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Sub-Nyquist radar receiver based on photonics-assisted compressed sensing and cascaded dictionaries

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A sub-Nyquist radar receiver based on photonics-assisted compressed sensing is proposed. Cascaded dictionaries are applied to extract the delay and the Doppler frequency of the echo signals, which do not need to accumulate multiple echo periods and can achieve better Doppler accuracy. An experiment is performed. Radar echoes with different delays and Doppler frequencies are undersampled and successfully reconstructed to obtain the delay and Doppler information of the targets. Experimental results show that the average reconstruction error of the Doppler frequency is 5.33 kHz using an 8-μs radar signal under the compression ratio of 5. The proposed method provides a promising solution for the sub-Nyquist radar receiver.

Keywords: compressed sensing; dictionary learning; sub-Nyquist radar; microwave photonics; Doppler frequency.

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1. Introduction

Radar, as the most mainstream target recognition and detection method, has been extensively studied[1]. Next-generation radars require higher frequencies and larger bandwidths to fit the smaller antennas and to obtain higher resolution[2]. However, the demand for large bandwidth and high frequency poses a great challenge to the receiver of the radar system. Therefore, there is an urgent need for a technology that can receive radar echoes at a low sampling rate. Sub-Nyquist radar that is capable of sampling radar echo signals at a sampling rate much lower than the Nyquist rate of the signal is thus proposed[3].

Compressed sensing (CS) is a technique that enables the reconstruction of an original signal from samples that have been captured at a sampling rate below the Nyquist rate of the signal[4], which is a feasible solution to sub-Nyquist radar receivers. Ref. [5] proposed a sub-Nyquist radar scheme with the CS technique, which uses a Doppler focusing method to process the Doppler information of the echoes and then obtains the delay of the targets in the focusing region. Some excellent results have been achieved. However, the use of a parallel structure is quite complicated. In addition, due to the electronic bottleneck, it is hard for a pure electronic system to handle the radar signals with high frequency and large bandwidth[6].

In recent years, the development of microwave photonics has provided a promising way for the processing of microwave signals with high frequency and large bandwidth[7]. At present, there have been many reports on photonics-assisted CS systems[8,9]. By converting the microwave signal to the optical domain for processing, the above system can easily process microwave signals with a larger bandwidth. However, the above systems can only handle multitone signals that are sparse in the frequency domain, and reports of processing large bandwidth signals that are widely used in radar systems are still rare. In Ref. [10], radar dechirped signals in linearly frequency-modulated (LFM) radar systems were reconstructed using photonics-assisted CS systems. Nevertheless, this method transforms the processing of wideband radar signals into sampling and processing of dechirped signals that are also sparse in the frequency domain, and the undersampling and reconstruction of wideband radar signals are not realized. The photonics-assisted CS method in Ref. [11] reconstructed the wideband LFM signals with the partial Fourier sparsity characteristics of the LFM signals. However, this method is still indirect and is not able to handle a radar waveform of arbitrary format. To realize the photonics-assisted CS of frequency domain non-sparse signal, we proposed a method based on dictionary learning and successfully reconstructed undersampled wideband radar echoes of different formats[12]. Nevertheless, this method can only handle radar echoes with delay information for range measurement. For a radar system, it is necessary to obtain the information on other dimensions besides the delay of the target, such as acquiring the Doppler frequency and delay of the radar echo at the same time.

To realize the sub-Nyquist reception of radar echoes with both delay and Doppler information, we propose and experimentally verify a sub-Nyquist radar receiver based on photonics-assisted CS. By designing cascaded dictionaries that can get sparse representations of the broadband signals with delay and Doppler information, the undersampled echo signal can be used to extract that information. After obtaining the delay and Doppler information of the target, we can utilize imaging algorithms to construct the simulated echo signals and obtain the image of the target. For instance, the range Doppler algorithm in ISAR imaging can be effectively employed for this purpose. A proof-of-concept experiment is carried out, and the experimental results show that the proposed method successfully reconstructs the delay and Doppler information of the target of interest, and a Doppler accuracy of 5.33 kHz is achieved at a compression ratio of 5 with a signal period of 8 μs, which is much better than the analysis resolution of the Fourier transform method[13]. Therefore, in addition to successfully acquiring the information of the target in an undersampling situation, the scheme has a certain advantage in accuracy when a very short period of the signal is used for detection. The experiment also shows that this method can deal with the radar signal in different formats, which is another advantage of the proposed scheme.

