Multimode photonic quantum memory could enhance the information processing speed in a quantum repeater-based quantum network. A large obstacle that impedes the storage of the spatial multimode in a hot atomic ensemble is atomic diffusion, which severely disturbs the structure of the retrieved light field. In this paper, we demonstrate that the elegant Ince-Gaussian (eIG) mode possesses the ability to resist such diffusion. Our experimental results show that the overall structure of the eIG modes under different parameters maintains well after microseconds of storage. In contrast, the standard IG modes under the same circumstance are disrupted and become unrecognizable. Our findings could promote the construction of quantum networks based on room-temperature atoms.

Keywords: elegant Ince-Gaussian mode; hot atomic ensemble; antidiffusion.
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2. Design of Experiment setup

2.1. Experimental setup

Our experiment is based on a Λ-type electromagnetically induced transparency (EIT) system, which consists of two ground states coupled to a common excited state of the $^{87}\text{Rb}$ atom, as is shown in Fig. 1(a). A 795 nm external-cavity diode laser is locked to the transition of $|5S_{1/2}, F = 1\rangle \rightarrow |5P_{1/2}, F' = 1\rangle$ as the probe beam (shown in Fig. 1(a) as $|1\rangle \rightarrow |3\rangle$). Simultaneously, another 795 nm laser used as the control beam resonates with the transition of $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{1/2}, F' = 1\rangle$ ($|2\rangle \rightarrow |3\rangle$).

In the experiment, the probe beam has an intensity of 10 μW and a waist radius of 2 mm. The control beam has a stronger intensity of 16 mW and a waist of 5 mm to fully cover the probe beam. Figure 1(c) depicts the experimental setup. The incoming probe beam is first modulated into an eIG or IG mode by a spatial light modulator (SLM). It then passes through a 4f imaging system with two lenses of 500 mm focal length so that the specific pattern can be imaged precisely at the center of the Rb vapor cell. The cell is kept at 60°C and placed in a magnetic shielded cavity to block the stray magnetic field. The control beam, which is orthogonally polarized to the probe beam, is combined with the probe beam using a polarizing beam splitter (PBS) in front of the cell and then isolated by another PBS after passing through the cell. The second 4f imaging system is placed after the second PBS, ensuring that the retrieved probe beam will be captured by an intensified charge-coupled device camera (ICCD, Andor iStar 334T). Because of the limited separation ability of the PBS, a heated Rb vapor cell is put in front of the ICCD as an atomic filter for further filtering. Atoms in this cell are pumped to an energy level of $|5S_{1/2}, F = 2\rangle$ by an extra laser that couples two states from $|5S_{1/2}, F = 1\rangle$ to $|5P_{1/2}, F' = 1\rangle$ to absorb the residual control beam. Furthermore, a phase-lock module (Vescent D2-135 Offset Phase Lock Servo) is utilized to lock the control beam to synchronize its phase with the probe beam, since the phase coherence of two beams is a prerequisite for obtaining a stable storage signal.

Precise control of the sequence is also required for collecting accurate experimental data. The time sequence of the experiment is shown in Fig. 1(b). First, the control beam is opened to pump the atoms to energy level $|1\rangle$. Then, we let a 2 μs pulse of the probe beam pass through the Rb vapor cell. As soon as the probe beam enters the cell, we turn off the control beam. Then the probe beam is stored as the atomic spin wave. In the experiment, the storage time ranges from 1 μs to 4 μs. After this time interval, the control beam is turned on to release the probe beam. At the same time, we open the ICCD for 2 μs, ensuring it only collects the retrieved beam.

2.2. Expression of standard and elegant IG modes

To derive the analytical expressions of the IG modes, we let $\Psi(r)$ be the slowly varying complex envelope of a paraxial field that satisfies the paraxial wave equation (PWE), $(\nabla^2 + 2ikz/\omega^2)\Psi(r) = 0$, where $\nabla^2$ is the transverse Laplacian, and $k$ is the wave number.

The elliptic coordinates are defined as $x = f(z) \cdot \cos \mu \cdot \cos \nu$, $y = f(z) \cdot \sin h \mu \cdot \sin \nu$, where $\mu \in [0, +\infty)$, $\nu \in [0, 2\pi)$, and $f(z) = f_0 \omega(z)/\omega_0$, in which $f_0$ is the semiconical separation with $\omega_0$ being the beam width at the waist plane $z_0$. Bandres and Gutiérrez-Vega have derived the solution of IG modes as

$$IG_{p,m}(r,e) = \begin{cases} \frac{C_{0m}}{\omega_0} C_{p}^{m}(\mu,e) C_{p}^{m}(\nu,e) \exp \left[ -\frac{r^2}{\omega_0^2} \right] \\ \times \exp \left[ ikz + i \frac{kr^2}{2R(z)} - i(p + 1) \arctan \left( \frac{z}{z_R} \right) \right], \end{cases}$$

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where $\omega_0$ is the beam waist at plane $z = 0$ and $e = 2f_0^2/\omega_0^2$ is the ellipticity parameter. The superscripts of IG refer to the even (e) and odd (o) modes. $C$ and $S$ are normalization constants, and $z_R = k\omega_0^2/2$ is the Rayleigh range. The even and odd Ince polynomials of order $p$ and degree $m$ are denoted by $C_{p}^{m}(\nu,e)$ and $S_{p}^{m}(\nu,e)$, respectively.

