Reconstruction of Fabry–Perot cavity interferometer nanometer micro-displacement based on Hilbert transform

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A novel reconstruction method of nanometer micro-displacement of Fabry-Perot (F-P) interference is proposed in this study. Hilbert transforms are performed for F-P interference fringes, and the obtained signal performs tangent operation with the original signal. Finally, the validity of the proposed algorithm and the structure are verified by simulation and several experimental measurements for vibration. Results from the experiments show that the maximum relative error is 4.9%.

Keywords: displacement reconstruction; phase unwrapping; Fabry–Perot interferometer. **D0I:** 10.3788/C0L202119.091202

1. Introduction

In recent years, optical fiber detection, especially the Fabry– Perot interferometer (FPI), is of great importance because of its high reliability, high resolution, immunity to electromagnetic interference, low fabrication cost, and ease of construction^[1]. Compared with other traditional interferometers, such as the Michelson interferometer and Mach–Zehnder interferometer, which need two relatively independent light paths, the FPI is composed of two half-reflective surfaces or a cavity between a half-reflective surface and a total reflective surface, which shows a high stability^[2]. Thus, Fabry–Perot (F-P) sensors are applied to many domains, such as the measurement of acceleration^[3], strain^[4], gas pressure^[5], temperature^[6], magnetic field^[7], mechanical vibration^[8], displacement^[9], and acoustic wave^[10].

In the field of micro-displacement reconstruction, the traditional demodulation method mostly uses interference fringe counting, which has low accuracy. Kent proposed the principle of normal phase-shifted outer F-P optical fiber sensors in 1990. Although this proposed method constructs orthogonal phase, it introduces complex hardware structure and requires careful operation^[11]. In 2002, Seat *et al.* proposed a structure composed of a partially coated gradient-index lens and a movable reflector to get the quadrature phase, making the ensemble expensive^[12]. Xia *et al.* proposed a phase interrogation method with two wavelengths in 2017. However, the method requires two monochromatic beams^[13]. Liao *et al.* proposed a construction with direct phase modulation by a built-in electro-optic modulator in 2017. Despite the improved accuracy, this method of adding phase modulation makes integrating the sensor difficult^[14]. In 2019, Domingues et al. presented the F-P interference by using an alternative high-rate dynamic acquisition methodology based on the conversion of frequency. Although this method can achieve dynamic data acquisition, the devices used are difficult to acquire^[15]. In the same year, Gomes et al. proposed an FPI system based on the optical Vernier effect to improve sensitivity and resolution, which requires the construction of two FPI cavities and creates a lot of trouble for measurement^[16]. Zhu et al. used this method to demodulate absolute displacement in 2017, which has higher requirements on the equipment^[17].

To simplify the model of the vibration measurement system, reduce the cost, and improve its precision, we proposed a new method to reconstruct the F-P interference signal of vibration based on the Hilbert transform. Then, we performed one and three Hilbert transforms, respectively, for the left and right of the F-P interference fringes' reverse point on one period. Through the tangent operation, we obtained the original signal from the phase information. Finally, the phase can be demodulated by an arctangent, and the reconstitution of vibration signal can be achieved by an algorithm.

2. Theoretical Analysis and Simulation

2.1. Principle of F-P interferometer and Hilbert transform

FPI was invented by physicists Fabry and Perot in 1897^[18]. In this paper, the structure of the FPI is based on the end face of the fiber and the surface of vibrating objects. The two beams of reflected light interfere, which is called the FPI, as shown in Fig. 1. Incident light P_i on the end face of the optical fiber returns part of the light, P_{o1} , and the transmitted light incident on target returns light, P_{o2} . The returned light meets on the end face of the optical fiber, resulting in interference.

In this study, we used the fiber end face of a port of the coupler and the mirror on the piezoelectric transducer (PZT) to construct the F-P cavity interference structure (see Fig. 2). Owing to the approximate 4% reflectivity of the optical fiber end face, as shown in Ref. [19], the F-P cavity interference structure in this experiment can be approximated as double beam interference^[20], because multiple reflected beams are so weak that they can be ignored.

