Measuring the topological charge of optical vortices with a single plate

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Measuring the topological charge (TC) of optical vortex beams by the edge-diffraction pattern of a single plate is proposed and demonstrated. The diffraction fringes can keep well discernible in a wide three-dimensional range in this method. The redundant fringes of the diffracted fork-shaped pattern in the near-field can determine the TC value, and the orientation of the fork tells the handedness of the vortex. The plate can be opaque or translucent, and the requirement of the translucent plate for TC measurement is analyzed. Measurement of TCs up to ±40 is experimentally demonstrated by subtracting the upper and lower fringe numbers with respect to the center of the light. The plate is easy to get, and this feasible measurement can bring great convenience and efficiency for researchers.

Keywords: optical vortex; orbital angular momentum; topological charge measurement.
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1. Introduction

In 1992, Allen et al. firstly, to the best of our knowledge, demonstrated a helical phase structure of light with wavefront singularities carrying orbital angular momentum (OAM)1. Such optical vortex (OV) beams characterized by the phase factor exp(i*l*θ), where l donates the OAM state or the topological charge (TC), and θ is the azimuthal angle, have been widely applied in various areas, including optical tweezers2, optical trapping3, optical communication4, and quantum information technology5. Most of these applications require a specific TC. As a consequence, the determination of TCs is of crucial importance.

Many methods are proposed to measure the TC of vortex beams, which can be basically divided into three techniques: interferometry, intensity analysis of OV beams, and diffractometry. Nevertheless, the first technique demands cumbersome interferometric setups and finely aligned optical elements6–11. The intensity analysis of the OV beam with complex algorithms12 is not intuitive enough to determine the TC. The present diffractometric methods usually transform OVs into identifiable patterns by specialized components such as apertures13–15, lenses16,17, and special gratings based on the mode conversion from Laguerre–Gaussian (LG) beams to near Hermite–Gaussian with phase-loaded spatial light modulators (SLMs)18–21.

Since the edge diffraction of OV beams was firstly investigated and demonstrated in 199822, to the best of our knowledge, increasing works have revealed its propagation properties due to the helical phase of the OV beam23–28. Masajda et al. showed that the OV beam is capable of self-reconstruction after edge truncation, no matter whether the OV core is cut off or not. Then, the ‘survived’ vortex core from the edge diffraction can shift to the propagation axis24,25. This phenomenon results from the propagation of the azimuthal component of the Poynting vector of the OV beam26, which is consistent with the transverse energy circulation (TEC) theory27 or the Gouy phase variation in LG modes28.

Furthermore, special attention should also be paid to the edge (or angular28,29, half-plane24,30) diffraction patterns, which can be applied for OV diagnostics and detection. However, the OV detection with angular diffraction29 is in the far-field diffraction, where the diffraction patterns are not as discernible as the simulated ones when |l| becomes higher, and the Fourier lens could result in additional aberration. In contrast, fork-shaped fringes in the near-field edge diffraction have a higher
tolerance for the lateral position of the plate and the longitudinal position of the observation plane.

In this paper, only one simple plate is utilized to conveniently measure the TC of OVs by its edge diffraction in the near-field no matter whether the plate is opaque or not. Analogous to the interferogram of an OV beam with a plane wave, the resultant fork-shaped diffraction fringes can be used to determine the TC value as well as the handedness of the OV beam. Tolerance for rotated off-axis plate and diffraction distance is demonstrated theoretically and experimentally. In addition, two methods to enhance the diffraction pattern are proposed: computational diffraction fringe enhancement by background deduction and the use of a translucent plate. The parallelism, transparency, and thickness required for the plate are also analyzed.

2. Theoretical Method
Assuming a paraxial monochromatic Gaussian-background vortex beam propagating along axis \( z \) is normally incident at a screen plate (depicted as Fig. 1), the complex amplitude of the beam at \( z = 0 \) can be described by

\[
u_0(x_0, y_0, 0) = u_0(x_0, y_0) = A(x_0 + i\sigma y_0)|l| \exp \left( -\frac{x_0^2 + y_0^2}{w_0^2} \right),
\]

where \( w_0 \) is the waist width of the Gaussian beam, \( \sigma = \text{sgn}(l) \), \( l \) is the TC, and \( A \) is a constant.

