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Photons can hide where they have been

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Recently, the nested Mach–Zehnder interferometer [Phys. Rev. Lett. 111, 240402 (2013)] was modified by adding Dove prisms in a paper [Quantum Stud.: Math. Found. 2, 255 (2015)], and an interesting result is that, after the Dove prisms were inserted, a signal at the first mirror of the nested interferometer was obtained. But, according to the former original paper, the photons have never been present near that mirror. In this work, we interpret this result naturally by resorting to the three-path interference method. Moreover, we find that even though the photons have been somewhere, they can hide the trace of being there.

Keywords: multi-path interference; quantum foundations.
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1. Introduction

In 2013, Danan and co-researchers presented creatively a nested Mach–Zehnder interferometer (MZI), which was built by nesting an inner MZI in one arm of an outer MZI[1], to trace the travel history of photons passing through it. They used the presence of a "weak trace" as the criterion of whether a photon was reflected by a certain mirror. This nested MZI produced some surprising experiment outcomes, which imply that a photon can pass through discontinuous trajectories. These counterintuitive outcomes were explained by the two-state vector formulation (TSVF)[2,3] of quantum mechanics in Ref. [1]. The experiment and interpretation stimulate lively academic discussions about "the past of a photon" among researchers in recent years[2–26]. These discussions are helpful in understanding the nature of the photon and developing photon-based technologies[27,28]. To explain the counterintuitive outcomes, we proposed a natural way by using three-path interference[29]. There are three possible propagating paths for a photon beginning from the light source and ending in the detector D in the nested MZI. According to our interpretation, all of these three paths contribute to the final output of the nested MZI, and the counterintuitive outcomes can be explained simply by the interference of the three paths.

Among such discussions, Alonso and Jordan proposed an interesting experimental scheme[30]. They modified the original nested MZI experiment by placing Dove prisms in its arms and found that the output signals changed, which seemed to suggest a change in the past of a single photon in the "weak trace" point of view. The TSVF interpretation given in Ref. [1] was also queried[30,31]. Alonso and Jordan analyzed their modified setup by considering the transverse momentum kicks of the photons, which come from the vibrating mirrors. In this paper, we revisit the discussion about the surprising outcomes in Ref. [30] by resorting to three-path interference interpretation. In the three-path interference point of view, all three possible paths, beginning at the photon source and ending up in the detector, do contribute to the final output, which means that the photons pass through all the paths simultaneously. The outcomes of the experiment are decided by the interference of the three paths. Our interpretation has a clear physical picture, and the surprising outcomes introduced by Refs. [1,30] can be explained in a simple and natural way. Moreover, we find it very interesting that the photons can hide where they have been in some certain situations.

Theory. Now, let us start by introducing the experimental setup in Ref. [30]. The nested MZI modified with Dove prisms is sketched in Fig. 1.

Photons emitted by a light source have three possible propagation paths to go through and reach detector D, which are shown in the following. Path I: LS → BS1 → Mirror E → iBS1 → Mirror A → iBS2 → Mirror F → BS2 → D; Path II: LS → BS1 → Mirror E → iBS1 → Mirror B → iBS2 → Mirror F → BS2 → D; Path III: LS → BS1 → Mirror C → BS2 → D.

According to the three-path interference method developed in Ref. [29], when a photon has passed through the first beam splitter (BS1), which is a $\frac{1}{3}:\frac{2}{3}$ BS, the state of the photon takes the form

$$|\psi_{\mathrm{BS1}}\rangle = \frac{1}{\sqrt{3}}|L\rangle + i\sqrt{\frac{2}{3}}|U\rangle, \tag{1}$$

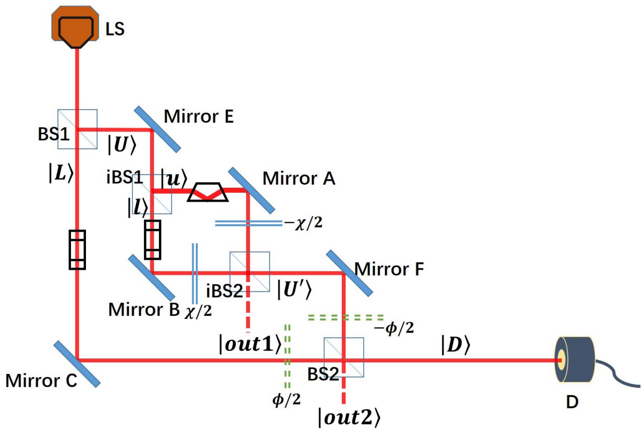

Fig. 1. Modified nested MZI[1] advised by Ref. [30]. An inner MZI is nested into one arm of an outer MZI. Three Dove prisms are placed in all three arms of the nested MZI, and the orientation of the prism near mirror A is different from that of the other two.

