The von Kries chromatic adaptation transform and its generalization

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Most viable modern chromatic adaptation transforms (CATs), such as CAT16 and CAT02, can trace their roots both conceptually and mathematically to a simple model formulated from the hypotheses of Johannes von Kries in 1902, known as the von Kries transform/model. However, while the von Kries transform satisfies the properties of symmetry and transitivity, most modern CATs do not satisfy these two important properties. In this Letter, we propose a generalized von Kries transform, which satisfies the symmetry and transitivity properties in addition to improving the fit to most available experimental visual datasets on corresponding colors.

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Our visual system attracts many researchers including medical, vision, optical, psychological, and color scientists. For clinical applications[1,2], medical, vision, and optical scientists investigated image formation by our eyes to correct dysfunctions such as astigmatism, presbyopia, and myopia. On a functional level, vision and color scientists try to understand how we generate color sensations from received images and try to model phenomena such as color difference[3], chromatic adaptation[4–6], and color appearance[7–9]. In this Letter, we model chromatic adaptation in order to predict corresponding colors when the predominant viewing environment is changed.

A chromatic adaptation transform (CAT) is capable of predicting corresponding colors. A pair of corresponding colors consists of a color observed under one illuminant (say, D65) and another color that has the same appearance when observed under a different illuminant (say, A). CATs are part of color appearance models (CAMs)[7–9], which are important for many industrial applications. These transforms have been extensively studied over several decades ever since Johannes von Kries[10] in 1902 laid down the foundation for modeling chromatic adaptation. Rather than giving a specific set of equations for the modeling, he instead simply outlined his hypothesis in words and described the potential impact of his ideas. Based on his hypothesis, chromatic adaptation in the visual system is considered the independent change in responsivity of the three types of cone photoreceptors. To present the von Kries hypothesis in terms of a chromatic adaptation model, we need a 3 by 3 matrix M, which transforms the tristimulus values (TSVs) $X_\beta, Y_\beta, Z_\beta$ under an

illuminant called β into the cone-like or sharper sensor spaces (R, G, B or L, M, S spaces). Here, we will use the R, G, B notation. Thus, we have

$$\begin{pmatrix} R_\beta \\ G_\beta \\ B_\beta \end{pmatrix} = M \begin{pmatrix} X_\beta \\ Y_\beta \\ Z_\beta \end{pmatrix}, \qquad (1)$$

where the matrix M can be the well-known HPE matrix[7], the CAT02 matrix[8], or the CAT16 matrix[9]. The entire chromatic adaptation is completed in the R, G, B space. The signals $R_\beta, G_\beta, B_\beta$ are considered to be the initial cone responses. According to the von Kries proportionality law, the von Kries post-adaptation signals $R_{a,\beta}, G_{a,\beta}, B_{a,\beta}$ are given by

$$\begin{pmatrix} R_{a,\beta} \\ G_{a,\beta} \\ B_{a,\beta} \end{pmatrix} = \begin{pmatrix} k_{R,\beta} R_\beta \\ k_{G,\beta} G_\beta \\ k_{B,\beta} B_\beta \end{pmatrix}, \qquad (2)$$

where the subscript a signifies adaptation, β represents the illuminant, and R, G, B indicate different channels. The von Kries adaptation factors or coefficients $k_{R,\beta}, k_{G,\beta}, k_{B,\beta}$ are independent of each other and are given by

$$k_{R,\beta} = \frac{1}{R_{w,\beta}}, \quad k_{G,\beta} = \frac{1}{G_{w,\beta}}, \quad k_{B,\beta} = \frac{1}{B_{w,\beta}}, \qquad (3)$$

where the subscript w signifies the sensor space signals transformed from the TSV of the illuminant β white point,

$$\begin{pmatrix} R_{w,\beta} \\ G_{w,\beta} \\ B_{w,\beta} \end{pmatrix} = M \begin{pmatrix} X_{w,\beta} \\ Y_{w,\beta} \\ Z_{w,\beta} \end{pmatrix}, \tag{4}$$