Considering that the screen plate located at the \( x_0-y_0 \) plane is rectilinear hard-edged, and the size of the plate is much larger than the beam waist (shown as Fig. 1), then the transmittance function of the plate can read

\[
T(x_0, y_0) = \begin{cases} 1, & x_0 \cos \theta_s + y_0 \sin \theta_s < r_s, \\ \alpha \exp(i\Phi), & \text{else} \end{cases}
\]

where \( \theta_s \) is the azimuthal angle of the plate edge (counted in the anti-clockwise direction), and \( r_s (\geq 0) \) is the distance from the origin to the plate edge. \( \Phi = nkd \) is the additional phase associated with the thickness of the plate, where \( \alpha, n, d \) are the transparency, refractive index, and thickness of the plate, respectively. Equation (2) can degenerate into an ordinary opaque plate when \( \alpha = 0 \) or a transparent homogeneous phase step with \( \alpha = 1 \). Generally, the front and rear surfaces are not technically parallel. Therefore, the phase step can be viewed as a prism with top angles of \( \phi_1 \) and \( \phi_2 \) (shown in Fig. 1), and then the thickness \( d \) should be replaced by

\[
d(x_s) = d_0 - \frac{\sin(\phi_1 + \phi_2)}{\sin(\phi_1) \sin(\phi_2)} (x_s - r_s),
\]

where \( d_0 \) is the thickness of the substrate, and \( (x_s, y_s) \) is the coordinate frame rotated with \( \theta_s \) from \( (x_0, y_0) \) giving relations

\[
x_s = x_0 \cos \theta_s - y_0 \sin \theta_s, \quad y_s = x_0 \sin \theta_s + y_0 \cos \theta_s.
\]

Particularly, when the cross section of \( S \) is an isosceles trapezoid, i.e., \( \phi_1 = \phi_2 \), the angle formed by the intersection of the extended lines on both sides of the trapezoid gives \( \beta = \phi_1 + \phi_2 - \pi \). Then, Eq. (3) evolves to

\[
d(x_s) = d_0 + 2 \tan(\beta/2)(x_s - r_s).
\]

After the edge diffraction, the light field gives the complex amplitude at the distance of \( z \) determined by the Fresnel diffraction integral in the Kirchhoff–Fresnel approximation

\[
u(x, y, z) = \frac{e^{-ikz}}{i\lambda z} \int_{-\infty}^{\infty} dy_0 \int_{-\infty}^{\infty} u_0(x_0, y_0) T(x_0, y_0) dx_0 \times \exp \left\{ \frac{ik}{2z} [(x - x_0)^2 + (y - y_0)^2] \right\}.
\]

Equation (6) can be changed to the form of Fourier transformation

\[
u(x, y, z) = \frac{e^{-ikz}}{i\lambda z} \exp \left[ \frac{ik}{2z} (x^2 + y^2) \right] \times F \left\{ u(x_s, y_s) T(x_0, y_0) e^{2i\pi z \eta} \right\} = E(x, y, z) \cdot [u_1(x, y, z) + au_2(x, y, z)],
\]

where

\[
E(x, y, z) = A e^{i\theta_0} \frac{e^{-ikz}}{i\lambda z} \exp \left[ \frac{ik}{2z} (x^2 + y^2) \right],
\]

\[
u_1(x, y, z) = \int_{-\infty}^{\infty} dy_0 \int_{-\infty}^{\infty} dx_0 [u_1(x_0, y_0)|l|] \times \exp \left[ \frac{ik}{2z} (x_0^2 + y_0^2) \right] \times \exp \left[ -\frac{ik}{z} (xx_s + yy_s) \right],
\]

\[
u_2(x, y, z) = \int_{-\infty}^{\infty} dy_0 \int_{-\infty}^{\infty} dx_0 [u_2(x_0, y_0)|l|] \times \exp \left[ \frac{ik}{2z} (x_0^2 + y_0^2) \right] \times \exp \left[ -\frac{ik}{z} (xx_s + yy_s) \right].
\]
where the number of the redundant fringes is equivalent to 4. For this paper are more convenient to simulate and analyze the evolution of the partially blocked OV beams.

and \( F\{ \cdot \} \) denotes the spatial Fourier transform. The intensity distribution can be calculated with \( I = (u \cdot u^*) \). The last line of Eq. (7) shows that the diffraction can be considered as the superimposition of \( u_1 \) and \( u_2 \), and \( \alpha \) represents the weight of \( u_2 \). In this consideration, \( u_1 \) [see Eq. (9)] leads to the edge diffraction of an opaque screen, and \( u_2 \) offers the blocked OV diffraction of a rectilinear phase step with the addition phase of \( nkd(x) \). Note that the integrals of Eqs. (9) and (10) can be deduced into analytical expressions that are pretty complicated and numerical methods such as the fast Fourier transform (FFT) algorithm used in this paper are more convenient to simulate and analyze the evolution of the partially blocked OV beams.

3. Simulation Results of the Edge Diffraction

3.1. Opaque screen

The edge diffraction by an opaque screen (\( \alpha = 0 \)) has verified that the creation, motion, and annihilation of phase singularities in the diffraction field may appear by varying the edge deviation and propagation distance\([25,28,30]\).