2. Principle

The schematic diagram of the proposed system is given in Fig. 1. The wideband radar echo signal $x(t)$ reflected by the target of interest is received by the receiving antenna (RX) and then fed to one arm of the dual-drive Mach–Zehnder modulator (DD-MZM). The other arm of the DD-MZM is modulated by a pseudo-random binary sequence (PRBS). The DD-MZM is biased at the minimum transmission point, and the required optical carrier is generated by a laser diode (LD). Meanwhile, the peak-to-peak amplitude of the PRBS should be set as the half-wave voltage of the DD-MZM. The modulated optical signal is converted into an electrical signal by a photodetector (PD), which can be expressed as[14]

$$I = \begin{cases} +A \times x(t) & \text{when PRBS} = 1 \\ -A \times x(t) & \text{when PRBS} = -1 \end{cases}, \tag{1}$$

where A is a parameter related to the optical link, which is determined by the feature parameters of the DD-MZM, the transmission loss of the optical link, etc. After the digital signal is obtained through the analog-to-digital converter (ADC), the low-pass filtering and downsampling are performed in the digital domain, and the radar echo signal can be reconstructed from the undersampled data by a sparse reconstruction algorithm. Further data processing can also be applied to extract the information that we need.

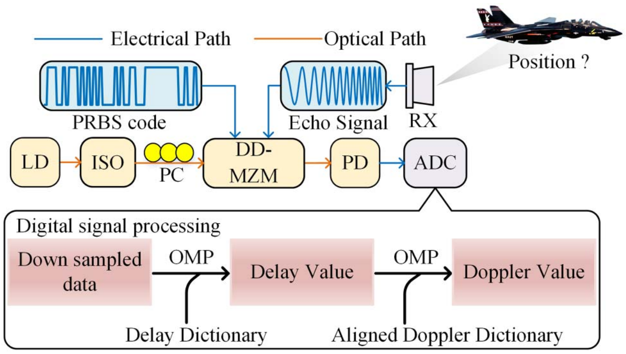

Fig. 1. Schematic diagram of the proposed sub-Nyquist radar receiver. LD, laser diode; ISO, optical isolator; PC, polarization controller; DD-MZM, dual-drive Mach–Zehnder modulator; PD, photodetector; PRBS, pseudo-random binary sequence; ADC, analog-to-digital converter; OMP, orthogonal matching pursuit; RX, receiving antenna.

According to the CS theory, the procedure to get a downsampled digital signal can be expressed as

$$\boldsymbol{y} = \boldsymbol{\phi x} = \boldsymbol{\phi \psi \theta}. \tag{2}$$

Here, $\boldsymbol{y}$ represents the downsampled digital signal; $\boldsymbol{\phi}$ comprises three components, a diagonal matrix containing the PRBS, a matrix of low-pass filtering, and a downsampling matrix; $\boldsymbol{x}$ represents the original digital signal; $\boldsymbol{\theta}$ is a sparse vector; and $\boldsymbol{\psi}$ represents a sparse dictionary that can get sparse representations of the radar signals. When the undersampled signal is obtained, after solving a minimization problem, the sparse vector can be obtained,

$$\hat{\boldsymbol{\theta}} = \arg\min \|\boldsymbol{\theta}\|_{l_1}, \quad \text{subject to } \boldsymbol{\phi\psi\theta} = \boldsymbol{y}. \tag{3}$$

The above minimization problem can be solved by the sparse reconstruction algorithm, and the algorithm selected in this work is the orthogonal matching pursuit (OMP) algorithm. When the sparse vector is obtained, we can reconstruct the original radar signal with the sparse dictionary. For active detection radars, because the transmitted signals are known in advance, in order to get a sparse representation of the transmitted signal, the sparse dictionary should be made of atomic signals with different features. The undersampled echo is first reconstructed using a delay dictionary, and the delay information of the target can be obtained by cross-correlation with the reference signal. Then the undersampled signal is reconstructed again using a Doppler dictionary whose atom signals have aligned in delay, and the Doppler information of the signal can be obtained according to the value of the reconstructed sparse vector obtained from the reconstruction algorithm. When the above information is obtained, the echo signals can be reconstructed, and the image of the target can be acquired with the help of imaging algorithms.