and $X_{w,\beta}$, $Y_{w,\beta}$, $Z_{w,\beta}$ are the TSVs of the illuminant β white point. Thus, if two stimuli s_β and s_δ are viewed under illuminants β and δ, respectively, and they are perceived with the same appearance, then we must have

$$\begin{pmatrix} R_{a,\beta} \\ G_{a,\beta} \\ B_{a,\beta} \end{pmatrix} = \begin{pmatrix} R_{a,\delta} \\ G_{a,\delta} \\ B_{a,\delta} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} k_{R,\beta}R_\beta \\ k_{G,\beta}G_\beta \\ k_{B,\beta}B_\beta \end{pmatrix} = \begin{pmatrix} k_{R,\delta}R_\delta \\ k_{G,\delta}G_\delta \\ k_{B,\delta}B_\delta \end{pmatrix}. \tag{5}$$

When Eq. (5) holds, the two stimuli are called corresponding colors.

Note that when we say stimulus s_β in the TSV space, we mean that s_β is a column vector formed by TSVs X_β, Y_β, Z_β. In this case, s_β can be written as $s_{XYZ,\beta}$. When we say stimulus s_β in the cone-like space, we mean that s_β is a column vector formed by cone sponse signals R_β, G_β, B_β obtained using Eq. (1). Similarly, in this case, s_β can be written as $s_{RGB,\beta}$. If we let diag (a, b, c) be a 3 by 3 diagonal matrix, the von Kries transform in the cone-like space, denoted by $\Gamma_{\delta,\beta}$, can be simply defined by

$$\Gamma_{\delta,\beta} = \text{diag}\begin{pmatrix} \dfrac{k_{R,\beta}}{k_{R,\delta}} & \dfrac{k_{G,\beta}}{k_{G,\delta}} & \dfrac{k_{B,\beta}}{k_{B,\delta}} \end{pmatrix}, \tag{6}$$

and the real von Kries transformation from stimulus s_β to stimulus s_δ is a simple matrix and vector multiplication:

$$s_{RGB,\delta} = \Gamma_{\delta,\beta}s_{RGB,\beta} \quad \text{or} \quad s_{XYZ,\delta} = M^{-1}\Gamma_{\delta,\beta}Ms_{XYZ,\beta}. \tag{7}$$

Note that the order of the symbols δ, β in the subscript of the von Kries transform $\Gamma_{\delta,\beta}$ is important. Here, δ, β mean that the von Kries transform maps stimulus s_β under illuminant β to stimulus s_δ under illuminant δ. Similarly, transform $\Gamma_{\beta,\delta}$ maps stimulus s_δ under illuminant δ to stimulus s_β under illuminant β. Note also that, if two stimuli s_β and s_δ are corresponding colors, then s_δ and s_β are also corresponding colors, with this property being called symmetry. Thus, we expect the von Kries transform to satisfy this property. In fact, it can be verified that

$$\Gamma_{\delta,\beta}\Gamma_{\beta,\delta} = I_3, \tag{8}$$

where I_3 is the 3×3 identity matrix. Equation (8) shows that the von Kries transform has the property of symmetry, as desired. Also, if s_β and s_δ are corresponding colors, and s_γ and s_δ are corresponding colors too, then s_γ and s_β must be corresponding colors, and this property is known as transitivity. Similarly, we also expect the von Kries transform to have transitivity. Fortunately, it is indeed the case, since

$$\Gamma_{\gamma,\delta}\Gamma_{\delta,\beta} = \Gamma_{\gamma,\beta}. \tag{9}$$

The von Kries transform can be further modified by introducing the modified von Kries adaptation factors:

$$k'_{R,\beta} = k_{R,\beta}q_{R,\beta}, \quad k'_{G,\beta} = k_{G,\beta}q_{G,\beta}, \quad k'_{B,\beta} = k_{B,\beta}q_{B,\beta}. \tag{10}$$

Based on the above new von Kries adaptation factors, we can have the modified von Kries transform, $\Gamma'_{\delta,\beta}$, which is defined by

$$\Gamma'_{\delta,\beta} = \text{diag}\begin{pmatrix} \dfrac{k'_{R,\beta}}{k'_{R,\delta}} & \dfrac{k'_{G,\beta}}{k'_{G,\delta}} & \dfrac{k'_{B,\beta}}{k'_{B,\delta}} \end{pmatrix}. \tag{11}$$

It can be shown that the modified von Kries transform also satisfies the symmetry and transitivity.