It reflects the results of two reflected powers, namely, R_1 and R_2 , which can be expressed as follows:

$$P_r(t) = P_i \Big[R_1 + R_2 - 2\sqrt{R_1 R_2} \cos \varphi(t) \Big],$$
(1)

where P_i and P_r are incident light power and reflected light power, respectively. R_1 and R_2 are reflected light power of the reflected surface $(R_1, R_2 \ll 1)$. $\varphi(t)$ denotes the total phase shift from one reflective surface to the other, which can be expressed as^[21]

$$\varphi(t) = \frac{4\pi n L(t)}{\lambda},\tag{2}$$



Fig. 1. FPI schematic diagram.



Fig. 2. Experimental schematic diagram.

where *n* is the refractive index (the medium in the cavity is air, n = 1), L(t) is the total variation of the cavity length of the FPI, and λ is the optical wavelength. Equation (1) shows that the reflected power *P* can be changed by changing the phase shift:

$$\varphi(t) = \varphi_{\text{initial}} + \Delta \varphi_{L(t)}, \qquad (3)$$

$$\Delta \varphi_{L(t)} = \frac{4\pi}{\lambda} \Delta L(t). \tag{4}$$

Thus, Eq. (4) can be converted to

$$\Delta L(t) = \frac{\lambda \Delta \varphi_{L(t)}}{4\pi}.$$
(5)

The DFB laser is a distributed feedback laser, the PD is a photodetector, and the laser trap is used to prevent the reflection light of the fiber end face. The PD filters out the high-frequency component, so it detects the interference term in Eq. (1). The coefficient before the interference term is filtered by the normalization function of the acquisition card. Because of the non-monotonicity of the cosine signal, the application range of the cosine signal demodulation is small.

To demodulate useful phase information with high precision, we propose a new algorithm based on the Hilbert transform. The expression for the Hilbert transform^[22] is

$$H[X(t)] = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{X(\tau)}{t - \tau} \mathrm{d}\tau.$$
(6)

By substituting the interference item of Eq. (1) into Eq. (6), the following expression can be obtained:

$$H\{\cos[\varphi_F(t)]\} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\cos[\varphi_F(\tau)]}{t - \tau} d\tau.$$
(7)

We calculated Eq. (7), and then we derived the following formula:

$$H\{\cos[\varphi_F(t)]\} = \cos[\varphi_F(t)]^* \frac{1}{\pi t}.$$
(8)

We take the Fourier transform of the above equation, given by the following expression:

$$\mathcal{F}\{H\{\cos[\varphi_F(t)]\}\}$$

$$= \mathcal{F}\{\cos[\varphi_F(t)]\} \cdot [-j \cdot \operatorname{sgn}(\omega)]$$

$$= \begin{cases} \mathcal{F}\{\cos[\varphi_F(t)]\} \cdot (-j), & \omega \ge 0, \\ \mathcal{F}\{\cos[\varphi_F(t)]\} \cdot j, & \omega < 0. \end{cases}$$
(9)

In terms of the spectrum, the positive frequency part of our original signal is multiplied by -j, which means that the phase is shifted by $\pi/2$. The imaginary part of the analytical signal after the Hilbert transform of the FPI signal has the phase difference of $-\pi/2$ compared with the original signal. When three Hilbert transformations are applied to the original signal, the virtual part

Chinese Optics Letters



Fig. 3. Reconstruction diagram of phase position.

and the original interference signal have a phase shift of $3(\pi/2)$ (mathematically, the phase shift is $-\pi/2$). As the inverted point of the interference fringe reflects the change of the vibration direction, the change of the reconstructive vibration direction is realized through three Hilbert transforms to avoid using symbolic functions. In the signal processing of harmonic motion, we use the midpoint of each period of the interference fringe as the reverse point. Through the algorithm, tan $[\varphi(t)]$ is derived.