3. Experiment Results

A proof-of-concept experiment is performed to verify the proposed system. An LD (ID Photonics CoBriteDX1-1-C-H01-FA) is used to generate a continuous-wave light wave, which is injected into a DD-MZM (Fujitsu FTM7937EZ200) with an operating bandwidth of 25 GHz. The PRBS is generated by

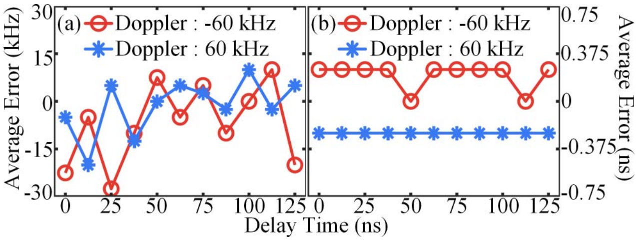

Fig. 2. Reconstruction errors for (a) Doppler frequency and (b) delay.

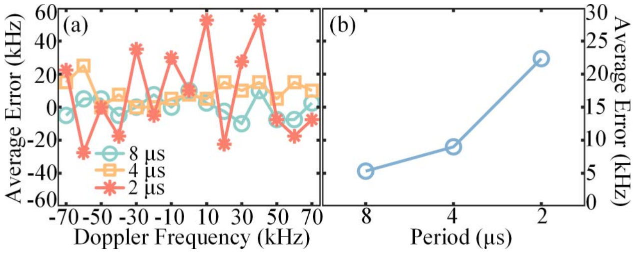

Fig. 3. (a) Reconstruction errors of Doppler frequency for different signal periods; (b) average error of all data points for different signal periods.

an arbitrary waveform generator (AWG) (Keysight M8190A), amplified by an electrical amplifier (Teledyne, Cougar AR3069B), and then sent to one arm of the DD-MZM. Since large Doppler frequency is difficult to simulate with real targets, we use the AWG to generate the predesigned echo signal. The echo signal is amplified by another electrical amplifier (Multilink, MTC5515) and then injected into the other arm of the DD-MZM. The optical signal from the DD-MZM is fed to a PD (Bookham PP-10G) for photoelectric conversion. The electrical signal from the PD is collected by an oscilloscope (R&S, RTO2032) and then further processed in MATLAB.

First, the effectiveness of the proposed method is verified by reconstruction experiments. The AWG generates a series of echo LFM signals with two specific Doppler frequencies (60 and −60 kHz) and different delays. The center frequency of the echo signal is 1.4 GHz, and the signal sweep bandwidth is 0.8 GHz. Its period is 4 μs, and the duty cycle is 4:1. The echo signal is mixed with a PRBS with a bit rate of 4 Gb/s in the microwave photonic system, and then sampled at a sampling rate of 4 GSa/s, resulting in 16,000 data points. Once the signal has undergone low-pass filtering in the digital domain, it is downsampled to 3200 points. As a result, the equivalent sampling rate decreases to 800 MSa/s, which is significantly lower than the Nyquist rate of the echo signal. The downsampled signal is initially reconstructed using a delay dictionary with a delay interval of one sampling point. Once the signal is reconstructed, it is cross-correlated with the reference signal, which allows the system to obtain the delay information. After obtaining the delay information, the downsampled signal is reconstructed again using a Doppler dictionary that has a Doppler interval of 5 kHz and has been aligned in delay. Then the Doppler frequency of the original signal is acquired by analyzing the value of the sparse vector resulting from the reconstruction process. The average error of the reconstructed Doppler frequency at different delays is shown in Fig. 2(a). Each of these data points is averaged by two independent reconstructions, and it can be found that most of the Doppler reconstruction error is within 15 kHz and does not show a correlation with the range, which means that the proposed method has a certain universality at different ranges. Figure 2(b) shows the delay reconstructed error, which is very small and lower than 0.375 ns.

It should be noted that the accuracy of Doppler frequency reconstruction can be greatly impacted by errors in delay reconstruction. Therefore, once the delay information is calculated, it is necessary to scan several adjacent delay values around the reconstructed delay value to obtain an accurate Doppler value. The delay value can then be corrected based on the Doppler value. Alternatively, multiple reconstructions of the delay can be performed to obtain an average result, which can help to achieve a more accurate delay value and avoid errors during the process. Moreover, this procedure can result in an average offset in the Doppler frequency, which necessitates calibration in practical applications. However, this calibration process only needs to be performed once.

