

Symmetry detection of rotating patterns based on rotational Doppler effect of light

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We propose a method for detecting the symmetry of rotating patterns based on the rotational Doppler effect (RDE) of light. The basic mechanisms of the RDE are introduced, and the spiral harmonic distribution of rotating patterns is analyzed. By irradiating the rotating pattern using a superimposed optical vortex and analyzing the amplitude of the RDE signal, the spiral harmonic distribution of the pattern can be measured, and then its symmetry can be detected. We demonstrate this method experimentally by using patterns with different symmetries and shapes. As the method does not need to receive the scattered light completely and accurately, it promises potential application in detecting symmetrical rotating objects at a long distance.

Keywords: rotational Doppler effect; optical vortex; spiral harmonic distribution; symmetry detection; detection of rotating objects.

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1. Introduction

An optical vortex is a structural beam with helical phase, which can possess both spin angular momentum (SAM) and orbital angular momentum (OAM)^[1,2], and its total angular momentum (TAM) can be precisely measured and controlled^[3]; it has been widely used in optical manipulation, quantum communication, microfabrication, and correlation imaging^[4–8]. In particular, the optical vortex can obtain more information than traditional plane waves when interacting with moving, especially rotating objects^[9] due to the unique phase structure. In recent years, it has been found that an optical vortex can produce the rotational Doppler effect (RDE) and detect the rotating objects, which makes up for the deficiency of traditional detection methods and becomes the current research hot spot.

The detection of rotating targets by using RDE has been widely studied since OAM was first confirmed, to the best of our knowledge, in a Laguerre–Gaussian (LG) beam in 1992^[2]. In 2013, a scheme of measuring angular velocity using RDE of an optical vortex was systematically reported by Lavery *et al.*^[10]. After that, Carmelo *et al.* proposed a method to measure both rotational speed and direction by using RDE in the following year^[11]. Later, studies that face the practical detection situation were carried out. Fu *et al.* confirmed that the method of angular

velocity measurement by RDE is still effective when there is an obstacle in the propagation path^[12]. Zhang *et al.* overcame the influence of atmospheric turbulence in the propagation process of an optical vortex to achieve angular velocity measurement at a distance of 120 m on the photon counting level^[13]. Qiu *et al.* discussed the RDE when the scatterer has both linear and rotational motion at the same time and realized the rotational speed measurement for a cylinder^[14]. Ding *et al.* studied the RDE under the condition that the optical axis of incident light does not coincide with the target's rotating axis^[15–18]. Zhai *et al.* investigated the measurement of the unsteady angular velocity and angular acceleration, respectively^[19,20]. All the above studies aimed at the motion information detection of the rotating target without concerning the geometric characteristics of the object.

However, the optical vortex and the RDE are closely related to the geometric characteristics of the rotating object. In 2005, Torner *et al.* proposed digital spiral imaging (DSI) and showed how the OAM spectrum of a light beam can be used to image a variety of encoded intrinsic and extrinsic properties^[21]. In 2020, Wei *et al.* performed imaging and image processing of a rotating object by analyzing the LG spectrum^[22]. As the RDE is closely related to optical vortices, it is possible to detect geometric characteristics of rotating objects by analyzing the RDE signals. In 2016, Zhou *et al.* analyzed the modulation effect of rotating

objects on incident light by means of mode expansion, which explained the phenomenon of topological charge change of scattered light and the generation of RDE^[23]. Furthermore, Qi *et al.* found in the experiment that the amplitude of the RDE signal reaches its maximum when the topological charge of the optical vortex matches the geometric symmetry of the object^[24]. The above researches proved the existence of a certain relationship between the RDE and the geometric characteristics of the rotating object, especially symmetry. Therefore, we can detect the symmetry of rotating objects by analyzing the RDE signal.

In this work, a new method of detecting symmetry of rotating patterns based on the RDE is proposed. Several patterns with different geometric characteristics are used to demonstrate this method. Firstly, the modulation effect of a rotating pattern on the incident optical vortex is introduced, its spiral harmonic distribution is defined, and the method of obtaining the spiral harmonic distribution based on RDE is presented later. Experiments are then carried out to obtain their spiral harmonic distribution, and the relationship between symmetry and spiral harmonic distribution is analyzed. This method detects the symmetry of rotating objects by recording RDE signals without the need to receive or regulate the scattered light completely. Therefore, it has potential applications in detection and recognition of symmetric rotating objects such as helicopter rotors, especially when the object rotates at high speed and over long distances. Since this method is sensitive to symmetry, it can also be used to detect the working state of rotating parts such as turbine blades without stopping.