Note that if the scaling factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ in Eq. (10) are all equal to one, the modified von Kries transform becomes the classical von Kries transform, i.e., $\Gamma'_{\delta,\beta} = \Gamma_{\delta,\beta}$. In fact, by different choices of the scaling factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$, the modified von Kries adaptation factors become some available adaptation factors in the literatures, such as the Fairchild factors (see page 177 in Ref. [11]) with

$$q_{R,\beta} = p_{R,\beta}, \tag{12}$$

$$p_{R,\beta} = \frac{1 + (L_\beta)^{1/3} + r_{E,\beta}}{1 + (L_\beta)^{1/3} + 1/r_{E,\beta}}, \tag{13}$$

$$r_{E,\beta} = \frac{3R_{w,\beta}/R_E}{R_{w,\beta}/R_E + G_{w,\beta}/G_E + B_{w,\beta}/B_E}; \tag{14}$$

CMCCAT2000[12], CAT02[13], and CAT16[4,9] factors with

$$q_{R,\beta} = q_{G,\beta} = q_{B,\beta} = Y_{w,\beta}; \tag{15}$$

or Smet *et al.*[5,6] factors with

$$q_{R,\beta} = R_E, q_{G,\beta} = G_E, q_{B,\beta} = B_E. \tag{16}$$

Here, R_E, G_E, B_E are obtained using Eq. (1) with the TSVs of the equal energy illuminant white point. L_β is the luminance of the adapting field and is about 20% of the absolute luminance of the illuminant β. Thus, it can be seen the modified von Kries adaptation factors [Eq. (10)] are more general. We will discuss next how to choose the factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$. Firstly, they should satisfy

$$\frac{q_{R,\beta}}{q_{R,\delta}} = \frac{q_{G,\beta}}{q_{G,\delta}} = \frac{q_{B,\beta}}{q_{B,\delta}} = c; \tag{17}$$

in this case, it can be shown from Eq. (11) that

$$\Gamma'_{\delta,\beta} = c \cdot \text{diag}\begin{pmatrix} \dfrac{R_{w,\delta}}{R_{w,\beta}} & \dfrac{G_{w,\delta}}{G_{w,\beta}} & \dfrac{B_{w,\delta}}{B_{w,\beta}} \end{pmatrix} = c\,\Gamma_{\delta,\beta}. \tag{18}$$

Hence,

$$s_{XYZ,\delta} = M^{-1}\Gamma'_{\delta,\beta} M s_{XYZW,\beta} = c s_{XYZW,\delta}. \qquad (19)$$

Thus, if the CAT maps the TSVs of $X_{w,\beta}$, $Y_{w,\beta}$, $Z_{w,\beta}$, the output is $cX_{w,\delta}$, $cY_{w,\delta}$, $cZ_{w,\delta}$, which is correct as expected. The CAT should make the chromaticity correct as discussed in the Letter[14] by Hunt *et al.* However, one may think it is better if the CAT can make the luminance correct as well. It is clear the constant c must be one under this condition, which one may want if the input is $X_{w,\beta}$, $Y_{w,\beta}$, $Z_{w,\beta}$ and then the output is $X_{w,\delta}$, $Y_{w,\delta}$, $Z_{w,\delta}$. So, the next condition for factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ is

$$\frac{q_{R,\beta}}{q_{R,\delta}} = \frac{q_{G,\beta}}{q_{G,\delta}} = \frac{q_{B,\beta}}{q_{B,\delta}} = c = 1. \qquad (20)$$

The condition of Eq. (20) means factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ are independent of illuminant. From Eqs. (12)–(14), the Fairchild factors are illuminant dependent, and they may not satisfy conditions of Eqs. (17) and (20). Factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ defined by Eq. (15) are illuminant dependent, but they satisfy condition Eq. (17). They also satisfy condition Eq. (20) if $Y_{w,\beta} = Y_{w,\delta}$. Factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ defined by Eq. (16) use a fixed illuminant; hence, then they will satisfy both conditions of Eqs. (17) and (20).

