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Statistical study on rogue waves in Gaussian light field in saturated nonlinear media

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The spatial rogue waves (RWs) generated by a wide Gaussian beam in a saturated nonlinear system are experimentally observed. Our observations show that RWs are most likely to occur when Gaussian light evolves to the critical state of filament splitting, and then the probability of RWs decreases with voltage fluctuations. The occurrence probability of RWs after splitting is related to the nonlinear breathing phenomenon of optical filament, and the statistics of RWs satisfy the long-tailed L-shaped distribution. The experiment proves that the presence of high-frequency components and the aggregation of low-frequency components can serve as a prerequisite for the occurrence of extreme events (EEs).

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1. Introduction

Optical rogue waves (RWs) were first discovered, to the best of our knowledge, in nonlinear fiber supercontinuum experiments^[1] and have become a research hotspot in the field of optics in the past decade. RWs have been found in different optical systems, such as nonlinear optical cavities^[2], photonic crystal fibers^[3], Bragg gratings^[4], erbium-doped fibers^[5], and optical waveguides^[6]. For the nonlinear modulation instability (MI) mechanism, the occurrence probability of RWs is different from the traditional normal distribution, and its probability presents an "L-shaped" heavy-tailed distribution. For the nonlinear Schrodinger equation (NLSE) solution^[7], it is a local wave solution in a finite plane wave background, including Akhmediev breathers (ABs)^[8], Kuznetsov-Ma solitons^[9], and Peregrine solitons (PSs)^[10], in which PS is considered to be the most basic model to describe RWs in nonlinear optics, with dual local characteristics of spatial distribution and temporal evolution. On the other hand, spatial RWs are rarely studied compared to temporal RWs. Experimental study on spatial RWs is still in its infancy. A highly nonlinear spatiotemporal turbulent region was observed on BaTiO₃:Co crystals^[11]. Reference [12] reported RWs on potassium lithium tantalate niobate (KLTN) crystals, where nanodisorders, giant nonlinearity (NL), and high temperature produce large intensity events (IEs). Ferroelectric-paraelectric transition also produces RWs due to thermally induced focusing and defocusing effects^[13], where the transition from a linear region to a highly nonlinear region promotes turbulent dynamics^[14]. The MI and patterning of strontium barium niobate (SBN) photorefractive crystals can

enable incoherent^[15] or coherent^[16] beams to produce different optical patterns, including fringes and filaments. This is due to the combined effect of different mechanisms such as crosstalk and NL, which are considered necessary to observe RWs. Recent studies have shown that the existence of a purely linear large refractive index is also possible due to the isolated caustics effect on the optical sea^[17]. When the incident beam is a standard Gaussian beam (GB), the incident beam will form a soliton or a respirator during the propagation process^[18]. When there are perturbations in GB, the splitting phenomenon occurs due to MI during the propagation process^[13]. At present, there have been studies on the statistics of RWs in split-filament phenomenon which used tight-focused GB^[19]. Wide GB and plane beam (PB) are only different in spatial distribution and using wide GB is more conducive to controlling variables.

2. Experiment

Therefore, we observe the phenomenon of wide GB splitting through experiments and numerical simulations to study the relationship between Gaussian MI and RWs and analyze the probability and spatial distribution of RWs. As shown in Fig. 1(a), we use an SBN crystal with a cross-sectional area of $5 \text{ mm} \times 5 \text{ mm}$ and a length of 10 mm (propagation coordinate *z*). The NL is indirectly controlled by the voltage applied in the direction of the crystal's optical axis. The incident beam is a 532 nm laser with a polarization direction along the voltage direction, and the beam width is about 300 µm. Keep the input optical power at 2.8 µW. The magnitude of the NL is only

controlled by the voltage (U) applied by the high-voltage, direct current (HVDC) source. To observe the light intensity distribution on the input and output sides of the crystal, we use a CCD camera.

We change the NL to study the change in the probability of RWs in the phenomenon of filament separation. Without applying any voltage, as shown in Fig. 1(b), the outline of the GB propagating at 10 mm in the crystal is the same as the input profile. It can be seen that the input light propagates parallel to the z axis inside the crystal, and the input light is a non-uniform GB with lots of perturbations. The perturbation in GB is provided by intensity mask A2. The perturbation intensity is approximately 20% of the light intensity. These perturbations are very important to our experiments, and they lead directly to filamentous processes under nonlinear conditions.