in which $|U\rangle$ refers to the state that the photon is reflected off BS1 and goes along the upper arm of the outer MZI, while $|L\rangle$ refers to the state that the photon passes through BS1 and goes along the lower arm of MZI. The 1/3 of the beam power goes to the lower arm, and 2/3 of the beam power passes through the upper arm. By the same way, we can conveniently write down the photon state as follows after it has been reflected by the mirror E and passed through the BS of the inner MZI (iBS1), a $\frac{1}{2}:\frac{1}{2}$ BS,

$$|\psi_{\text{iBS1}}\rangle = \frac{1}{\sqrt{3}}|L\rangle + i\frac{e^{i\varphi_e}}{\sqrt{3}}|l\rangle - \frac{e^{i\varphi_e}}{\sqrt{3}}|u\rangle, \tag{2}$$

where φ_e is a complex phase that comes from mirror E, with its real and imaginary parts marking the phase change and the strength change of the beam, respectively. When the mirror E is vibrating, it causes a tiny change in the optical path and a small motion of the light spot on the surface of a position sensitive photodetector[1]. The former brings in a small change in the phase of the photon, while the latter makes a little change in the strength of the detected signal. State $|u\rangle$ refers to the photon reflected off iBS1 and going along the upper arm of the inner MZI, while state $|l\rangle$ refers to the photon passing through iBS1 and going along the lower arm of the inner MZI.

In Ref. [30], the nested MZI introduced by Ref. [1] is reformed. Three Dove prisms are placed into the three arms of the nested interferometer, but the orientation of the Dove prism in path I is orthogonal to that of the other two. Half-wave plates are introduced in every path for polarization correction[30], but they are not shown here in Fig. 1 for simplicity. After passing through the three Dove prisms, the state of photon becomes

$$|\psi_{\text{Dove}}\rangle = \frac{1}{\sqrt{3}}|L\rangle + i\frac{e^{i\varphi_e}}{\sqrt{3}}|l\rangle - \frac{e^{i\tilde{\varphi}_e}}{\sqrt{3}}|u\rangle, \tag{3}$$

in which $\operatorname{Re}\varphi_e = \operatorname{Re}\tilde{\varphi}_e$ and $\operatorname{sign}(\operatorname{Im}\varphi_e) = -\operatorname{sign}(\operatorname{Im}\tilde{\varphi}_e)$. Re is the real part function, and Im is the imaginary part function. sign(x) is sign function, $\operatorname{sign}(x) = -1$ for $x < 0$, and $\operatorname{sign}(x) = 1$ for $x \geq 0$. As Dove prisms are placed in all the three paths, they bring the same phase delays and thus do not change the interference of the three paths, so we do not need to add any phase factor or change the real part of φ_e in Eq. (2). (This is why $\operatorname{Re}\varphi_e = \operatorname{Re}\tilde{\varphi}_e$.) But, we must pay attention to the imaginary part of φ_e. The orientation of the Dove prism in path I is orthogonal to that in paths II and III. As a result, the Dove prism changes the tilting direction of the laser ray going in path I, as shown in Fig. 2, while the prisms in paths II and III do not.

For example, if the mirror E tilts to left, the laser ray reflected by mirror E also turns to the left. In paths II and III, the laser rays passing through the Dove prisms with mirror E tilting (green dotted line) also tilt to the left of the original rays (red solid line). But, the condition is different in path I. Because the orientation of the Dove prism in path I is orthogonal to that in the other two paths, if the laser ray before entering the Dove prism tilts to the left of the original rays (red solid line), the ray going out of the Dove prism will go right (green dotted line). The swing direction of the laser ray going in path I is reversed. As the tilting direction of laser ray changes, the motion of the light spot on the photodetector also changes, which reverses the imaginary part of φ_e, that is why we make $\varphi_e \to \tilde{\varphi}_e$ in front of state $|u\rangle$.