Up to now, it seems that if the factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ satisfy the condition of Eq. (20), the modified von Kries transform is in fact the classical von Kries transform. Yes, it is the case. However, one will see the reason for introducing the modified von Kries transform.

However, neither the (classical) von Kries [see Eq. (6)] nor the modified von Kries transform [see Eq. 11)] with factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ given by any set of Eqs. (12)–(16) shows a tight fit with the experimental visual data sets on corresponding colors (see test results below). To solve this problem, researchers have proposed various linear and nonlinear extensions, as detailed by Fairchild[11]. The linear extensions related to the International Commission on Illumination (CIE) CAMs[8,9], such as CAT02[13] and CAT16[9], with factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ defined by Eq. (15), can be expressed as

$$\Gamma_{\delta,\beta,\text{CATxx}} = D_{\text{xx}}\Gamma'_{\delta,\beta} + \left(1 - D_{\text{xx}}\right)I_3, \qquad (21)$$

where xx in the subscript can be 02 for CAT02[13] and 16 for CAT16[9]; although, in fact, D_{02} and D_{16} are the same. The incomplete adaptation factor D_{xx} is between 0 and 1. When D_{xx} is one, $\Gamma_{\delta,\beta,\text{CATxx}}$ becomes $\Gamma'_{\delta,\beta}$ in such a way that CAT02 and CAT16 can be considered as extensions to the modified von Kries transform. However, when D_{xx} is different from 1 or 0, they no longer satisfy the symmetry and transitivity properties. That is, in general, $\Gamma_{\delta,\beta,\text{CATxx}}$ does not satisfy Eqs. (8) and (9). Hence, an inverse CAT is needed for $\Gamma_{\delta,\beta,\text{CATxx}}$, which is simple for linear CATs and is given by $(\Gamma_{\delta,\beta,\text{CATxx}})^{-1}$ mapping stimulus s_δ to

stimulus s_β. The CAT $\Gamma_{\delta,\beta,\text{CATxx}}$ is normally called a one-step CAT, which directly maps stimulus s_β to stimulus s_δ.

Recently, Smet *et al.*[5,6] [using factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ defined by Eq. (16)] and Li *et al.*[4] [using factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ defined by Eq. (15)] proposed two-step CATs via the intermediate CIE illuminant E, which is defined by the equi-energy spectrum (see Fig. 1 in Ref. [5]). Firstly, a one-step CAT such as $\Gamma_{E,\beta,\text{CATxx}}$ is applied to map stimulus s_β to stimulus s_E. In this stage, the adaptation to the illuminant β for our visual system is referred to as illuminant E. Similarly, for the adaptation to the illuminant δ in the second stage, the illuminant E is also used, and a one-step CAT $(\Gamma_{E,\delta,\text{CATxx}})^{-1}$ is applied to map stimulus s_E to stimulus s_δ. The end result is the two-step CAT, denoted by $\Pi_{\delta,\beta,2\text{Step}}$, defined by

$$\Pi_{\delta,\beta,2\text{Step}} = \left(\Gamma_{E,\delta,\text{CATxx}}\right)^{-1}\Gamma_{E,\beta,\text{CATxx}}. \qquad (22)$$

Note that the incomplete adaptation factor D_{xx} in each of the one-step CATs in Eq. (22) may be different. Fortunately, the two-step CAT satisfies the symmetry and transitivity properties[4]. Furthermore, the two-step CAT performs equally well or better than the one-step CAT in predicting the visual datasets on corresponding colors[4]. However, the derivation of the two-step CAT is debatable. Why does the adaptation for our visual system always refer to an illuminant (illuminant E) that does not exist in the real world? We should recall that the derivations of the von Kries and modified von Kries transforms do not need an intermediate illuminant.