The algorithm block diagram is shown in Fig. 3. The black arrow in Fig. 3(a) represents the inverted point. The original interference waveform $\cos [\varphi_F(t)]$ and the target vibration inverted point are shown in Fig. 3(a). The right side of reversal point in the interference pattern performs one Hilbert transformation, and the left side performs three Hilbert transformations, as shown in Fig. 3(b). It is written as $\sin [\varphi_F(t)]$. Finally, $\tan [\varphi(t)]$ can be obtained. Then, we can get the phase information $\varphi_F(t)$ of the interference signal, as shown in Fig. 3(c). By using the relationship between phase and cavity length, as given in Eq. (5), the target vibration can be realized without the use of symbolic functions in Fig. 3(d).

2.2. Results of simulation

To obtain convincing results, we first simulate the FPI signal and the reconstruction signal, as shown in Fig. 4. The motion is driven by zero initial phase and a peak-to-peak value of 6.2 μ m. The simulated wavelength of the light source is 1550 nm, and the frequency of the vibration is 5 Hz. The sampling frequency is 50 kHz, and 4000 sampling points exist. The values of the parameters and variables in the simulation are listed in Table 1.

Figure 4(a) shows the simulation fringe of F-P interference. As shown in Fig. 4(b), the red fringes present the FPI signal, and the blue ones present the signal after Hilbert transforms. Figure 4(c) shows the simulated harmonic motion in red and reconstruction vibration in blue. Figure 4(d) shows the absolute errors between reference and vibration reconstruction. Dividing the maximum absolute error by the peak-to-peak value, the



Fig. 4. Simulation of the FPI signal and the reconstruction signal.

Table 1. Values of the Variables in Simulations.

Variables	Definition	Values
λ	Laser initial wavelength	1550 nm
L	Original external cavity length between the laser and the target	0.1 m
f	Frequency of the vibration	5 Hz
ΔL	Peak-to-peak value of the vibration	6.2 µm

maximum relative error is obtained. Thus, the maximum relative error of simulation is approximately 1.48%.

3. Experimental Results and Discussion

3.1. Experiment

The experimental device for micro-vibration measurement is shown in Fig. 5. The platform used in this experiment is the optical vibration isolation platform (THORLABS, PTR52509), which can effectively isolate the influence of environmental vibration. The light source is provided by a DFB with a wavelength of 1550 nm (THORLABS, S3FC1550). A well-performing isolator is built into the laser to prevent the reflected light from entering the laser cavity and forming self-mixing interference,



Fig. 5. Schematic of the experimental setup.



Fig. 6. Experimental signal and reconstruction of sine wave vibration.

which affects the interference signal. At the same time, the DFB has a temperature control function, which can accurately control the laser temperature at 25°C. The laser is divided into two beams of light by a 2×2 coupler (THORLABS, TN1550R5A2) with a 50/50 ratio. One beam of light is connected to the light trap (THORLABS, FTFC1). The laser trap is used to avoid the interference of the light reflected from the end of the coupler. The other beam enters the F-P cavity formed by the end face of the fiber with smooth processing and the reflective mirror on the PZT to interfere. Among them, 4% of the light reflected from the end face of the fiber is used as the reference arm, and the beam carrying vibration information reflected back to the end face of the emitted fiber by the mirror on PZT is used as the measurement arm. The interference signal that returns to the input direction through the coupler is collected by the PD collector (THORLABS, DET01CFC) on the other port and is then converted into a digital signal through the 24 bit data acquisition card (NI, USB-4431). Finally, the digital signal is processed through collection program to recover the vibration of the object on the PC. All experiments are carried out at the room temperature of 25°C.