When it comes to the elegant IG modes, a complex parameter $c = c(z) = k/2iq(z)$ is introduced with $q(z) = z - i\varepsilon_R$. The form of the elliptic coordinates remains the same but $f(z)$ is rewritten as $f(z) = f_0/\omega_0 c(z)^{1/2}$. The general expression of the eIG beams is given by

$$IG_{p,m}(r,c) = \begin{cases} \frac{C_{0m}}{\omega_0} C_{p}^{m}(\mu,c) C_{p}^{m}(\nu,c) \exp \left[ -\frac{r^2}{\omega_0^2} \right] \\ \times \exp \left[ ikz + i \frac{kr^2}{2R(z)} - i(p + 1) \arctan \left( \frac{z}{z_R} \right) \right], \end{cases}$$

$$IG_{p,m}(r,c) = \begin{cases} \frac{S_{0m}}{\omega_0} S_{p}^{m}(\mu,c) S_{p}^{m}(\nu,c) \exp \left[ -\frac{r^2}{\omega_0^2} \right] \\ \times \exp \left[ ikz + i \frac{kr^2}{2R(z)} - i(p + 1) \arctan \left( \frac{z}{z_R} \right) \right], \end{cases}$$ (1)

where $\omega_0$ is the beam waist at plane $z = 0$ and $e = 2f_0^2/\omega_0^2$ is the ellipticity parameter. The superscripts of IG refer to the even (e) and odd (o) modes. $C$ and $S$ are normalization constants, and $z_R = k\omega_0^2/2$ is the Rayleigh range. The even and odd Ince polynomials of order $p$ and degree $m$ are denoted by $C_{p}^{m}(\nu,e)$ and $S_{p}^{m}(\nu,e)$, respectively.
elG_{p,m}(r,ε) = C_{p} \left( \frac{q_{0}}{q} \right)_{\frac{1}{2}+1} C_{p}^{m}(\mu,ε) \times C_{p}^{m}(\nu,ε) \exp(-cr^2),

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3. Experimental Result

To investigate the storage behavior of the elG mode undergoing coherent diffusion, we modulate the probe beam into even and odd elG_{10,2,2} modes, and the corresponding standard modes are also employed for comparison. The results are shown in Fig. 2. The first and third rows in Fig. 2 show the storage results of the even and odd elG_{10,2,2} modes. Correspondingly, the second and fourth rows give the intensity distributions of the retrieved standard IG_{10,2,2} modes. The theoretical intensity distributions before storage are presented in column (1), and the images captured by the ICCD are shown in columns (2) to (5), corresponding to the original patterns and the patterns after a storage time of 1 μs to 3 μs. In the experiment, the measured storage efficiency is 15% when the storage time is 1 μs. For the elG beams, the retrieved patterns keep their profiles intact and suffer little from atomic diffusion as the storage time increases, regardless of whether they are in the even or odd mode. However, the standard IG modes lose their internal spatial structures, and we can hardly recognize the original appearance as the storage time reaches 3 μs.

Additional experiments are carried out to explore whether the order and the ellipticity of the elG mode would affect the fidelity of the results. We first modulate the probe beam into odd and even elG_{20,4,2} modes, and the results are shown in Fig. 3. The top row presents the even modes, and the bottom row presents the odd modes. Then, we store the elG_{20,4,ε} beams with ε ranging from 0 to 200. When the ellipticity comes to 0, the elG mode converts to the eLG mode, and as ellipticity increases, it gradually converts to the eHG mode. The storage results are shown in Fig. 4. We find that the similarity and sharpness of the retrieved images do not change when the order and ellipticity vary.