Can we have a CAT that satisfies symmetry and transitivity without referring to an intermediate illuminant and fits the visual datasets as good as or better than the one-step CAT? The answer is yes. To this end, we have introduced the incomplete adaptation factor D into the modified von Kries adaptation factors rather than into the modified von Kries transform $\Gamma'_{\delta,\beta}$ [see Eq. (21)]. Thus, the new incomplete adaptation factors under illuminant β are

$$\begin{aligned}
k''_{R,\beta} &= D_\beta k'_{R,\beta} + (1 - D_\beta), \\
k''_{G,\beta} &= D_\beta k'_{G,\beta} + (1 - D_\beta), \qquad (23) \\
k''_{B,\beta} &= D_\beta k'_{B,\beta} + (1 - D_\beta).
\end{aligned}$$

The new incomplete adaptation factors under illuminant δ can be similarly defined. As with the derivation of the von Kries or the modified von Kries transform, we have a new CAT, called the generalized von Kries (GvK) transform, which is denoted as $\Gamma''_{\delta,\beta}$ and uses the new incomplete adaptation factors defined in Eq. (23). Thus, the GvK transform $\Gamma''_{\delta,\beta}$ is given by

$$\Gamma''_{\delta,\beta} = \text{diag}\left(\begin{matrix} \dfrac{k''_{R,\beta}}{k''_{R,\delta}} & \dfrac{k''_{G,\beta}}{k''_{G,\delta}} & \dfrac{k''_{B,\beta}}{k''_{B,\delta}} \end{matrix}\right). \qquad (24)$$

It can be shown that $\Gamma''_{\delta,\beta}$ satisfies Eqs. (8) and (9). Thus, the GvK transform indeed satisfies the properties of symmetry and transitivity.

Note that the GvK transform has two adaptation factors, D_β and D_δ. The D_β factor in CAT02 and CAT16 depends only on the luminance level of illuminant β, and hence, D_β and D_δ are the same if the luminance levels of the two illuminants are the same. Recently, several papers[5,6,15] have reported that the D factor affects the performance of CATs and have guessed that the D factor may also depend on correlated color temperature (CCT).

Note also that when we consider the von Kries, modified von Kries, and GvK transforms in TSV space, an associated matrix M mapping the stimulus in TSV to the cone-like space [see Eq. (1)] is necessary. For example, as noted before, the von Kries transform in TSV space is simply given by $(M^{-1}\Gamma_{\delta,\beta}M)$, where the matrix M can be the CAT02, CAT16, or HPE matrices.

Before we test the performance of the GvK transform, we need also to specify the factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ in Eq. (10). According to the discussion above, it is better that they are independent of illuminant. A simple choice is

$$q_{R,\beta} = q_{G,\beta} = q_{B,\beta} = c_2. \tag{25}$$

Here, c_2 is a constant again. According to Eqs. (23) and (24), performance of the GvK transform is also dependent on c_2. For the testing below, c_2 is set to be 100. The reason for it is explained below.

Performance of the proposed von Kries transform $\Gamma''_{\delta,\beta}$ with the CAT02, CAT16, and HPE matrices has been tested using the available corresponding color datasets[16–22], which were used for developing CAT02 and CAT16. The formula employed for the D factor was the one used for CAT02 and CAT16. Comparisons with the von Kries transform $\Gamma_{\delta,\beta}$, (one-step) CAT02, and CAT16 were also made. The results found are summarized in Table 1, in terms of mean, weighted mean, minimum (Min), and maximum (Max) CIELAB color differences between the predicted and experimental TSVs for each pair of corresponding colors in datasets. There are 21 datasets, and each dataset has a different number of pairs of corresponding colors. The values in the row of Table 1 labeled with Mean are the average of the mean color differences

for the different datasets, while the values in the row labeled with Weighted Mean are the weighted mean color difference; the weight for each dataset is the ratio of the number of pairs in this dataset and the number of pairs in all the datasets. The values in the second to last row are the Max of Max color differences for the different datasets. The values in the last row are the Min of Min color differences for the different datasets. The lower the values in Table 1, the better the performance of the corresponding model.

