Nonreciprocal transmission of multi-band optical signals in thermal atomic systems

Shengfa Fan (范圣法)1,2, Yihong Qi (祁义红)1*, Yueping Niu (钮月萍)1, and Shangqing Gong (龚尚庆)1

1 School of Physics, East China University of Science and Technology, Shanghai 200237, China
2 School of Materials Science and Engineering, East China University of Science and Technology, Shanghai 200237, China

*Corresponding author: qiyihong@ecust.edu.cn
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Multi-band signal propagation and processing play an important role in quantum communications and quantum computing. In recent years, optical nonreciprocal devices such as an optical isolator and circulator are proposed via various configurations of atoms, metamaterials, nonlinear waveguides, etc. In this work, we investigate all-optical controlled non-reciprocity of multi-band optical signals in thermal atomic systems. Via introducing multiple strong coupling fields, non-reciprocal propagation of the probe field can happen at some separated frequency bands, which results from combination of the electromagnetically induced transparency (EIT) effect and atomic thermal motion. In the proposed configuration, the frequency shift resulting from atomic thermal motion takes converse effect on the probe field in the two opposite directions. In this way, the probe field can propagate almost transparently within some frequency bands of EIT windows in the opposite direction of the coupling fields. However, it is well blocked within the considered frequency region in the same direction of the coupling fields because of destruction of the EIT. Such selectable optical nonreciprocity and isolation for discrete signals may be greatly useful in controlling signal transmission and realizing selective optical isolation functions.

Keywords: optical nonreciprocity; atomic thermal motion; electromagnetically induced transparency; multi-band.
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1. Introduction

Similar to their electronic counterpart, optical nonreciprocal devices play an important role and possess the fundamental function in photonic devices and quantum circuits, urging great research interest in an optical isolator1–3, circulator4–6, and router7,8. These optical nonreciprocal devices allow photon transportation in one direction, while blocking it in the reverse direction. In laser systems, the optical isolator, which blocks the back transmission of light, provides effective protection for the lasers. Optical nonreciprocal devices also have promising applications in all-optical quantum networking and computing. Transmission and processing of multi-wavelength/band optical signals is an important issue in optical communications and processing. Optical devices for multi-wavelength applications have attracted intensive attention in recent years, such as multi-wavelength lasers9–13, multiplexing and communications14,15, imaging and sensors16–19, and photovoltaic devices20. It is also a very important subject to develop and design optical nonreciprocal devices such as optical isolators suitable for multi-wavelength applications21,22. In this work, we are committed to the study of controllable multi-band optical nonreciprocal transmission by using atomic systems.

The key to realize optical nonreciprocal devices is the nonreciprocal or asymmetric transmission of light. Magneto-optical materials were used to produce optical nonreciprocal propagation via the Faraday rotation effect23–27. However, responses of the magnetic materials are often very weak, implying requirements of the large size bulk magnetic media or the strong magnetic fields, which may bring about some unfavorable impacts28. Then, for avoiding the use of magnetic materials, schemes based on optical nonlinearity28–31 or photonic crystal heterojunctions32 were proposed. Recently, a number of works dedicated to realizing nonmagnetic optical nonreciprocal transmission via different schemes, such as frequency conversion33,34, angular momentum biasing35–37, optoacoustic effects34,38, artificial gauge field39–41, parity-time-symmetry breaking42–45, and moving medium46,47.

Generally, random thermal motion of atoms has a negative impact on coherence of the quantum system, resulting in the decoherence effect or thermal noise48–50. It is fortunate that via a smart design people can also actively utilize the atomic thermal motion in some particular fields. Based on the electromagnetically induced transparency (EIT) effect and the thermal motion of atoms, Zhang et al. proposed a novel mechanism to
achieve optical propagation of the probe field in a three-level \( \Lambda \)-type atom-cavity coupling system\(^{[51]} \). Then, Xia et al. investigated the direction-dependent cross phase modulation (XPM) in an \( N \)-type thermal atomic system and utilized the XPM to achieve an optical isolator and circulator\(^{[52]} \). Later, Gong et al. also investigated directional optical amplification\(^{[53]} \), optical isolation by the pumping effect\(^{[54]} \), and broadband optical nonreciprocity\(^{[55]} \) in multi-level atomic systems. Manipulation of multiple optical signals may have potential applications in optical communications and quantum information processing. Utilizing dynamically induced photonic band gaps, Yang et al. have proposed a scheme to generate two-color optical nonreciprocity in a cold tripod-type atomic system\(^{[56]} \). In this work, stimulated by these works, we investigated all-optical controlled nonreciprocal propagation of multi-band optical signals in a \( \Lambda \)-type-like multi-level hot atomic system. By introducing multiple strong coupling fields, some nonreciprocal bands with separated frequencies can be generated for the weak probe field. Moreover, these separated nonreciprocal frequency bands can be flexibly controlled by the coupling fields. This work may have potential applications in multi-band signal detection, discrimination, and processing.

