# Design method for freeform reflective－imaging systems with low surface－figure－error sensitivity 

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#### Abstract

Freeform surfaces are difficult to manufacture due to their lack of rotational symmetry．To reduce the requirements for manufacturing precision，a design method is proposed for freeform reflective－imaging systems with low surface－ figure－error sensitivity．The method considers both the surface－figure－error sensitivity and optical specifications， which can design initial systems insensitive to surface figure errors．Design starts with an initial planar system；the surface－figure－error sensitivity of the system is reduced during construction．The proposed method and another that is irrelevant to figure－error sensitivity are used to design a freeform off－axis three－mirror imaging system． Comparison of the sensitivities of the two systems indicates the superiority of our proposed method．


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Tolerance analysis is an essential part in optical system design $[1,2]$ ．Surface－figure－error sensitivity is the perfor－ mance degradation of the system caused by the surface figure errors，which reflects the narrowness of surface tol－ erance and can evaluate the difficulty of manufacturing the system．

Freeform off－axis reflective－imaging systems have numer－ ous advantages．First，compared with refractive systems， reflective ones have a wider working－wavelength range without chromatic aberrations ${ }^{[3-6]}$ ．Secondly，in comparison with coaxial reflective systems，off－axis reflective ones can enlarge fields of view（FOVs）and eliminate obscuration ${ }^{[6,7]}$ ． Thirdly，freeform surfaces have more degrees of freedom to reduce the asymmetric aberrations to achieve high－ performance imaging systems ${ }^{[8-13]}$ ．However，they are not rotationally symmetric，which increases the difficulty of manufacturing ${ }^{[6]}$ ．Therefore，designing freeform reflective systems with lower surface－figure－error sensitivity is impor－ tant for reducing manufacturing－precision requirements and expanding the application of freeform surfaces．

In the traditional design method for freeform imaging systems，a spherical or aspherical initial system with similar performance to the final design goals is first selected．The freeform surfaces are obtained by optimization－${ }^{[9]}$ However， a limited number of initial systems are available．Recently some direct design methods have been developed，including the partial－differential－equation $(\mathrm{PDE})^{[14,15]}$ method，the simultaneous－multiple－surface（SMS）${ }^{[16,17]}$ method，and the construction－iteration（CI－3D）$)^{[18-21]}$ method．However，these methods have not considered surface－figure－error sensitivity while solving the initial system．If the tolerance is tight， designers must make modifications，dramatically reducing design efficiency．

In this Letter，a design method for freeform reflective－ imaging systems with low surface－figure－error sensitivity is proposed．The proposed method considers reducing
surface－figure－error sensitivity while solving the initial sys－ tem．It is well known that the design of freeform optical systems lacks initial systems and has much fewer insensi－ tive initial systems．A good initial system can be easily op－ timized for achieving good design results，which could improve design efficiency significantly．Design starts with an initial planar system，in which the minimum angle of incidence（AOI）on each surface is given．In the construc－ tion process，the system sensitivity is reduced by control－ ling the minimum AOI on each surface by rotating the mirrors and shifting image points with both reverse and forward ray tracing．Two freeform off－axis three－mirror imaging systems are designed－one using the proposed method and one using another design method．The two designs operate at $F / 1.5$ with a 100 mm focal length and a $3^{\circ} \times 4^{\circ}$ FOV．The root－mean－square（RMS）wave－ front errors（RMSWFEs）of the two systems are below $0.02 \lambda$ at $10 \mu \mathrm{~m}$ ．The surface－figure－error sensitivity of the two systems is analyzed．The results illustrate that the proposed method is effective in designing freeform reflective－imaging systems with low surface－figure－error sensitivity．
To elaborate on the proposed method in this Letter， some related concepts are first introduced．

We use the Cartesian coordinate system（ $X Y Z$ ）to de－ scribe the geometry of the optical system．The projections of ray $\boldsymbol{R}$ of field $\boldsymbol{\psi}$ in the $X O Z$ and $Y O Z$ planes are $\boldsymbol{R}_{x}$ and $\boldsymbol{R}_{y}$ ，respectively．The field $\boldsymbol{\psi}$ is depicted as $\boldsymbol{\psi}=\left(\varphi_{x}, \varphi_{y}\right)$ in this Letter．The angles $\varphi_{x}, \varphi_{y}$ ，and $\varphi$ are rotating from the $Z$ axis to rays $\boldsymbol{R}_{x}, \boldsymbol{R}_{y}$ ，and $\boldsymbol{R}$ ．Note that all angles in this Letter are measured with an acute angle and are negative if rotating clockwise．

The construction（C－3D）method ${ }^{[18]}$ is a direct design method for freeform imaging systems．To ensure the object－image relationships，all data points $P_{s, m}^{(k)}$ on the unknown surface are first calculated point by point，
obeying the nearest ray principle, Fermat's laws, and Snell's law $-\underline{[18-20]}$. The subscripts $s$ and $m$ indicate the serial numbers of the surface and the field, respectively. The superscript $k$ indicates the serial number of the ray of field $\boldsymbol{\psi}_{m}$. Then, these data points are fitted into freeform surfaces, considering both the coordinates and the normal ${ }^{[21]}$.

Forward ray tracing is a process by which rays are traced from the object space to the image space. Reverse ray tracing is based upon the reversibility of ray paths: the rays are traced from the image plane to the object space ${ }^{[22]}$.