Then, the Doppler reconstruction error is further reduced by increasing the signal period. Various echo signals with different periods (2, 4, and 8 μs) and Doppler frequencies (−70 to 70 kHz) are reconstructed in the experiment. In this experiment, the echo delays of the echo signals are all set to 25 ns and the compression ratio is set to 5, which means the equivalent sampling rate is still 800 MSa/s. Figure 3(a) displays the results of the reconstruction error for the Doppler frequency, while Fig. 3(b) exhibits the average absolute error value for each curve in Fig. 3(a). It can be seen from Fig. 3 that as the signal period is increased from 2 to 8 μs, the average Doppler reconstruction error is decreased from 22.33 to 5.33 kHz. Our experiment and simulation results consistently demonstrate that the average reconstruction error of Doppler frequency is consistent for signals with identical periods. In contrast, analyzing the Doppler frequency of signals with a signal period of 8 μs using the Fourier transform would have a frequency resolution limit of 125 kHz. Therefore, the proposed system can greatly increase the accuracy of the recovered Doppler frequency without increasing the signal period. Thus, the advantage of this scheme is that the Doppler frequency analysis can be performed without accumulating signals in several periods, which is a significant advantage when dealing with high-speed targets.

Compared with microwave photonic CS methods reported by other groups that can only handle frequency domain sparse signals, another distinct advantage of the proposed system is that it can receive wideband radar signals other than LFM signals. It is well known that radar signals other than LFM signals cannot be dechirped, so their waveform needs to be fully sampled at the radar receiving end. Using the proposed system, the frequency domain non-sparse signals can be sampled at a sub-Nyquist rate, which can greatly reduce the sampling rate requirement of the radar receiving end. Then, the delay and Doppler frequency reconstruction of other kinds of radar signals is demonstrated,

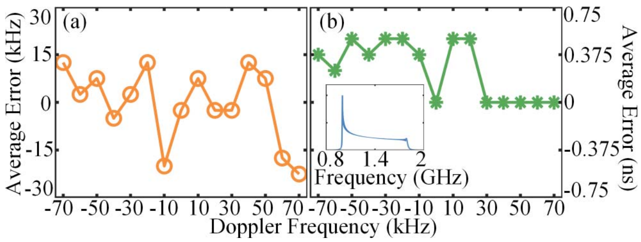

Fig. 4. Reconstruction errors for (a) Doppler frequency and (b) delay, with a constant delay and Doppler frequency ranging from −70 to 70 kHz. The inset in (b) is the spectrum of the NLFM signal used.

which is verified by using nonlinearly frequency-modulated (NLFM) signals.

The transmitted NLFM signal has a center frequency of 1.4 GHz, a bandwidth of 0.8 GHz, a period of 4 μs, and a duty cycle of 4:1. The simulated NLFM echoes with different Doppler frequencies are used for reconstruction at a compression ratio of 5. The reconstruction error of the Doppler frequency is shown in Fig. 4(a), the reconstruction error of the delay is shown in Fig. 4(b), and the inset in Fig. 4(b) shows the spectrum of the NLFM signal used. It can be observed that the system is capable of accurately extracting the Doppler frequency. Additionally, when a different transmitted signal is used, the system's performance remains consistent, demonstrating its universality and applicability in radar applications. In addition, the average Doppler error of all data points at this time is about 9.17 kHz, which is still much better than the resolution obtained by the Fourier transform with a signal period of 4 μs.

Next, dual-target detection is further demonstrated. In this experiment, the simulated echo signals are LFM signals that have the same parameters as those used for obtaining Fig. 2. The compression ratio is also set at 5. The delays of the two targets are set at 25 and 100 ns, while the Doppler frequencies are set at 30 and −50 kHz, respectively. The reconstruction result of the two targets is shown in Fig. 5(a). As can be seen, the two delays, as well as the two Doppler frequencies, are well recovered (25 and 100.5 ns, 30 and −50 kHz) in this experiment. When the above information is obtained, the image of the targets could be obtained by imaging algorithms such as the range Doppler algorithm. It should be noted that the conventional range Doppler algorithm requires accumulating multiple signal periods of the echo signals to improve the azimuth/Doppler accuracy. Compared with the conventional method, the proposed method can obtain a much better Doppler accuracy in one signal period. Then, the delay and Doppler information of the two targets are changed to 25 and 200 ns, and −50 and 40 kHz. The reconstruction result is shown in Fig. 5(b). The delays are also very accurately recovered (25.5 and 199.75 ns), while the Doppler frequency errors are 5 kHz and 0, respectively.