As mentioned above, the vibrating mirrors A, B, and C also introduce tiny complex phases to the state of the photon, and we write the state down in the same way,

$$|\psi_{\text{ABC}}\rangle = \frac{e^{i\varphi_c}}{\sqrt{3}}|L\rangle + i\frac{e^{i(\varphi_b+\varphi_e)}}{\sqrt{3}}|l\rangle - \frac{e^{i(\varphi_a+\tilde{\varphi}_e)}}{\sqrt{3}}|u\rangle. \tag{4}$$

After the photon passes through the inner MZI, its state becomes

$$|\psi_{\text{iBS2}}\rangle = \frac{e^{i\varphi_c}}{\sqrt{3}}|L\rangle + i\frac{1}{\sqrt{6}}[e^{-\frac{i\chi}{2}}e^{i(\varphi_b+\varphi_e)} - e^{\frac{i\chi}{2}}e^{i(\varphi_a+\tilde{\varphi}_e)}]|U'\rangle$$
$$-\frac{1}{\sqrt{6}}[e^{-\frac{i\chi}{2}}e^{i(\varphi_b+\varphi_e)} + e^{\frac{i\chi}{2}}e^{i(\varphi_a+\tilde{\varphi}_e)}]|\text{out1}\rangle. \tag{5}$$

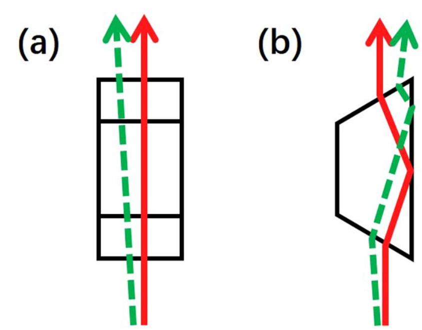

Fig. 2. (a) The photon traces passing through the Dove prisms along paths II and III, in which the relative position of the laser ray (green dotted line) with mirror E tilting does not change compared to the original ray (red solid line) without mirror E tilting. (b) The photon traces passing through the Dove prism along the path I, in which the relative position of the laser ray (green dotted line) with mirror E tilting changes compared to the original ray (red solid line) without mirror E tilting.

State $|U'\rangle$ refers to the photon that has passed through the inner MZI and propagates toward mirror F, while state $|\mathrm{out1}\rangle$ refers to the photon that leaves the nested MZI after iBS2, a $\frac{1}{2}:\frac{1}{2}$ BS. In Ref. [1], the situation of interference is adjusted by "slightly shifting mirror B". For the convenience of discussion, we equivalently introduce phase tuners in Fig. 1. χ is the relative phase between paths I and II, or in other words, between two arms of the inner MZI. By changing χ, we can control the condition of the inner MZI. After the photon passing through BS2, a $\frac{1}{3}:\frac{2}{3}$ BS, the final output state becomes

$$\begin{aligned}
&|\psi_{\mathrm{BS2}}\rangle \\
&=\frac{1}{3}\left[e^{i\phi/2}e^{i\varphi_c}-e^{-i\phi/2}e^{-i\chi/2}e^{i(\varphi_b+\varphi_e+\varphi_f)}-e^{-i\phi/2}e^{i\chi/2}e^{i(\varphi_a+\tilde{\varphi}_e+\varphi_f)}\right]|D\rangle \\
&\quad+\frac{i}{3}\left[\sqrt{2}e^{i\phi/2}e^{i\varphi_c}+\frac{e^{-i\phi/2}e^{-i\chi/2}}{\sqrt{2}}e^{i(\varphi_b+\varphi_e+\varphi_f)}-\frac{e^{-i\phi/2}e^{i\chi/2}}{\sqrt{2}}e^{i(\varphi_a+\tilde{\varphi}_e+\varphi_f)}\right]|\mathrm{out2}\rangle \\
&\quad-\frac{1}{\sqrt{6}}\left[e^{-i\chi/2}e^{i(\varphi_b+\varphi_e)}+e^{i\chi/2}e^{i(\varphi_a+\tilde{\varphi}_e)}\right]|\mathrm{out1}\rangle,
\end{aligned} \tag{6}$$