2. Theory

Based on the mechanism of decomposition of the spiral phase, the amplitude modulation function of the pattern, which is determined by its geometric characteristics, can be expressed as $a(r, \varphi)$ and expanded in terms of the spiral harmonic factor $\exp(il\varphi)$ ^[25] in the cylindrical coordinate system as

$$a(r, \varphi) = \sum_n B_n(r) \exp(in\varphi), \quad (1)$$

where n represents the n th-order spiral harmonic of the pattern, B_n denotes the complex expansion coefficient of the n th-order harmonic whose intensity $|B_n|$ satisfies $\sum |B_n|^2 = 1$.

When the object rotates at angular velocity Ω , the modulation function under the complex amplitude representation can be written as

$$a(r, \varphi - \Omega t) = \sum_n B_n(r) \exp(in\varphi) \exp(-in\Omega t). \quad (2)$$

Consider an optical vortex with frequency f and topological charge l irradiating on the rotating pattern. The electrical field distribution of incident optical vortex can be expressed as $E(r) \exp(-i2\pi f t) \exp(il\varphi)$, and then the scattered light modulated by the rotating pattern can be expressed as

$$E_{\text{sca}} = \sum_n B_n(r) E(r) \exp[i(l+n)\varphi] \exp[-i(2\pi f + n\Omega)t]. \quad (3)$$

It can be seen that the topological charge of the incident optical vortex is changed by the amplitude modulation of the rotating pattern. Moreover, the mode components of scattered light only depend on the spiral harmonic distribution of the rotating pattern and each component of scattered light with topological charge $l+n$ acquires an RDE frequency shift $\Delta f = n\Omega/2\pi$. Since the changes of mode components and the corresponding RDE frequency shift are only related to the spiral harmonic distribution of the pattern, which is determined by its geometric characteristics, the geometric characteristics of the rotating pattern, including symmetry, can be detected by analyzing the RDE signals in the spectrum of the scattered light.

When a single optical vortex irradiates a rotating object, interference between the incident and the scattered light is necessary to obtain the RDE signal. In practice, it is difficult for the scattered light to interfere with incident light because the scattered light has complex mode components and low intensity. Using a superimposed optical vortex can effectively solve this problem. At this time, any mode of scattered light is the coherent superposition by multiple modes of incident light modulated by different-order spiral harmonics of the rotating pattern. We consider a superimposed optical vortex with two different topological charge number components, which can be expressed as

$$E_{\text{inc},s} = E_1(r) \exp(-i2\pi f t) \exp(il_1\varphi) + E_2(r) \exp(-i2\pi f t) \exp(il_2\varphi). \quad (4)$$

When it coaxially irradiates a rotating object, the scattered light can be expressed as

$$E_{\text{sca},s} = \sum_n \{B_n(r) E_1(r) \exp[-i(2\pi f + n\Omega)t] \exp[i(l_1 + n)\varphi] + B_n(r) E_2(r) \exp[-i(2\pi f + n\Omega)t] \exp[i(l_2 + n)\varphi]\}. \quad (5)$$

Considering the fundamental mode (whose topological charge number is zero) component in the scattered light, the $-l_1$ and $-l_2$ -order spiral harmonics of the rotating pattern modulate the components with topological charge l_1 and l_2 in the incident light into the fundamental mode, respectively. Thus, the fundamental mode component of the scattered light can be expressed as

$$E_{\text{sca},f} = B_{-l_1}(r) E_1(r) \exp[-i(2\pi f - l_1\Omega)t] + B_{-l_2}(r) E_2(r) \exp[-i(2\pi f - l_2\Omega)t]. \quad (6)$$