2. Model and Equations

We consider interaction of laser fields and the atomic system of \( N + 2 \) levels, as shown in Fig. 1, in which the weak probe field and \( N \) strong coupling fields couple corresponding energy levels in Fig. 1(a) and propagate along different directions, respectively, as shown in Figs. 1(b) and 1(c). The weak probe field \( \Omega_p \) of frequency \( \omega_p \) couples states \(| g \rangle \) and \(| m \rangle \), and the transitions \(| m \rangle \leftrightarrow | n \rangle \) \((n = 1, \ldots , N)\) are driven by the strong coupling fields \( \Omega_n \) of frequency \( \omega_n \), where \( \Omega_p = |\mu_{gm}|E_p/2\hbar \) and \( \Omega_n = |\mu_{mn}|E_n/2\hbar \) are corresponding half-Rabi frequencies of the fields with the electric dipole momentum \( \mu_{ij} \) \((i, j = g, m, 1, \ldots , N)\) and the electric field amplitudes \( E_p \) and \( E_n \). The atomic gas is loaded in a cell, and its temperature is controlled by a temperature control system. In general, the atoms are in constant thermal motion following the Maxwell velocity distribution. Under the electric dipole and rotating-wave approximations, the Hamiltonian of the system can be written in the interaction picture as

\[
H_{\text{int}} = -\hbar |\Delta_p| |m\rangle \langle m| + \sum_{n=1}^{N} (\Delta_n + \Delta_p)|n\rangle \langle n| + (\Omega_p)|g\rangle \langle g| + \sum_{n=1}^{N} \Omega_n|n\rangle \langle m| + \text{H.C.} \]  

(1)

where \( \Delta_p = \tilde{\omega}_{mg} - \omega_p \) \((\Delta_n = \tilde{\omega}_{nm} - \omega_n)\) denotes the detuning of the probe field (coupling fields) for the corresponding transition \(| g \rangle \leftrightarrow | m \rangle \) \(| |m\rangle \leftrightarrow | n\rangle \) with transition frequency \( \tilde{\omega}_{mg}(\tilde{\omega}_{nm}) \).

From the Liouville equation, we can obtain the following motion equations for the density-matrix elements:

\[
\dot{\rho}_{gg}(t) = i\Omega_p^* \rho_{mg}(t) - i\Omega_p \rho_{gm}(t) + \sum_{n=1}^{N} \gamma_{mn} \rho_{mn}(t) + \Gamma_{mg} \rho_{nm}(t),
\]

(2)

\[
\dot{\rho}_{mm}(t) = i\Omega_p^* \rho_{gm}(t) + i\sum_{n=1}^{N} \left( \Omega_n \rho_{nm}(t) - \Omega_n^* \rho_{mn}(t) \right)
\]

\[
- i\Omega_p^* \rho_{mg}(t) - \gamma_{mg} \rho_{mg}(t) + \sum_{n=1}^{N} \Gamma_{mg} \rho_{nm}(t),
\]

(3)

\[
\dot{\rho}_{nn}(t) = i\Omega_p^* \rho_{mn}(t) - i\Omega_p \rho_{nm}(t) + \sum_{n=1}^{N} \gamma_{nm} \rho_{nm}(t) - \gamma_{mg} \rho_{m}(t),
\]

(4)

\[
\dot{\rho}_{mg}(t) = (i\Delta_p - \gamma_{mg}) \rho_{mg}(t) + i\sum_{n=1}^{N} \Omega_n \rho_{mg}(t)
\]

\[
+ i\Omega_p \rho_{gg}(t) - i\Omega_p^* \rho_{gm}(t),
\]

(5)

\[
\dot{\rho}_{ng}(t) = (i\Delta_n + \gamma_{mg}) \rho_{ng}(t) + i\Omega_p \rho_{mg}(t) - i\Omega_p^* \rho_{gm}(t),
\]