4. Analysis and Discussion

From all these results, it is clearly seen that the elG mode is robust to atomic diffusion. For quantitative analysis, we use the Pearson correlation coefficient (PCC) as a statistical tool to judge the relevance between the retrieved images and the original images\(^{[28,29]}\). PCC is widely used in measuring the correlation between two images. Its expression is

\[ PCC = \frac{\sum_{i=1}^{g} \sum_{j=1}^{h} (A_{ij} - \bar{A})(B_{ij} - \bar{B})}{\sqrt{\sum_{i=1}^{g} \sum_{j=1}^{h} (A_{ij} - \bar{A})^2} \sqrt{\sum_{i=1}^{g} \sum_{j=1}^{h} (B_{ij} - \bar{B})^2}}, \quad (3) \]

where \(A_{ij}\) and \(B_{ij}\) correspond to the grayscale value of every pixel of the image before storage and after storage, respectively. \(\bar{A}\) and \(\bar{B}\) are the average grayscale value of all pixels in these two images, and \(g\) and \(h\) are the number of pixels in every row and column. Since the PCC itself is normalized, its value ranges from –1 to 1. PCC = 1 means the two images have a perfectly linear correlation, while PCC = –1 indicates an anti-correlation, and two unrelated images will lead to a PCC value close to 0.
We calculate the PCC values of every mode in Figs. 1 and 2, and the results are shown in Fig. 5. As the storage time increases, the PCC values of all modes decrease. However, whether in odd or even mode, the PCC values of the elegant beams, eIG10,2,2 and eIG20,4,2, remain above 0.9 after 3 μs of storage, which illustrates that they preserve their intensity distributions quite well. Regarding the standard IG modes, the PCC values drop below 0.3 after a storage time of 3 μs, indicating that the information carried by the original modes basically disappears.

The calculation results of the PCC value further illustrate that the eIG mode is invariant to coherent diffusion. In order to analyze this phenomenon, we adopt the theory proposed by Shuker et al.\textsuperscript{[18,19]}. Based on this theory, the retrieved beam \( E(r, t + \tau) \) has been stored as

\[
|E(r, t + \tau)|^2 \propto \int d^2r' E(r', t)e^{-|r-r'|^2}, \tag{4}
\]

where \( E(r', t) \) is the probe field and convolves with a Gaussian function \( \exp^{-|r-r'|^2} \). According to the convolution theorem, we can rewrite Eq. (4) as

\[
|E(r, t + \tau)|^2 \propto |\mathcal{F}^{-1}\{\mathcal{F}[E(r, t)] \cdot \mathcal{F}(e^{-|r|^2})\}|^2, \tag{5}
\]

where \( \mathcal{F} \) represents the Fourier transform, and \( \mathcal{F}^{-1} \) represents the inverse Fourier transform. \( \mathcal{F}(e^{-|r|^2}) \) remains as a Gaussian function, which acts as a low-pass filter, filtering out most of the high-frequency components of the probe beam in the frequency domain.

The distribution of probe field in the spatial and frequency domains is shown in Fig. 6. The first and third rows present eIG10,2,2 modes, while the second and fourth rows are the standard IG modes for comparison. The Gaussian function \( \mathcal{F}(e^{-|r|^2}) \) decreases in value from the center to the edge, resulting in better preservation of the low-frequency components of the probe beam which are at the center of the frequency domain. However, the high-frequency components at the periphery of the frequency domain are greatly reduced when multiplied by \( \mathcal{F}(e^{-|r|^2}) \). From the second column of Fig. 6, we can find that the standard IG mode has a wide range of angular spectra in the frequency domain. Once it is Gaussian modulated by \( \mathcal{F}(e^{-|r|^2}) \), its high-frequency components are filtered, resulting in the loss of light field information after storage. In comparison, the distribution of the elegant modes in the frequency domain is more concentrated and is symmetrically centered about the origin of the coordinates, which means that its angular spectrum only undergoes a uniform attenuation and maintains the overall structure unchanged. Therefore, the frequency domain distribution of the retrieved beam is well preserved. So the corresponding spatial patterns are well preserved as a result. Furthermore, by upgrading the Rb cell to a paraffin-coated cell filled with inert gas and by using a combination of probes and control beams with Zeeman coherence \( \Delta m = 1 \)\textsuperscript{[30]}, longer storage time of the eIG mode can potentially be achieved, potentially extending into the millisecond range.

5. Conclusion

To summarize, we have studied the storage performance of eIG modes in a warm Rb vapor cell. We record the retrieved patterns of eIG10,2,2 and eIG20,4,2 modes after storage times of 1, 2, and 3 microseconds; quantify their fidelity using PCC values; and compare their storage performance to the IG10,2,2 modes. The experiment results show that after storing for 3 μs, PCC values of all elegant modes are higher than 0.9, while the values of the standard modes are less than 0.3. Moreover, eIG beams with
different ellipticity are stored for 3 μs, which is found to have little effect on the storage results. Our demonstrations indicate that the eIG modes could be faithfully stored in room temperature atoms, which is helpful for building a high-dimensional and multiplexed quantum repeater. By combining it with a fiber network[31], high-fidelity and long-distance quantum communication is expected to be achievable in the future.

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References