Photodetectors (PDs) are used to receive time-domain signals of scattered light intensity. Due to the small facula width of the superimposed optical vortex, the radial difference of the rotating pattern in the area covered by the circular facula can be approximately ignored. In other words, we treat B_n as a constant rather than changing with r . Thereupon, an RDE beat signal can be obtained as

$$I_{\text{sac},f}(t) = |B_{-l_1}|^2 I_1 + |B_{-l_2}|^2 I_2 + 2|B_{-l_1} B_{-l_2}| \sqrt{I_1 I_2} \cos[(l_1 - l_2)\Omega t + \theta], \quad (7)$$

where $\theta = \text{angle}[E_1 B_{-l_1} E_2^* B_{-l_2}^*]$. It can be seen that the amplitude of the RDE beat signal with frequency value $(l_1 - l_2)\Omega/2\pi$ is proportional to the intensity of the object's $-l_1$ and $-l_2$ -order spiral harmonic.

According to the results of Eq. (7), we can irradiate the rotating pattern with a series of superimposed optical vortices to obtain its spiral harmonic distribution. In the superimposed incident light, one of the components has fixed topological charge $l_1 = k_0$, which is called reference mode, and its light intensity is I_1 . Another component is called scanning mode, whose topological charges $l_2 = k$ ($k \in \mathbf{Z}$) are evaluated one by one within a certain range. After each irradiation on the rotating pattern, the amplitude C_k of the beat signal with frequency value $|k_0 - k|\Omega/2\pi$ in the spectrum of scattered light is recorded. When the light intensity of the scanning mode I_2 remains constant, the normalized intensity of spiral harmonics corresponding to the rotation pattern can be obtained according to the normalized amplitude of beat signal as

$$\frac{|B_{-k}|}{\sum |B_{-k}|} = \frac{2\sqrt{I_1 I_2} |B_{-k_0} B_{-k}|}{\sum 2\sqrt{I_1 I_2} |B_{-k_0} B_{-k}|} = \frac{C_k}{\sum C_k}. \quad (8)$$

Since the superimposed optical vortex only irradiates a part of the pattern, and the radial difference was ignored, the spiral harmonic distribution obtained at this time could not reflect all the geometric characteristics of the rotating pattern or reproduce its appearance. However, azimuthal information such as symmetry could still be interpreted from the measured spiral harmonic distribution.

3. Experiment

The experimental setup is shown in Fig. 1. A fundamental mode Gaussian beam with a wavelength of 632.8 nm is emitted by the He-Ne laser, which is incident on the spatial light modulator (SLM) after beam expansion and collimation with lenses L_1

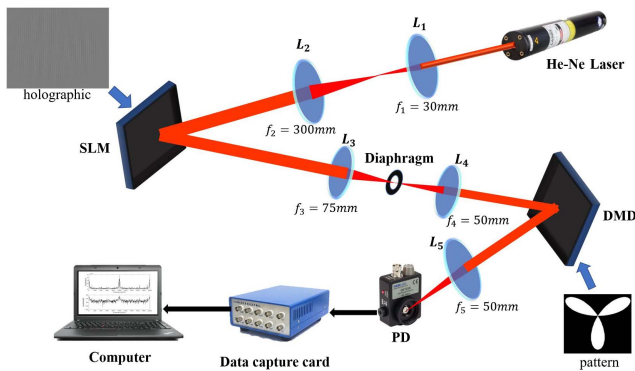


Fig. 1. Schematic of the experimental setup.

and L_2 . By loading the designed hologram on the SLM, the incident light can be modulated with complex amplitude, and the required superimposed vortex light can be obtained by filtering the first-order diffraction light through the $4f$ system composed of lenses L_3, L_4 , and the diaphragm. A digital micromirror device (DMD) is used to load the rotating patterns to be tested. The scattered light is collected by lens L_5 and received by a PD connected to the data capture card and a computer. The fast Fourier transform of the collected temporal signal is performed to obtain the spectrum of scattered light.

The reference mode of the incident superimposed optical vortex is $LG_{0,0}$, while scanning modes range from $LG_{0,1}$ to $LG_{0,15}$ successively, and the light intensity of the reference mode and the scan mode is always constant. After each irradiation to the rotating pattern, we find the RDE signal in the spectrum of scattered light according to Eq. (7) and record its amplitude. Normalizing the amplitude of the all 15 RDE signals according to Eq. (8), we finally obtain the spiral harmonic distribution spanning from order -1 to -15 .