(6)

\[
\dot{\rho}_{nm}(t) = i(\Delta_n - \gamma_{mg}) \rho_{nm}(t) + i\Omega_p \rho_{mg}(t) - i\Omega_p^* \rho_{gm}(t)
\]

\[
- i\Omega_n \rho_{ng}(t) - i\sum_{l=1}^{N} \Omega_l \rho_{nl}(t),
\]

(7)

\[
\dot{\rho}_{nl}(t) = i(\Delta_n - \Delta_l + i\gamma_{nl}) \rho_{nl}(t) + i\Omega_p \rho_{mg}(t) - i\Omega_p^* \rho_{gm}(t)
\]

\[
- i\Omega_n \rho_{ng}(t) - i\sum_{l=1}^{N} \Omega_l \rho_{nl}(t),
\]

(8)

with \( \sum_{n=1}^{N} \rho_{nm}(t) + \rho_{gg}(t) + \rho_{mm}(t) = 1, \rho_{ij}(t) = \rho_{ji}(t), i \neq j \), and \( n,l = 1 - N \). We denote the radiative decay rate of the populations from level \(| n \rangle \) to \(| m \rangle \) \(| |m\rangle \rangle | g\rangle \rangle \) by \( \Gamma_{nm} \) \( (\Gamma_{ng}) \) and the decoherence rate by \( \gamma_{nl} \), respectively. Assuming \( \Omega_p < \Omega_n, \gamma_{ij} \), the atoms are mainly populated on state \(| g\rangle \).

Then, by solving the density matrix of Eqs. (2)–(8) in steady state, we can obtain the linear susceptibility for the weak probe field as
Then, the normalized transmissivity for the probe field is obtained from the Maxwell equations as follows:

$$\chi \approx \frac{-N_D|\mu_{gm}|^2}{\Delta_p + i\gamma_{mg} - \sum_{n=1}^{N} \Omega_n^2/\left(\Delta_p + \Delta_n + i\gamma_{mg}\right)},$$  \hspace{1cm} (9)$$

with the atom density $N_D$.

Due to the irregular thermal motion, atoms in the hot atomic system move with various velocities in different directions. Under this condition, both frequencies of the lasers and frequency shift arising from the Doppler effect take effect on the interaction between lasers and atoms. Then, the detuning $\Delta_i$ ($i = p, n$) in Eq. (9) should be rewritten as $\Delta_i \pm k_i v_j$ with the atom of velocity $v_j$ and the wavevector $k_i$ of the laser $\Omega_i$.

Assuming all the coupling lasers $\Omega_n$ propagate along the same direction, the effective macro susceptibility for the probe field $\Omega_p$ of co-/counter-propagation with the coupling lasers should be integrated on all the atoms of different velocities by

$$\chi_{\text{co}(\text{cou})} = \int_{-\infty}^{+\infty} D(v) \, dv,$$  \hspace{1cm} (10)$$

in which $D(v) = e^{-v^2/(\sqrt{\pi} v_p)}$ indicates the Maxwell–Boltzmann distribution of the atoms, and $v_p = \sqrt{2k_B T/M}$ is the most probable velocity with the Boltzmann constant $k_B$, the absolute temperature $T$, and the atom mass $M$. Then, susceptibilities for the co-/counter-propagating probe field can be obtained via the following integrations:

$$\chi_{\text{co}} = \int_{-\infty}^{+\infty} \frac{-N_D|\mu_{gm}|^2 D(v)}{\Delta_p - k v + i\gamma_{mg} - S_1} \, dv,$$  \hspace{1cm} (11)$$

$$\chi_{\text{cou}} = \int_{-\infty}^{+\infty} \frac{-N_D|\mu_{gm}|^2 D(v)}{\Delta_p + k v + i\gamma_{mg} - S_2} \, dv,$$  \hspace{1cm} (12)$$

with

$$S_1 = \sum_{n=1}^{N} \Omega_n^2/\left(\Delta_p + \Delta_n - 2 k v + i\gamma_{mg}\right),$$

$$S_2 = \sum_{n=1}^{N} \Omega_n^2/\left(\Delta_p + \Delta_n + i\gamma_{mg}\right),$$

where we have assumed $k_p \approx k_i = k$ ($i = 1, \ldots, N$) for simplicity. Transmission of the probe field in the medium can be obtained from the Maxwell equations as follows:

$$t_{\text{co}(\text{cou})} = e^{i\alpha L \text{co}(\text{cou})},$$  \hspace{1cm} (13)$$

with the transmission coefficient $\alpha = \frac{\gamma_{mg} L}{\Delta_p}$ and medium length $L$.