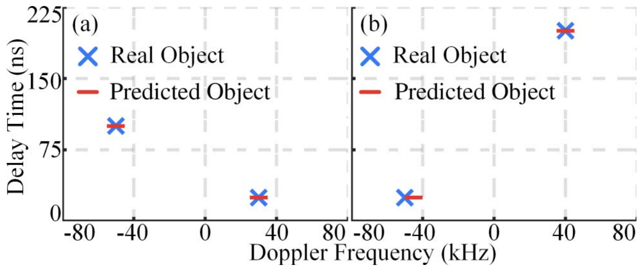

Fig. 5. Reconstruction of two targets. The delay and Doppler frequency of the two targets are (a) 25 ns and 30 kHz, 100 ns and −50 kHz; (b) 25 ns and −50 kHz, 200 ns and 40 kHz.

4. Discussions

By using the proposed sub-Nyquist radar receiver, the requirement of sampling rate at the receiving end of the radar can be greatly reduced, and the delay and Doppler information of radar echoes can be recovered and extracted by undersampling at the reduced sampling rate. The cost of these advantages is that more computing resources are required. In this experiment, the delay and Doppler information of the original signal are recovered by the undersampled signal and the dictionary through MATLAB. If the above signal processing is realized through hardware, the signal recovery time will be greatly reduced. In addition, also limited by the computing power, the two dictionaries used in the system are not trained because the data points of the dictionaries are 16,000 and much longer than those in Ref. [12]. If greater computing power is employed, dictionaries that can get better sparse representations of the transmitted signal can be obtained through a dictionary-learning algorithm to reduce the reconstruction error. Although the process of training a dictionary requires a significant amount of computing resources, once a dictionary is trained for a transmitted signal, it can be utilized multiple times across a variety of environments.

An expensive AWG is used to generate the PRBS to simplify the experiments. Although the sampling rate of the receiver is much decreased, a high-data-rate PRBS is indispensable to the CS system. In practical application, a low-cost shift register can be used to generate the required PRBS with a high data rate. In this work, because the quality of PRBS directly determines the quality of the reconstructed signal, and the PRBS generated by the AWG can still ensure good signal quality at 4 Gb/s, a 4 Gb/s PRBS is used in our experiment. According to the theory of CS, the maximum frequency of the echo signal is limited to within 2 GHz.

Finally, the application scenario of the proposed system is further discussed. The proposed approach for undersampled signal recovery involves constructing a dictionary with different delays and Doppler frequencies. While this method offers some advantages over conventional Fourier analysis for extracting Doppler information from a single signal period, its overall accuracy for Doppler recovery is inferior to Fourier analysis methods that allow for multi-period accumulation over longer intervals. This is due to the limitations of the proposed method in dictionary size and inability to accumulate information over multiple periods. Although theoretically, it is possible to achieve higher Doppler resolution by further increasing the signal period, it should be noted that an excessively large signal period can introduce difficulties in data processing. Therefore, compared with low-speed moving objects, the proposed scheme is more suitable for detecting ultrahigh-speed moving targets.

5. Conclusion

In summary, a sub-Nyquist radar receiver based on photonics-assisted CS and cascaded dictionaries is proposed and experimentally demonstrated. Through the cascaded dictionaries, the undersampled echo signals with delay and Doppler information are successfully reconstructed, and the corresponding information is extracted by using only one period of signal. The advantage of this scheme is that the Doppler frequency analysis can be performed without accumulating signals in several periods, which is a significant advantage when dealing with high-speed targets. Experimental results indicate that the Doppler reconstruction error is as low as 5.33 kHz for a signal period of 8 μs and a compression ratio of 5 when only a single period of echo is used. This performance is significantly better than the frequency analysis resolution achieved by conventional Fourier analysis when only a single period is considered. The proposed sub-Nyquist radar receiver is a promising solution for reducing the signal-processing burden in microwave photonic radar systems that employ high-frequency, large-bandwidth radar signals. This approach is especially advantageous when detecting ultrahigh-speed moving targets.

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