First, Table 1 indicates that when using any of the three matrices, the proposed GvK transform (see results under columns $\Gamma''_{\delta,\beta}$) is better than the von Kries transform (see results under columns $\Gamma_{\delta,\beta}$). Note the von Kries and modified von Kries transforms are the same since factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ satisfy Eq. (25). Second, the proposed GvK transform is equally well as or better than the (one-step) CAT02 with one exception being under Min measure with negligible 0.1 color difference unit (see results under column CAT02) and (one-step) CAT16 (see results under column CAT16). Third, both the von Kries and the proposed GvK transforms perform best using the CAT02 matrix, second best using the CAT16 matrix, and worst using the HPE matrix. However, we should note that the CAT02 matrix has the "yellow–blue" and "purple" problems[23–25]. The CAT16 matrix[9,26] was derived for the aim of fitting visual datasets and overcoming the "yellow–blue" and "purple" problems. Therefore, we recommend that the CAT16 matrix should be used for the von Kries, modified von Kries, and proposed GvK transforms.

Note that if the D factors D_β and D_δ are set to one, any c_2 value does not affect the performance of the GvK transform. In fact, in this case, it is simply the von Kries transform, i.e., $\Gamma''_{\delta,\beta} = \Gamma'_{\delta,\beta} = \Gamma_{\delta,\beta}$. However, when we use the D factor to be the D factor of CAT02 (CAT16), the c_2 value indeed affects the performance of the GvK transform. It was found that when $c_2 = 100$, the GvK transform performs the best; when c_2 deviates from 100, the GvK transform performs worse. This may come from three facts. First, all visual datasets tested here, $Y_w = 100$. Second, the Y_w factor was introduced into the CAT02, CAT16, and CMCCAT2000 for being consistent with the nonlinear CMCCAT97[27], which was built in CIECAM97s[7]. As discussed above, the Y_w factor is just related to the scaling

Table 1. Performance of von Kries ($\Gamma_{\delta,\beta}$) and Generalized von Kries Transforms ($\Gamma''_{\delta,\beta}$) Together with the (One-Step) CAT02 and the (One-Step) CAT16, in Terms of Mean, Weighted Mean, Maximum (Max), and Minimum (Min) CIELAB Color Differences from Visual Datasets on Corresponding Colors[16–22]

	CAT02 Matrix			CAT16 Matrix			HPE Matrix	
	$\Gamma_{\delta,\beta}$	CAT02	$\Gamma''_{\delta,\beta}$	$\Gamma_{\delta,\beta}$	CAT16	$\Gamma''_{\delta,\beta}$	$\Gamma_{\delta,\beta}$	$\Gamma''_{\delta,\beta}$
Mean	7.6	6.3	6.2	8.1	6.3	6.2	9.4	7.4
Weighted Mean	6.4	5.5	5.5	6.9	5.6	5.6	8.1	6.7
Max	34.2	26.7	24.4	39.9	27.0	25.5	50.0	35.5
Min	0.5	0.3	0.4	0.3	0.3	0.3	0.4	0.3

factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ [see Eqs. (10), (15)]. Finally, the matrix and D factor of CAT02/CAT16 were derived based on fitting all the visual datasets[16–22] as best as possible. Hence, it is recommended to use the $c_2 = 100$ together with the D factor of the CAT02/CAT16 for the GvK transform before a better D factor is developed.

When one of the two illuminants is illuminant E, the GvK transform $\Gamma''_{\delta,\beta}$ becomes a one-step CAT [see Eq. (21)] with $c_2 = Y_w = 100$. In fact, since the CAT02, CAT16, and HPE matrices are normalized according to the illuminant E, we have

$$R_{w,E} = G_{w,E} = B_{w,E} = X_{w,E} = Y_{w,E} = Z_{w,E} = 100. \tag{26}$$

Thus, from Eqs. (3), (10), and (14), we have

$$k'_{R,E} = k'_{G,E} = k'_{B,E} = k''_{R,E} = k''_{G,E} = k''_{B,E} = 1. \tag{27}$$