The harmonic wave is set as a sinusoidal wave. The peak-topeak value is set as $1.55-4.65 \mu m$, and $1.55 \mu m$ is used as the step size to carry out the experiment, data acquisition, and algorithm reconstruction. The sampling frequency of the experimental signals is 100 kHz.

Table 2. Experimental Relative Error

Error at Various Peak-to-Peak Values			
1.55 μm	3 .1 μm	4.65 μm	
2.7%	2.3%	1.5%	
2.4%	3.0%	2.2%	
3.8%	2.8%	2.4%	
4.9%	3.2%	2.1%	
	Error at 1.55 μm 2.7% 2.4% 3.8% 4.9%	Error at Various Peak-to-Pea 1.55 μm 3.1 μm 2.7% 2.3% 2.4% 3.0% 3.8% 2.8% 4.9% 3.2%	

The experimental signal and reconstruction are shown in Fig. 6. The vibration frequency of the PZT is 3 Hz, and the vibration peak-to-peak value is 3.1 μ m. In Fig. 6(a), the red is the interference signal obtained by the experiment, and the blue stripe is the Hilbert transformation performed on the signal. In Fig. 6(b), the red is the set vibration track, and the blue is the reconstructed vibration track. In Fig. 6(c), the curve is the error. The experimental error is shown in Table 2.

3.2. Error analysis

Many reasons can explain the errors in the vibration measurement methods in this study. A difference between simulated and experimental results can be mainly caused by three factors, such as mechanical noise, light noise, and electromagnetic noise. First, the optical fiber is sensitive to environmental disturbance. This error can be reduced by fixing the optical fiber to the optical isolation table with tape and using a high-performance optical vibration isolation platform. Second, the light emitted by the light source is not a complete single-frequency signal, so the signals with Hilbert transform may interfere with each other. Finally, the section of photoelectric conversion, which includes PD and data acquisition card, can produce electromagnetic noise. A grounding operation can reduce the error effectively. In addition, the difference of laser wavelength produces error. Thus, the DFB with temperature control and the controlling of room temperature are necessary.

4. Conclusion

A novel micro-displacement reconstruction method based on Hilbert transform for the FPI signal is proposed. The whole vibration measuring system has a simple structure, low cost, and convenient reconstruction process. The FPI is constructed of a coupler with a fiber optic end face and a total reflector. Then, the tangent signal is constructed using the Hilbert transforms of the interference signal repeatedly to realize the reconstruction of micro-displacement. This paper provides a useful exploration for micro-displacement measurement and interference signal reconstruction.

Acknowledgement

This work was supported by the Natural Science Foundation of Northeast Petroleum University (No. 2018GPYB-03).

References

- Y. Y. Yang, Y. X. Zhang, Z. H. Liu, J. Yang, and L. B. Yuan, "Twin-core fiberbased sensor for measuring the strain and bending simultaneously," in *International Symposium on Photoelectronic Detection and Imaging 2013: Fiber Optic Sensors and Optical Coherence Tomography* (2013).
- A. D. Kersey, D. A. Jackson, and M. Corke, "A simple fiber Fabry–Perot sensor," Opt. Commun. 45, 71 (1983).