Then, the normalized transmissivity for the probe field is

$$T_{\text{co}(\text{cou})} = |t_{\text{co}(\text{cou})}|^2.$$  \hspace{1cm} (14)$$

So, transmission of the probe field in the co-/counter-propagation direction can be regulated to pursue high asymmetric transmission in the two opposite directions by controlling the coupling fields.

### 3. Results and Discussion

Figure 2 shows transmissions of the co-propagating (red dash-dotted line) and the counter-propagating (blue solid line) probe fields in the three, four, five, and six-level atomic systems. In the calculation, we consider the atomic medium length $L = 5.0$ cm, the atomic density $N_p = 5.0 \times 10^{16}$ m$^{-3}$, the temperature $T = 70.0^{\circ}\text{C}$, and the other parameters are normalized by $\gamma = \gamma_{mg}$. It is clear that multiple nonreciprocal windows with separated frequency bands are generated in the multi-level atomic systems. When the probe field propagates along the opposite direction of the coupling fields, effects of the atom motion on the probe field and the coupling fields can be offset in the proposed configuration, which leads to the construction of the EIT under two-photon resonance ($\Delta_p + \Delta_n = 0$) and thus high transmissivity of the probe field in the EIT windows.

Then, it can be seen that one or several separated high transmission bands (blue solid lines) are created in the transmission spectrum for the probe field, depending on the number and detuning of control fields. However, for the co-propagating probe field, frequency shifts induced by the atom motion produce remarkable two-photon detunings for the probe field and the coupling fields, which destruct the EIT effect and make large absorption of the probe field. Under this condition, the weak probe field interacts with an effective two-level atomic system, and probe photons are greatly absorbed by a large number of atoms.

The following spontaneous emission can never generate a field along the incident direction of the probe field. Transmission of the co-propagating probe field is almost vanishing.

![Fig. 2. Transmission of the probe field in multi-level atomic systems as a function of the probe detuning $\Delta_L$.](image)
Tunable nonreciprocal frequency bands in the five-level atomic system are achieved in this atomic system by introducing multiple coupling lasers driving corresponding transitions.

In this scheme, each band for nonreciprocal propagation of light can be well controlled and shifted individually by changing the detuning of the corresponding coupling field, which brings us great convenience for optical signal or information processing. Figure 3 shows the transmissions of the co-propagating and counter-propagating probe fields with different detunings of the coupling fields in the five-level atomic system. It can be seen in Figs. 3(a)–3(c), with the fixed detunings Δ1 and Δ3, the central frequency of the middle nonreciprocal band is shifted independently by tuning Δ2. Clearly, the left and right nonreciprocal bands can also be controlled by changing Δ1 and Δ3, respectively. Such frequency-tunable multi-band nonreciprocity may be very helpful in the processing of optical multi-band signals.

Bandwidth of optical nonreciprocal devices plays an important role in applications\cite{57,58}. Broad and tunable widths of nonreciprocal windows are often desirable for nonreciprocal devices such as an optical isolator and circulator. In this scheme, these nonreciprocal windows can be controlled individually or simultaneously by adjusting intensities of the coupling fields. To further examine the dependence of transmissivity and nonreciprocal bandwidth of the probe field on Rabi frequencies of the coupling fields, we calculated transmissions of co-propagating and counter-propagating probe fields in the five-level atomic system by changing the Rabi frequencies of the coupling fields. In Figs. 4(a) and 4(b), for simplicity, we adjust simultaneously the Rabi frequencies of the coupling fields by Ω1 = Ω2 = Ω3 = Ω0. It is shown that, for the counter-propagating probe field, bandwidths of the central three separated high transmission bands increase simultaneously with the Rabi frequencies of the coupling fields, while transmission of the co-propagating probe field is well suppressed in corresponding frequency windows, and the total width of the absorption window is also broadened with the Rabi frequency of the coupling fields. In Figs. 4(c) and 4(d), we fix Ω1 = 40γ and Ω3 = 20γ and only change Ω2 to control transmission of the probe field. In this case, only the bandwidth of the central nonreciprocal window is enlarged with the increase of Ω2, but the left and right ones are almost unchanged.