Note that the radius of the OV beam at the maximum of intensity is associated with its TC (\( l \)) and waist radius (\( w_0 \) at \( z = 0 \)) of the fundamental mode, given by

\[
r_{\text{max}} = \sqrt{\left| l \right|/2w_0}.
\]  

(11)

Here, we define the normalized deviation of the plate edge as \( \tilde{r} = r_{\text{max}}/r_{\text{max}} \) and the normalized diffraction distance as \( \tilde{z} = z/z_R \), where \( z_R = \pi w_0^2/\lambda \) is the Rayleigh distance of the Gaussian beam.

Figure 2 calculated via Eq. (8) shows the simulated intensity distributions of the edge diffraction of a blocked OV beam with an opaque screen. The wavelength used in the simulation is 1064 nm, and the waist radius is 1 mm, giving \( z_R = 5.91 \text{ m} \). Diffraction patterns of varied TCs are shown in Fig. 2(a). It is verified that OV beams embedded with phase singularities exhibit redundant fringes in contrast with the straight fringes of the conventional Gaussian beam. For \( l > 0 \), the redundant fringes appear at the top when the screen blocks partial light on the right, i.e., the ‘fork’ orients to the upper and vice versa, where the number of the redundant fringes is equivalent to \( |l| \). Note that the deviation of the edge may result in different diffraction patterns. It is recommended from simulation results [shown in Fig. 2(b)] that the rightmost fringe should be preserved in order to determine which fringes are redundant. On the other hand, as the fifth column of Fig. 2(b) shows, the diffraction fringes vanish when the screen edge locates more than \( 2r_{\text{max}} \) away from the center, which shows the weak influence of the edge. Therefore, to obtain distinct fringes, the edge of the screen plate can deviate about 0.5\( r_{\text{max}} \)–1.5\( r_{\text{max}} \) from the center of the OV beam.

It is shown in Fig. 2(c) that the pattern rotates as the edge rotates due to the rotational symmetry of the OV beam. Thus, the orientation of the fork-shaped pattern should be defined relative to the screen edge. The evolution of the diffraction pattern along the \( z \) axis with varied diffraction distances \( z \) is depicted in Fig. 2(d) with the normalized values of \( \tilde{z} = 0.05, 0.1, 0.2, 0.4, 1 \) for column 1–5, respectively. It turns out that the number of the fringes decreases as the edge-diffracted beam propagates further, and the fringes are finally deformed into a symmetric structure, in which the symmetry axis is parallel to the edge of the screen. This phenomenon is consistent with the theoretical and experimental results in early works\([25,27]\). The overall requirement to discernibly and effectively measure the TC and the handedness of the incident OV beam is that the diffraction distance and the edge deviation are suggested to be controlled at \( z = (0.05–0.4)z_R \) and \( r = (0.5–2)r_{\text{max}} \), respectively.

3.2. Translucent plate

As the screen plate becomes transparent (Fig. 3), the blocked part goes through an additional phase \( \Phi = nkd \) [see also Eq. (2)]. Figure 3(b) shows the varied normalized thickness \( \tilde{d} = 8 \cdot \text{mod}(nd, \lambda)/\lambda = 0–4 \), giving the additional phase \( \Phi = 2\pi m + \pi \tilde{d}/4 = 2\pi m + (0–1)\pi \), where \( m \) is an integer. A phase step is formed between the perturbed and ‘survived’ segments of the vortex beam and exhibits strong perturbation when the
phase step value approaches \( \pi \) or \( \bar{d} = 4 \) in Fig. 3(b)\(^{[32]} \). In this case, the two segments of the vortex beam interfere to the maximum extent. At the same time, the fork-shaped fringes can be more discernible. Therefore, the angle \( \beta \) between two surfaces of the plate can be used to adjust the position of the perturbed segment (see Fig. 1). As Fig. 3(c) shows, the fringes are further distinct when the perturbed segment is at the center of the OV beam, for instance, \( \beta = -3^\circ \) at \( \bar{z} = 0.1 \) in Fig. 3(c).

4. Experiment Results and Discussions

The edge-diffraction-based TC measurement is also experimentally demonstrated, and the setup is shown in Fig. 4(a). A neodymium-doped yttrium aluminum garnet (Nd:YAG) laser is used to produce the fundamental Gaussian mode at \( \lambda = 1064 \) nm with the waist radius \( w_0 = 0.685 \) mm and then expanded into 1.5 mm by an expander, giving the Rayleigh distance \( z_R = 6.643 \) m. The incident beam illuminates a pure phase SLM loaded with fork-shaped blazed gratings [shown in Fig. 4(b)], generating the desired OV beam at the first diffraction order, and the other orders are filtered with an iris diaphragm.