Therefore, from Eq. (15), considering also Eqs. (11), (14), and (17),

$$
\begin{aligned}
\Gamma''_{E,\beta} &= \mathrm{diag}\left(\frac{k''_{R,\beta}}{k''_{R,E}} \quad \frac{k''_{G,\beta}}{k''_{G,E}} \quad \frac{k''_{B,\beta}}{k''_{B,E}} \right) \\
&= \mathrm{diag}\left(k''_{R,\beta} \quad k''_{G,\beta} \quad k''_{B,\beta} \right) \\
&= D_\beta \mathrm{diag}\left(k'_{R,\beta} \quad k'_{G,\beta} \quad k'_{B,\beta} \right) + (1 - D_\beta)I_3 \\
&= D_\beta \Gamma'_{E,\beta} + (1 - D_\beta)I_3.
\end{aligned}
$$

Hence, if we let $D_\beta = D_{xx}$ in Eq. (12), we have

$$\Gamma''_{E,\beta} = \Gamma_{E,\beta,\mathrm{CATxx}}. \tag{28}$$

Equation (28) means that the proposed GvK transform mapping stimulus under illuminant β to stimulus under illuminant E is just the normal one-step CAT from stimulus under illuminant β to stimulus under illuminant E. Remember that the one-step CAT $\Gamma_{E,\beta,\mathrm{CATxx}}$ is used in the forward mode in CIECAM02/CAM16. Similarly, it can be proved that

$$\Gamma''_{\delta,E} = \left(\Gamma_{E,\delta,\mathrm{CATxx}} \right)^{-1}. \tag{29}$$

Equation (29) means that the GvK transform mapping stimulus under illuminant E to stimulus under illuminant δ is just the inverse of the normal one-step CAT from the stimulus under illuminant δ to the stimulus under illuminant E. We should bear in mind that the inverse of one-step CAT $\Gamma_{E,\delta,\mathrm{CATxx}}$ is used in the inverse mode in CIECAM02/CAM16. Therefore, we conclude that the proposed GvK transform can be used in the current CIECAM02/CAM16.

In conclusion, the von Kries transform was reviewed, and then the modified von Kries transform was derived based on the modified von Kries adaptation coefficients [see Eq. (10)]. The factors $q_{R,\beta}$, $q_{G,\beta}$, $q_{B,\beta}$ in Eq. (10) were shown to be better if they satisfy condition Eq. (20),

resulting in it being better if the Y_w factor in CAT02, CAT16, and CMCCAT2000 is a constant of 100. The Y_w factor was introduced into CMCCAT2000 first, later into CAT02 and CAT16 to be consistent with CIECAM97s and CMCCAT97, and further justification was discussed in the Letter given by Hunt et al.[14]. Since 2000, there was a debate about the Y_w factor. There is nothing wrong with a CAT including the Y_w factor like CMCCAT2000 and CAT02, since the main purpose of a CAT should make the chromaticity correct. However, when the Y_w factor is fixed to a constant like 100, the CAT can make both chromaticity and luminance correct.

It was found that the current linear CAT02 and CAT16 can be considered to be the extension of the modified von Kries transform. However, while the von Kries and modified von Kries transforms satisfy symmetry and transitivity, CAT02 and CAT16 do not satisfy these two properties in general. Finally, a GvK transform has been proposed. The proposed GvK transform, similar to the von Kries and modified von Kries transforms, satisfies the symmetry and transitivity of the properties. Performance evaluation using the available visual datasets[16–22] showed that the proposed GvK transform performs better than the von Kries and modified von Kries transforms and performs equally well as or better than the (one-step) CAT02 and CAT16. Furthermore, the proposed GvK transform does not need an inverse transform and can be used in CIECAM02/CAM16.

Finally, we note that recently Kerouh et al.[28] used a CAT to convert the (input) image of a scene captured under one illuminant to the (output) image of the same scene captured under another illuminant. Their results have shown that the CAT affects image content such as edges, texture, and homogeneous area differently. Image-content-based CATs were developed. Comparison results based on multispectral images have shown that the image-content-based CATs perform better than other CATs including the von Kries and Bradford transforms. Our proposed GvK model is evaluated here using the visual corresponding color datasets and may be further evaluated in the future using image data.

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