For patterns, clover, four-leaf clover, six-leaf clover, equilateral triangle, and square are chosen to study the detection results when patterns have different symmetry and how to distinguish patterns with the same symmetry but different shapes. Triangles at different angles are set up to research the changes in detection results caused by changes in geometric characteristics. In addition, binary clover patterns with different sizes, clover pattern with random noise, and clover pattern with soft edges are set up to explore the influence on detection results caused by other factors, including the noise, the different sizes of patterns, and the blurred edges.

Figure 2 shows the received RDE signal under different conditions. It can be seen that an obvious rotational Doppler signal

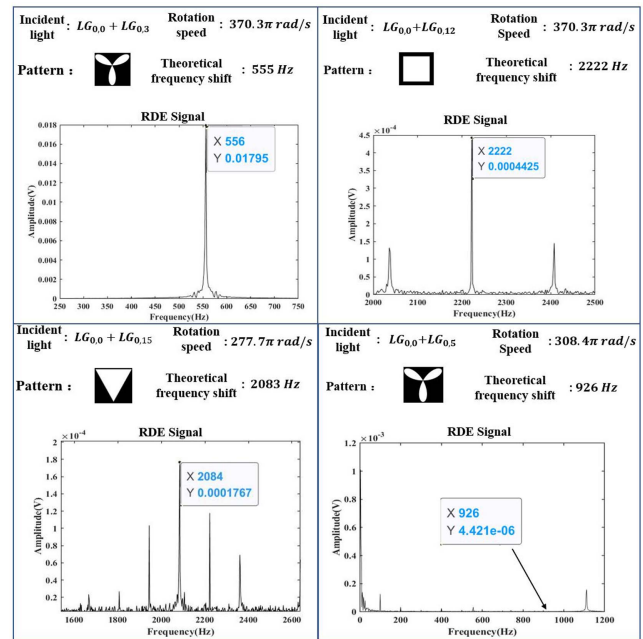


Fig. 2. RDE signal of scattered light under different conditions.

can be observed in the spectrum of scattered light when the superimposed optical vortex irradiates on the rotating pattern in most cases. Only when the rotating pattern has n -fold symmetry, and the topological charge of the incident light scanning mode is not an integer multiple of n , the RDE signal cannot be observed in the spectrum, which indicates that the intensity of spiral harmonics at corresponding order is weak. In this case, we still record the amplitude of the signal at the theoretical frequency value as part of the detection result. The frequency shift signal of the RDE appears to be widened, especially when its amplitude is low. This is caused by the fact that the optical axis of the incident light does not coincide strictly with the rotating axis of the pattern^[9]. The misalignment has little influence on the results of the experiment and can be accurately measured and corrected^[26].

Experimental results of the spiral harmonic distribution of typical patterns such as clover are shown in Fig. 3. When the pattern has rotational symmetries, the spiral harmonic distribution of the pattern has missing-order phenomenon, i.e., the values of the spiral harmonic distribution orders are integer multiples of the symmetry times, and the intensity of spiral harmonics gradually decreases with the increase of order. The orders and intensity of spiral harmonics can reflect the symmetry of the pattern; therefore, the symmetry of a rotating pattern can be directly inferred once the spiral harmonic distribution is obtained.

In order to further study the spiral harmonic distribution of rotating patterns with the same symmetry but different shape and find out how to distinguish them, we carried out an experiment on clover, four-leaf clover, equilateral triangle, and square, and their spiral harmonic distribution is shown in Fig. 4. The same three-fold rotational symmetry results in the same order of spiral harmonics distribution between the clover and equilateral triangle, and so do four-leaf clover and square. The spiral harmonics of the equilateral triangle and square are more concentrated in orders -3 and -4 , and the other harmonics are very low in comparison, while the spiral harmonics of orders -6 and -8 of clover and four-leaf clover pattern are still relatively high. It can be seen that the intensity of spiral harmonics in different levels can tell the difference of shape characteristics between the patterns with the same symmetry.