To further examine transmissivity and contrast η = |Tco−Tcou|, for the case of Figs. 3(c) and 3(d), we calculate and plot the transmissivity and corresponding transmission contrast at the central frequencies of the three nonreciprocal bands in Fig. 5. As shown in Fig. 5(a), transmission of the probe field at Δp = 0 is enhanced obviously with the increase of Ω2, while at the other two nonreciprocal bands the probe fields have little change. This provides us with a way to flexibly control transmission of signals in need in the nonreciprocal windows. It is anticipated that high transmissions of the probe fields at different nonreciprocal bands can be achieved via increasing the corresponding intensities of the coupling fields. Figure 5(b) shows high transmission contrasts at the center of the three nonreciprocal bands, implying excellent isolation performance of them.

![Fig. 3. Tunable nonreciprocal frequency bands in the five-level atomic system with Δ2 = 0, Ω2 = Ω3 = 40γ, Δ1 = −30γ, Δ3 = 30γ and (a) Δ1 = 20γ, (b) Δ1 = 0γ, (c) Δ1 = −20γ. The other parameters are the same as in Fig. 2.](image)

![Fig. 4. Variation of transmission of probe fields with (a, b) Δp and Ω2 or (c, d) Ω2, where (a, c) are the results for the counter-propagating probe field and (b, d) for the co-propagating probe field. In the calculation, Δ1 = −30γ, Δ2 = 0, Δ3 = 30γ, and the other parameters are the same as in Fig. 2.](image)

![Fig. 5. Variations of (a) transmissivity T of the counter-propagating probe field and (b) corresponding transmission contrast η with Rabi frequency of the coupling field Ω2 at the center frequencies of the three nonreciprocal bands, corresponding to the cases of Figs. 4(c) and 4(d).](image)
Even for \( k_p \) the transitions \( 5P_{3/2}, F = 2 \rightarrow 5D_{3/2}, F = 1 \), \( 5D_{3/2}, F = 1 \rightarrow 7S_{1/2}, F = 1 \), and \( 8S_{1/2}, F = 1 \). In this case, \( \alpha_p = 2\pi \times 384.23034 \text{ THz} \) and \( \alpha_{l, 2, 3, 4} = 2\pi \times (386.25231, 386.3411, 404.5667, 486.58499) \text{ THz} \) are used for calculation. It can be found that, as long as \( k_p \) and \( k_i \) are not too different, the property of nonreciprocity can be well kept. The only difference is that the transmission of the probe field in the counter-propagating direction may be suppressed slightly, or part of the multi-band signals cannot be well separated (as shown in Fig. 7). Therefore, multi-band nonreciprocity can also be achieved by using similar multi-level transitions in alkali-metal atoms, such as rubidium and cesium. For example, transitions of \( (5S_{1/2} \leftrightarrow (5P_{1/2}, 5P_{3/2} \leftrightarrow (6S_{1/2}, 7S_{1/2}, 8S_{1/2}, \ldots, 5D_{3/2}, 6D_{3/2}, 7D_{3/2}, 8D_{3/2}, \ldots) \) in rubidium provide the possibility of cascade-like transitions. In addition, small tilt angles between the probe and coupling fields may also be arranged for matching the condition of \( k_p \approx k_i \) in experiment.

4. Conclusions

In conclusion, based on the EIT effect, we have investigated controllable multi-band nonreciprocal propagation of optical signals in the thermal multi-level cascade atomic systems. By use of multiple strong coupling fields, the weak probe field can propagate with several separated high transmission bands in the opposite direction of the coupling fields due to the EIT effect, while the counter-propagating probe field can be well absorbed in the same frequency domain. This provides the possibility of generating and flexibly controlling multi-band nonreciprocal propagation of optical signals. Moreover, separation, bandwidth, and center frequencies of these nonreciprocal transmission bands can be well adjusted and controlled by changing the Rabi frequencies and detunings of the coupling lasers. Simultaneously, high transmission contrast can be maintained in these nonreciprocal bands, guaranteeing excellent optical isolation performance. This work may provide references for related optical isolation devices such as an optical diode and circulator. Other probable functions of the separated nonreciprocal bands may be extracting and discriminating optical signals, which may find application in optical information processing and optical networking.

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