As shown in Fig. 4(a), in the case of \( \alpha = 0 \), an opaque screen \( S \) is employed to realize the hard edge diffraction. For \( \alpha = 1 \), as a consequence of the additional phase \( \Phi = 2\pi nd/\lambda \) varying with a period of \( 2\pi \), it is challenging to control the plate thickness in the wavelength scale (the effective optical thickness \( d = d\lambda/4n \), \( \bar{d} = 0-4 \)). Therefore, in this experiment, \( S \) is substituted by another SLM (SLM2) loaded with a phase step [shown in Fig. 4(c)] to mimic the transparent plate. On the other hand, the phase pattern in SLM2 is supposed to be a whiteboard when switching to the opaque screen diffraction. Finally, a CCD camera set at the distance of \( z \) from \( S \) or SLM2 is used to capture the diffraction profiles.

Figure 5 shows the experimental results of the truncated OV beam of \( l = 3 \) with varied edge deviations of an opaque or a transparent plate. Note that there are several ‘ripples’ in the generated beam, and the ring width is narrower than that desired in Fig. 2. It may result from the properties of the Kummer beam\(^{[33]} \), which is usually generated by fork-shaped gratings and can be degraded into a sum of standard LG beams with different TCs and weights. Regardless, the experimental fringes in Fig. 5(b), to a certain extent, show good agreement with the simulation ones in Fig. 2(b). Figures 5(a)–5(c) show that the fringes can remain discernible at \( \bar{r} = 0.5-1.5 \) and \( \bar{z} = 0.05-0.2 \), i.e., \( r_s = 0.48-1.44 \) mm and \( z = 33.22-132.86 \) cm in our experiment, which is a wide range in three-dimensional space.
In addition, the edge-diffraction patterns of the transparent plate at \( \bar{z} = 0.1 \) with fixed \( \bar{d} = 4 \) and varied \( \bar{r} \) are shown in Fig. 5(d). By contrast with those of the opaque screen in Fig. 5(b), it can be seen that the actual interference of the two segments of the vortex beam mentioned in Section 3.2 is not obvious enough to improve the fringe clarity. It may still result from the narrow ring width of the Kummer beam so that the light from the blocked segment is not enough to interfere with the other.

The experimental results for the measurement of higher TCs of OV beams with \( l = \pm 40 \) (compared to \( |l| \leq 3 \) illustrated above) by an opaque screen are shown in Fig. 6. Since the central dark area of OV light becomes more expansive with the increase of TC, coupled with the ratchet-shaped initial light-field background [shown as Fig. 6(a)] caused by insufficient SLM resolution, fringes of the edge diffraction become more inconspicuous. Therefore, for measurements of high TCs, the background profile of the generated OV beam is removed from the fringes of the edge-diffraction pattern to enhance the fringe contrast.

Figure 6(c) shows the corresponding enhanced fringes where one can explicitly count the number of bright stripes at the upper and lower cambered cross section of the intensity, and specific values are shown in Fig. 6(d). As the plate is set to the right of the OV beam, the TC value can be determined by subtracting the number of lower fringes from the number of upper fringes. In this case, the subtracted number’s sign coincides with the chirality of the vortex beam to be measured.

This method can remain steady for much higher TC measurements but is limited by the resolution and field of view of the CCD due to the increasing ring size and fringe density at the end away from the plate edge. In this case, one can extend the diffraction distance or reduce the edge deviation from the center to increase the stripe spacing.

5. Conclusion

In conclusion, it is demonstrated theoretically and experimentally that the edge of a plate can be used to measure the TC of OV beams at a proper diffraction distance in the near-field. The number of redundant fringes in the diffraction fork-shaped pattern is equal to the TC value, and the orientation of the fork relative to the plate edge indicates the handedness of the OV beam. Simulated results of the opaque screen indicate that the diffraction fringe contrast increases when the screen edge moves closer to the center of the OV beam, and the fringe density decreases as the diffraction distance increases, forming a three-dimensional space to control the diffraction fringes. It turns out that the edge diffraction of translucent plates can also be used to form fork fringes based on the self-interference of the OV beam with a rectilinear phase step. The transparency of the plate affects the degree of interference, and the angle between two surfaces of the plate determines the interference angle. However, experimental results do not show obvious self-interference from the simulation due to the intensity differences between the generated Kummer beam and the standard LG beam. Since the generated OV beam with high TCs is accompanied by the intrinsic large central dark area and low purity limited by the SLM resolution, the additional computational diffraction fringe enhancement by removing the background profile of the undiffracted beam from the diffraction pattern is recommended and applied in our analysis for \( l = \pm 40 \).

This TC measurement method for OV beams takes good advantage of using only one simple and easily available screen whether it is opaque or not. As the edge-diffraction pattern is relevant to the phase of the incident light field, the method could be used to diagnose the phase structure and energy flow for other OV beams, such as composed vortices (a method to measure fractional TC has been proposed in Ref. [34]) and vector vortices [35,36].

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