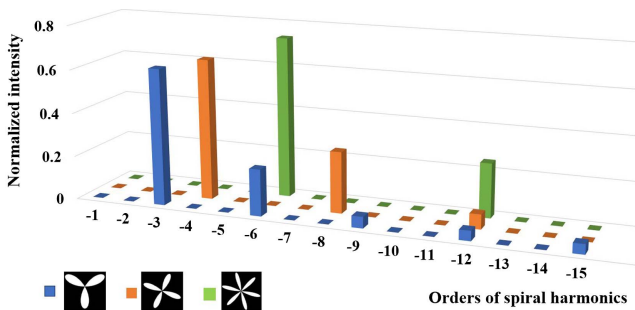


Fig. 3. Experimental spiral harmonic distribution of clover, four-leaf clover, and six-leaf clover.

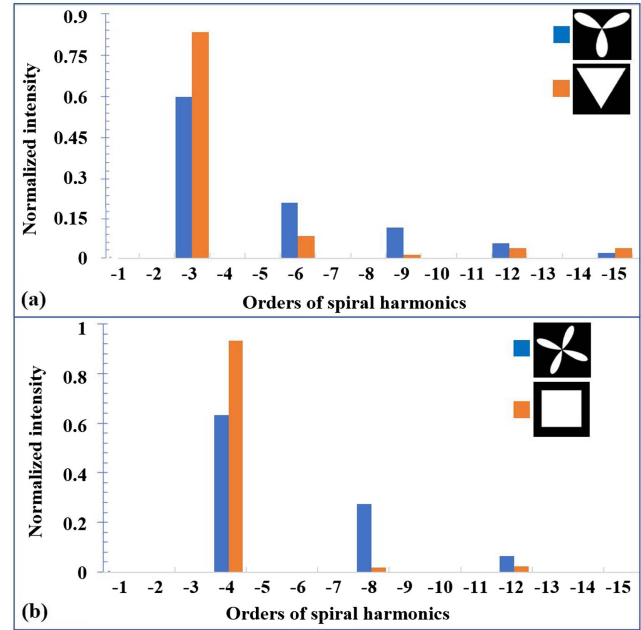


Fig. 4. Experimental results of patterns with same symmetry but different shape: (a) clover and equilateral triangle; (b) four-leaf clover and square.

Small changes in the geometric features of the pattern may not be perceived by the naked eye or the imaging system. However, if the pattern has symmetry, the symmetry detection method based on the RDE can keenly capture the changes of geometric features of the pattern. In the experiment, we measured the spiral harmonic distribution of triangles whose shapes changed from 1 deg to 5 deg compared to equilateral triangles. Figure 5 shows the experimental results of triangles with different angles. Spiral harmonics of the equilateral triangle, which has triple symmetry, are mostly concentrated in order 3. When the shape changes by only 1 deg, we can notice that the intensity of spiral harmonic at order -3 is reduced by about 12% compared with equilateral triangles, although the pattern is still very close to triple symmetry, and the spiral harmonics distribution conforms

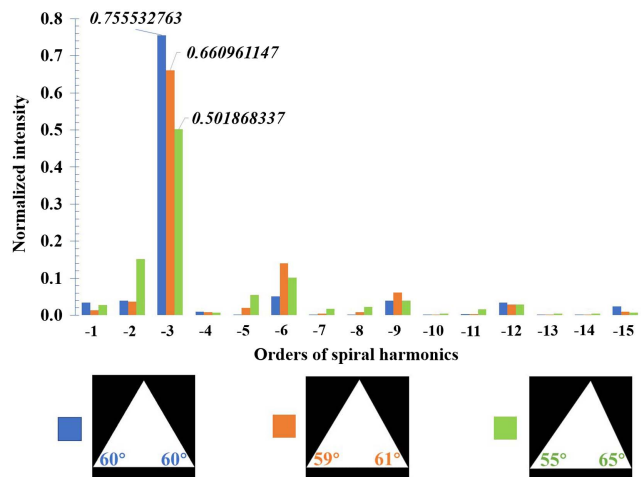


Fig. 5. Experimental results of triangles with different angles.

