energy bands is depicted in Fig. 6. When above the curve, all eigenvalues of the
system are completely imaginary. When on the curve, symmetry breaking occurs. Below the line, the eigenvalues of the system are complex, representing a complex spectrum with both real and imaginary components. It is evident that the curve exhibits symmetry with respect to the $\varphi = \pi/2$ axis.

4. Conclusion

In summary, we constructed an anti-PT symmetric synthetic photonic lattice using two coupled fiber loops. We presented the corresponding equivalent lattice diagram and derived the pulse amplitude expression for light pulses in the double loops. Then, we obtained the dispersion relation of the system using discrete plane wave solutions and plotted the band structure by varying phase values and gain-loss coefficients. Furthermore, we analyzed and studied the Talbot effect in the anti-PT symmetric synthetic photonic lattice under both Hermitian and non-Hermitian phase modulation. The research revealed that when the input period is $N = 2, 4, 6, 8$, the Talbot distance can be flexibly controlled by changing the gain and loss factors or linearly modulating the input pulse sequence. Additionally, at $\varphi = \pi/2$, when the gain-loss factor is greater than or equal to 0.33, the Talbot effect disappears. If the pulse input period, gain, and loss coefficients are the same, the band structures corresponding to the pulse phases $\varphi = 0$ and $\varphi = \pi$ exhibit the same patterns, resulting in the same Talbot effect and associated energy oscillations. This work may find promising applications in pulse repetition-rate multiplication, temporal cloaking, and tunable intensity amplifiers.

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