- Z. G. Wang, W. T. Zhang, J. Han, W. Z. Huang, and F. Li, "Simplified design of diaphragm-based fiber optic extrinsic Fabry–Perot accelerometer," Proc. SPIE 9274, 927417 (2014).
- X. K. Yu, N. F. Song, and J. M. Song, "A novel method for simultaneous measurement of temperature and strain based on EFPI/FBG," Opt. Commun. 459, 6 (2020).
- Y. G. Liu, D. Q. Yang, Y. X. Wang, T. Zhang, M. Shao, D. K. Yu, H. W. Fu, and Z. N. Jia, "Fabrication of dual-parameter fiber-optic sensor by cascading FBG with FPI for simultaneous measurement of temperature and gas pressure," Opt. Commun. 443, 166 (2019).
- H. Li, Q. Zhao, S. Jiang, J. Ni, and C. Wang, "FP cavity and FBG cascaded optical fiber temperature and pressure sensor," Chin. Opt. Lett. 17, 040603 (2019).
- E. Kendir and S. Yaltkaya, "Variations of magnetic field measurement with an extrinsic Fabry–Perot interferometer by double-beam technique," Measurement 151, 107217 (2020).
- L. C. Zhang, Y. Jiang, J. S. Jia, P. Wang, S. M. Wang, and L. Jiang, "Fiber-optic micro vibration sensors fabricated by a femtosecond laser," Opt. Lasers Eng. 110, 207 (2018).
- 9. K. Tian, J. Yu, X. Wang, H. Zhao, D. Liu, E. Lewis, G. Farrell, and P. Wang, "Miniature Fabry–Perot interferometer based on a movable microsphere reflector," Opt. Lett. 45, 787 (2020).
- Z. Qu, P. Lu, Y. Li, X. Fu, W. Zhang, D. Liu, and J. Zhang, "Low-frequency acoustic Fabry-Pérot fiber sensor based on a micromachined silicon nitride membrane," Chin. Opt. Lett. 18, 101201 (2020).
- K. A. Murphy, M. F. Gunther, A. M. Vengsarkar, and R. O. Claus, "Quadrature phase-shifted, extrinsic Fabry-Perot optical fiber sensors," Opt. Lett. 16, 273 (1991).
- H. C. Seat, E. Ouisse, and V. Metivier, "Demonstration of a dual-cavity extrinsic fiber Fabry–Perot interferometer for vibration displacement measurements," in *Proceedings of SPIE the International Society for Optical Engineering* (2002).

- W. Xia, C. Li, H. Hao, Y. Wang, X. Ni, D. Guo, and M. Wang, "High-accuracy vibration sensor based on a Fabry–Perot interferometer with active phasetracking technology," Appl. Opt. 57, 659 (2018).
- 14. H. Liao, P. Lu, L. Liu, D. M. Liu, and J. S. Zhang, "Phase demodulation of Fabry-Perot interferometer-based acoustic sensor utilizing tunable filter with two quadrature wavelengths," Proc. SPIE **10110**, 101101J (2017).
- M. F. Domingues, C. Tavares, N. Alberto, A. Radwan, P. Andre, and P. Antunes, "High rate dynamic monitoring with Fabry–Perot interferometric sensors: an alternative interrogation technique targeting biomedical applications," Sensors 19, 4744 (2019).
- A. D. Gomes, M. S. Ferreira, J. Bierlich, J. Kobelke, M. Rothhardt, H. Bartelt, and O. Frazo, "Optical harmonic Vernier effect: a new tool for high performance interferometric fibre sensors," Sensors 19, 5431 (2019).
- C. Zhu, Y. Z. Chen, Y. Du, Y. Y. Zhuang, F. X. Liu, R. E. Gerald, and J. Huang, "A displacement sensor with centimeter dynamic range and submicrometer resolution based on an optical interferometer," IEEE Sens. J. 17, 5523 (2017).
- M. R. Islam, M. M. Ali, M. H. Lai, K. S. Lim, and H. Ahmad, "Chronology of Fabry–Perot interferometer fiber-optic sensors and their applications: a review," Sensors 14, 7451 (2014).
- S. Pullteap and H. C. Seat, "An extrinsic fiber Fabry–Perot interferometer for dynamic displacement measurement," Photon. Sens. 5, 50 (2014).
- L. Yuan and L. Zhou, "Fiber optic differential interferometer," IEEE Trans. Instrum. Meas. 49, 779 (2000).
- T. Ito, "Precise measurement of the change in the optical length of a fiberoptic Fabry-Perot interferometer," Appl. Opt. 25, 1072 (1986).
- Z. Zhang, C. Li, and Z. Huang, "Vibration measurement based on multiple Hilbert transform for self-mixing interferometry," Opt. Commun. 436, 192 (2019).