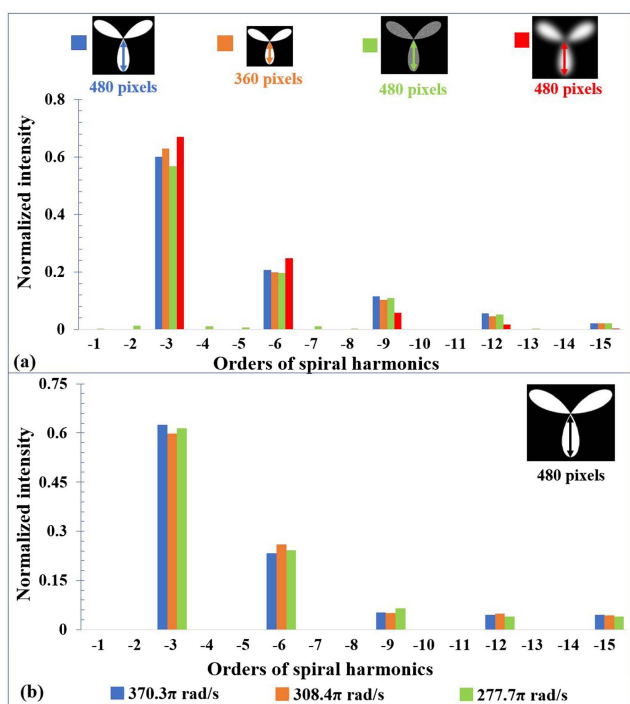


Fig. 6. (a) Experimental results of binary clover with different sizes and clover with grayscale; (b) experimental results of clover at different rotating speeds.

to the result of a pattern with triple symmetry. When its shape changes by 5 deg, the spiral harmonic distribution has a very obvious change compared with that of the equilateral triangle, indicating that its triple symmetry has been seriously damaged.

An effective detection method should be insensitive to influencing factors. In the experiment, we measured the spiral harmonic distribution of the same pattern with different rotational speeds, different sizes, and whether there is noise to verify the effectiveness of the method. Figure 6 shows the experimental results of clover with different sizes, random noise, soft edges, and different rotational speeds. The results show that the rotational speed, size, and noise of the pattern have no significant influence on the detection results, indicating that spiral harmonics distribution is only related to the geometric characteristics of the pattern itself. Therefore, it is feasible and effective to detect the symmetry of rotating patterns based on spiral harmonics distribution and the RDE.

4. Conclusion and Discussion

To summarize, this paper presents a new method to detect the symmetry of rotating patterns based on RDE. Firstly, the theoretical model is established, and the feasibility of the method is verified from the perspective of theoretical analysis. Then, several different patterns are set up, and the detection methods are verified by experiment. It is clear that the spiral harmonic distribution is closely related with the geometric characteristics of the rotating pattern, especially symmetry, and is rarely affected

by other factors. As different patterns possess different spiral harmonic distribution, once the spiral harmonic distribution is obtained, the symmetry of the pattern can be inferred accordingly. Since the spiral harmonics of the rotating patterns are related with the RDE signals directly, the symmetry of the rotating pattern can be detected by analyzing the RDE signals.

In the experiment, the superimposed incident light consists of a fundamental mode Gaussian beam and an LG beam. When the topological charge of the LG beam increases, its waist radius increases accordingly. Therefore, when the topological charge is high, the interference between two components is inefficient, resulting in a weak RDE signal and affecting the measurement accuracy of high-order spiral harmonics. However, the high-order spiral harmonics of the pattern have low intensity, so the influence on the detection result is not serious. In order to make the detection method more accurate, the waist radius of the Gaussian beam can be adjusted to fit that of the LG beam.

Compared with the existing scheme, the method proposed in this paper is unique in that the detection of symmetry of the rotating pattern is based on the RDE, without the need to receive and regulate the scattered light completely and accurately, which determines that this method is particularly suitable for detection at a long distance such as remote sensing. Although the current result has a certain gap to quantitatively measure all the complex expansion coefficients of spiral harmonics and reconstruct an image of the measured rotating pattern, we can qualitatively detect its geometric characteristics such as symmetry. In practice, the amplitude modulation effect of any real object on the incident light field can be replaced by an equivalent amplitude pattern, so this method is expected to be applied to the detection of real objects. The proposed method can detect the symmetry of unknown rotating objects to distinguish or identify different rotating objects. It can also be used to monitor whether rotating parts with symmetry are in a normal working state without stopping.

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