Our experiments were performed by adjusting the voltage to control the NL of the crystal at a larger incident size. As shown in Figs. 1(b)-1(e), this allows light to be concentrated in very specific regions due to the initial non-homogeneous intensity distribution and MI, resulting in high-intensity spots. Since wire splitting is not obvious under low voltage, we set the following rules for the applied voltage: the voltage is increased from 350 V to 1300 V, each time increasing by 50 V, and the image of the exit surface is collected after the crystal modulation is stabilized.

By the standard of space strange waves^[5], we consider those waves whose intensity is greater than twice the effective intensity (I_e) as extreme events (EEs), and define I_e as the average of the first third of the waves with the highest intensity. In order to determine the probability of RW, a wave whose intensity reaches the anomaly index AI = $I/I_e > 2$ is considered to be an odd wave. We carried out 12 experiments under the voltage of 300–1300 V/cm, and the statistical results of the probability of strange waves are shown in Fig. 2. It can be seen that the strange wave appears after the beam is split, and the probability of the appearance of the strange wave is not positively correlated with the voltage. As the voltage increases, the probability of strange waves fluctuates, reaching a peak at the first probability increase, and subsequent statistical results are lower than this



Fig. 1. (a) Experimental setup for observing RWs. Inset: input profile. (b)-(d) Output beam profiles for indicated voltages.



Fig. 2. Statistical chart of RWs probability with indicated voltage in experiments.

peak. The statistical properties of the wide Gaussian ghost wave are significantly different from the conclusions in Ref. [19], that is, the occurrence probability of the ghost wave for tightly focused Gaussian light is positively related to the applied voltage.

3. Simulation

To verify the experimental phenomenon and find out the main reason for the generation of RWs under GB, we carried out a numerical simulation on the RWs appearing on SBN crystals and GB and compared it with the results on PB. The propagation of incident light in SBN follows the NLSE^[20]:

$$i\frac{\partial}{\partial z}\psi(x,y,z) + \frac{1}{2k_0}\nabla_{\perp}^2\psi(x,y,z) + \frac{k_0}{n_e}\Delta n\psi(x,y,z) = 0.$$
(1)

In the above formula, ∇_{\perp}^2 is the Laplacian operator, *z* is the propagation distance, $\psi(x,y,z)$ is the slowly varying light wave envelope, (x,y) are the coordinates, the wave vector size is $k_0 = 2\pi/\lambda$, λ is the wavelength, and n_e is the refractive index of the extraordinary polarized beam in the SBN crystal. The refractive index change due to saturated NL in SBN crystal Δn is

$$\Delta n = \frac{1}{2} n_e^3 \gamma_{33} E \frac{1}{1 + |\psi(x, y, z)|^2}.$$
 (2)

In the above formula, γ_{33} is the electro-optic coefficient, and *E* is the applied voltage. In the experiment, the NL is proportional to the applied voltage.

We add a noise seed with a certain width and intensity to the formula, the size of which is related to the GB intensity at the corresponding position; the average value is zero, and the maximum value of the noise seed is 21% of the original GB amplitude at the corresponding position. The beam transmission in the SBN is obtained by the stepped beam propagation method^[21].

We set the input width to be 300 µm, the propagation distance to be 10 mm, the lateral width of the crystal to be 5 mm × 5 mm, the refractive index (n_e) to be 2.33, and the electro-optic coefficient to be 280×10^{-12} cm/V. We use the normalized variance of scintillation index-filament intensity [S(U)] to characterize the intensity gap between filaments at different voltages on the exit surface of the SBN crystal, $S(U) = \langle I^2(U) \rangle / \langle I(U) \rangle^2 - 1$; the intensity dispersion between filaments is positively correlated



Fig. 3. (a) Scintillation index of the exit surface at different voltages.(b) Statistical probability plot of Gaussian RWs versus voltage in numerical simulation. Inset: probability distribution of Al under 700 V.

with S(U), and the higher S(U) represents the higher probability of RWs appearing. As shown in Fig. 3(a), when the applied voltage is ~400 V, the light on the exit surface is divided into filaments.