in which ϕ represents the relative phase between two arms of the outer interferometer. State $|D\rangle$ refers to the photon that will be detected by the photon detector D, and the other state $|\mathrm{out2}\rangle$ refers to the photon that leaves the nested MZI without being detected. Now, we have the probability of detecting a photon in the photodetector D, which is written as

$$P_D=\frac{1}{9}\left|e^{i\phi}e^{i\varphi_c}-e^{-i\phi/2}e^{-i\chi/2}e^{i(\varphi_b+\varphi_e+\varphi_f)}-e^{-i\phi/2}e^{i\chi/2}e^{i(\varphi_a+\tilde{\varphi}_e+\varphi_f)}\right|^2. \tag{7}$$

This expression is general, and, by changing the relative phases χ and ϕ, the output signals can be achieved in different interference conditions.

2. Analysis and Discussion

Based on the general expression of Eq. (7), the probability of detecting a photon in the photodetector D, now we analyze two situations. First, we consider the situation where the inner MZI is aligned to achieve a "complete destructive interference of the light propagating towards mirror F"[1]. When the inner MZI is adjusted "in such a way that the beam of light passing through it does not reach the photodetector", the probability of finding a photon near mirror F is zero. In this situation, the relative phases should be $\phi=\chi=0$[29], and the probability of detecting a photon by the photodetector is

$$\begin{aligned}
P_D&=\frac{1}{9}|e^{i\varphi_c}-e^{i(\varphi_b+\varphi_e+\varphi_f)}+e^{i(\varphi_a+\tilde{\varphi}_e+\varphi_f)}|^2 \\
&\approx\frac{1}{9}[1-2\mathrm{Im}(\varphi_a-\varphi_b+\varphi_c-\varphi_e+\tilde{\varphi}_e)].
\end{aligned} \tag{8}$$

In this "weak trace" experiment, the amplitudes of mirror vibrations must be tiny to avoid the influence on the interference of the arms of nested MZIs. For this reason, the complex phases are all very small, and we can only keep the linear terms of the complex phases and drop all of the higher terms. The approximation $e^{\varphi}\approx 1+\varphi\ (|\varphi|\ll 1)$ has been utilized in Eq. (8). The tilt angles of laser rays are also very small, and thus we have $\mathrm{Im}\,\tilde{\varphi}_e=-\mathrm{Im}\,\varphi_e$ and substitute $\tilde{\varphi}_e=\varphi_e^*$, into Eq. (8),

$$P_D=\frac{1}{9}[1-2\mathrm{Im}(\varphi_a-\varphi_b+\varphi_c-2\varphi_e)]. \tag{9}$$

Equation (9) contains the complex phases φ_a, φ_b, φ_c, and φ_e, except for φ_f, which means that the signals of mirrors A, B, C, and E are detected by the detector D, while that of the mirror F is not. The strength of the mirror E signal is two times as strong as the others. This result agrees with Ref. [30] exactly. To go back to the original result in Ref. [1], the Dove prisms should be removed, and we can simply replace $\tilde{\varphi}_e$ by φ_e in Eq. (8). Then, Eq. (9) reverts back to $P_D=\frac{1}{9}[1-2\mathrm{Im}(\varphi_a-\varphi_b+\varphi_c)]$, which is exactly the result introduced by Ref. [1]. Using the three-path interference method, we can derive naturally and simply the results, which agree with that introduced by Refs. [1,30] very well.

Then, we discuss the second situation, in which the outcome is even more interesting. If we let "all the photons end up in detector"[1], each of the three paths is constructively interfering with the other two, and the relative phases should be $\phi=-\frac{\pi}{2},\chi=\pi$[29]. Substituting the relative phases into the general expression of Eq. (7) yields

$$\begin{aligned}
P_D&=\frac{1}{9}|-ie^{i\varphi_c}-e^{i(\varphi_b+\varphi_e+\varphi_f)}-ie^{i(\varphi_a+\tilde{\varphi}_e+\varphi_f)}|^2 \\
&\approx\frac{1}{9}[9-6\mathrm{Im}(\varphi_a+\varphi_b+\varphi_c+\varphi_e+\tilde{\varphi}_e+2\varphi_f)].
\end{aligned} \tag{10}$$