The voltage was increased from 300 to 1300 V in 50 V increments. In the case of random perturbation, each voltage was repeated 400 times. As shown in Fig. 3, it can be clearly seen that the variation trend of the RWs occurrence probability is consistent with the experimental results and is more detailed. The occurrence probability of RWs has a peak point, and the peak appears at the first highest point of the RWs probability fluctuation. In both experiments and simulations, this maximum value appears around 450 V. We have counted the intensity distribution of light filament. From the inset of Fig. 3(b), it can be clearly seen that there is a 10^{-5} probability of appearing waves with an intensity greater than seven times that of I_e , which satisfies the characteristic that RWs have a long-tailed distribution.

It can be seen from Figs. 2 and 3(b) that the numerical simulation shows a distribution trend close to the experiment. In order to test whether this phenomenon is unique to GB itself, we numerically simulate the probability of RWs on a Gaussian amputation PB under the same conditions. As shown in Fig. 4, PB also has the statistical characteristics where the occurrence probability of RWs fluctuates with voltage, and the highest probability is located at the second peak after the occurrence of RWs. It can also be seen from Figs. 3(b) and 4 that the split light spots satisfy the long-tailed distribution statistically and meet the criterion of RWs.

This phenomenon is inconsistent with the traditional concept of RWs formation. To explain this phenomenon, we carried out numerical statistics on the maximum light intensity and wave number at different voltages. The statistical results are shown



Fig. 4. Statistical probability plot of plane RWs with voltage in numerical simulation. Inset: probability distribution of ratio of emitted light intensity to I_e under 600 V.

in the Fig. 5; when the propagation distance, light intensity, beam width, and perturbation range are fixed, RWs are more likely to appear at a specific voltage. At a suitable voltage, Gaussian light with corresponding width, intensity, and perturbation level is more likely to evolve to a high-intensity filament. Since the wave number decreases only slightly with increasing voltage, the number of high-intensity filaments directly determines the probability of RW occurrence. Voltage, or the magnitude of the NL, is also a parameter that has a complex effect on the RW phenomenon, rather than the simple positive correlation commonly thought.

Although GB and PB have a similar trend in statistics, it can be seen that the probability of RWs appearing in the GB is much higher than that in the PB. To explain this result, we performed a numerical simulation of the propagation process under GB and PB. As shown in Fig. 6, compared with PB, the overall light intensity distribution trend of GB after splitting is basically the same as that at incidence, and the intensity at the middle position is significantly higher than that at the margin. The intensity difference caused by the initial distribution will cause a larger gap between the large peak wave and I_e , making it easier to satisfy the RWs condition.

4. Supplementary Experiment

We use spatial frequency as the basis for judging the phenomenon of filament separation. As shown in Fig. 1(a), we added two beam splitters (BSs) to the original imaging system to image part



Fig. 5. Blue line shows the statistics of the maximum light intensity varying with voltage under GB. Red line shows the statistical graph of the wave number whose intensity is greater than I_e varies with voltage.



Fig. 6. (a)–(c) Light intensity distribution during GB propagation at different voltages. (d)–(f) Light intensity distribution during PB propagation at different voltages.



Fig. 7. (a)-(c) Output beam profiles for indicated voltages. (d)-(f) Spatial spectra of output beams.

of the light on the front focal plane of the Fourier lens MO3 [see Fig. 1(a)], so that the CCD is precisely on the Fourier plane. In this way, we get the spatial frequency of the back light of SBN. As shown in Figs. 7(a)-7(f), the discrete high-frequency components caused by RWs and low-frequency components caused by the PB background appear at the same time along with the filaments.

5. Conclusion

In summary, the formation mechanism of broad-GB excited RWs under saturation NL is much more complicated than generally thought. An increase in NL will cause the increase in average light intensity, so even with increasing dispersion, the odds of RWs still decrease. It is not simply reduced, but determined by the breathing behavior after the filaments separation. Statistically, the probability of occurrence of RWs excited by wide GB is much higher than that excited by PB. This is caused by the intensity gradient of GB itself. The weak light part of GB reduces the average light intensity after filament splitting. It can also be seen from Fig. 6 that the occurrence probability of waves with different propagation lengths and large amplitudes is not just positively correlated. We are designing an experiment that we hope to confirm it experimentally.

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