Same as above, by substituting $\tilde{\varphi}_e=\varphi_e^*$, into Eq. (10), we have

$$P_D=\frac{1}{9}[9-6\mathrm{Im}(\varphi_a+\varphi_b+\varphi_c+2\varphi_f)]\approx 1. \tag{11}$$

In contrast to Eq. (9), Eq. (11) contains φ_f but no φ_e! This means that the photodetector receives the signal of the mirror F, but not that of the mirror E. Equation (11) can also be reverted back to the original outcome introduced by Ref. [1]. If the Dove prisms are removed, we can simply replace $\tilde{\varphi}_e$ in Eq. (10) by φ_e and get

$$P_D=\frac{1}{9}[9-6\mathrm{Im}(\varphi_a+\varphi_b+\varphi_c+2\varphi_e+2\varphi_f)], \tag{12}$$

which agrees exactly with the outcome given by Ref. [1].

The result given by Eq. (11) is counterintuitive. In the condition that the nested MZI has been aligned to make "all the photons end up in detector"[1], our derivation gives out $P_D\approx 1$, which means that the photons being directed into the nested MZI must have passed through all three paths. In other words, the photons must pass through the arm of the outer MZI nested with the inner MZI. To go through this nested arm, the photons must be reflected off mirror E, and the possibility of finding a photon near mirror E is 2/3 according to Eq. (1). Surprisingly, according to Eq. (11), the signal of mirror E is absent from the output. That is to say, photons hide the trace that they have passed through mirror E. Even if the signal relating to a certain mirror is absent from the output, we cannot judge that the photon has not been there.

Equations (10) and (11) are both comprehensible from the viewpoint of three-path interference. The famous Young's double-slit experiment indicates that the photons pass two slits simultaneously, and the same is true for a three-path interferometer case. No matter what condition the nested MZI is in, the photons emitted by the photon source have three possible paths to go through before reaching detector D, and they go past all of these three paths simultaneously.

When paths I and II are in destructive interference, the inner MZI of the nested interferometer seems to be "blocked", and, according to Eq. (9), $P_D \approx 1/9$, which agrees with the passing rate of path III. But paths I and II are not really blocked, they still interfere with path III, and the signals of mirrors A and B are brought to the detector; that is why we have φ_a, φ_b, and φ_c in Eq. (9). Because paths I and II are in destructive interference, the signs of complex phases related to these two paths are inverse. Without the Dove prisms, as φ_e and φ_f relating to both paths I and II, these two complex phases will cancel each other. But in the presence of the Dove prisms, the imaginary part of φ_e relating to path I is reversed ($\varphi_e \to \tilde{\varphi}_e$), so only φ_f is canceled, while the imaginary part of φ_e remains unchanged. On the other hand, if we indeed block the nested arm of the outer interferometer with a non-transparent plate, for example, put it near mirror E or F, then paths I and II are truly cut off. As a result, the three-path interference will fail, and all of the complex phases except φ_c will disappear from the output.

When paths I, II, and III are all in-phase, the signs of complex phases relating to the three paths are all the same. Without the Dove prisms, as φ_e and φ_f relate to both paths I and II, the constructive interference between them makes the signals of mirrors E and F twice the strength of that of mirrors A, B, and C. This is exactly the outcome introduced by Ref. [1]. But, after the Dove prisms are placed, as the imaginary part of φ_e relating to path I is reversed, it cancels out φ_e relating to path II, but φ_f is not affected by the prisms. That is exactly what we see in Eq. (11).

3. Conclusion

We have revisited the nested MZI modified with Dove prisms and give out three-path interference interpretation of the outcomes. All three possible paths, which begin at a light source and end up in the detector, do contribute to the final outcomes. The presence or absence of the mirror signals is decided by the interference of these paths. The presence of the mirror signals is a sufficient but not necessary condition for the fact that the photon has been reflected off the corresponding mirrors. Our three-path interference interpretation can explain the surprising experiment outcomes introduced by those previous works in a simple and natural way. Most interestingly, even if the photon has been reflected by all mirrors, the final output may not contain all of the signals of these mirrors in certain conditions. Thus, we conclude that the photons may hide where they have been.

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