Figure 1 illustrates the relationship between the change in the wavefront errors ( $\Delta \mathrm{WE}$ ) caused by surface perturbations and AOIs upon the reflecting surfaces ${ }^{[23]}$. The curves $P Q$ and $P^{\prime} Q^{\prime}$ represent the designed reflecting surface and the perturbed surface, respectively. A ray, $\boldsymbol{L}_{i}$, is incident upon point $B$ on curve $P Q$. When a surface perturbation is applied to the designed surface, ray $\boldsymbol{L}_{i}$ is incident at point $B^{\prime}$ on curve $P^{\prime} Q^{\prime} . B B^{\prime}$ is the surface displacement along ray $\boldsymbol{L}_{i}$, and $B H$ is the displacement along the designed reflecting surface normal at point $B$.

Since the surface figure error is small, $B H$ is assumed to be perpendicular to $B^{\prime} H$. The change in the optical path length of ray $\boldsymbol{L}_{i}$ caused by surface perturbation is calculated by Eq. (1):

$$
\begin{align*}
\Delta \mathrm{WE} & =B B^{\prime}-B G=B B^{\prime}[1-\cos (\pi-2 \theta)] \\
& =2 B B^{\prime} \cos ^{2} \theta \approx 2 B H \cos \theta \tag{1}
\end{align*}
$$

where AOIs $\theta$ are the angles rotating from the normal to the rays. Equation (1) indicates that enlarging the absolute values of the AOIs on reflective surfaces can diminish $\Delta W E$, that is, reduce the system's sensitivity to surface perturbation.

The proposed design method for freeform reflectiveimaging systems with low surface-figure-error sensitivity reduces the system's sensitivity by controlling the minimum value of the absolute value of AOI (min $|\mathrm{AOI}|$ ) on each surface. Taking freeform off-axis three-mirror imaging systems as an example, this method is divided into five steps: (1) establish the initial planar system with expected AOIs on each plane; (2) construct freeform surfaces using the C-3D method (however, the AOIs on each surface are changed after construction); (3) rotate and reconstruct the primary mirror (PM) via reverse ray tracing to ensure the $\min |\mathrm{AOI}|$ on the PM is greater than the expected AOI; (4) rotate and reconstruct the secondary mirror (SM) via


Fig. 1. Optical-path-length change caused by figure errors.
forward ray tracing to ensure the min $|\mathrm{AOI}|$ on the SM is greater than the expected AOI; (5) shift image points and reconstruct the tertiary mirror (TM) via forward ray tracing to ensure the min $|\mathrm{AOI}|$ on the TM is greater than the expected AOI.

In step (1), the AOIs on each flat mirror are considered when establishing the initial planar system. Biased fields and tilted planes are used to eliminate obscuration. Assume that the system is symmetrical about the YOZ plane. Therefore, the FOV of the system is biased in the tangential direction, and the planes are tilted in the YOZ plane.

With the FOV fixed, changing the biased field or tilt angles of mirrors can modify the initial AOI on each plane, as shown in Fig. 2. Ray $\boldsymbol{R}$ of field $\boldsymbol{\psi}=\left(\varphi_{x}, \varphi_{y}\right)$ is successively reflected by PM, SM, and TM. The AOIs on each plane are calculated by Eq. (2):

$$
\left\{\begin{array}{l}
-\theta_{1}=-\varphi+\alpha_{1}  \tag{2}\\
\theta_{2}=-\varphi+2 \alpha_{1}-\alpha_{2} \\
-\theta_{3}=-\varphi+2 \alpha_{1}-2 \alpha_{2}+\alpha_{3}
\end{array}\right.
$$

where angles $\alpha_{i}(i=1,2,3$, representing PM, SM, and TM, respectively) are the tilt angles of these mirrors (rotating from the $+Y$ axis to the plane $)$, angles $\theta_{i}(i=1,2$, 3) are the minimum AOI on each mirror (rotating from the normal to the rays), and $\varphi$ is the angle rotating from the $Z$ axis to ray $\boldsymbol{R}$.

To reduce system sensitivity, an initial planar system is established, in which the min $|\mathrm{AOI}|$ on each mirror is greater than the expected angles $w_{i}(i=1,2,3)$, respectively. Decanters of all mirrors are adjusted to eliminate obscuration.

In step (2), taking the initial planar system established in step (1) as input, a freeform surfaces system is designed using the C-3D method ${ }^{[18]}$ with forward ray tracing to ensure that parallel rays of each object field $\boldsymbol{\psi}_{m}=\left(\varphi_{x}^{(m)}, \varphi_{y}^{(m)}\right)$ can image at the corresponding ideal image point $T_{m}$. Here, $m$ indicates the serial number of each field. In the surface construction process, feature rays defined by the polar-ray grids


Fig. 2. Establish initial planar system.
are sampled ${ }^{[18]}$. After construction, the $\min |\mathrm{AOI}|$ on each surface $\left|\theta_{i}\right|$ can be less than the expected angles $w_{i}$. In the next steps, angles $\left|\theta_{i}\right|$ are increased successively.

In step (3), the $\min |\mathrm{AOI}|$ on the $\mathrm{PM},\left|\theta_{1}\right|$, is increased to be greater than angle $w_{1}$ by rotating the PM and resetting the biased-object fields. The PM is then reconstructed using the C-3D method with reverse ray tracing.

If the rays of the system are traced in forward ray tracing, as shown in Fig. 3(a), the rotation of the PM can affect the AOIs on the mirrors behind it. To keep the AOIs on SM and TM unchanged, the PM is reconstructed via reverse ray tracing following its rotation. As shown in Fig. 3(b), in the reverse ray tracing process, the rays traced from the image plane arrive at the PM after being reflected by the SM and TM. Therefore, the change of the PM does not affect the AOIs on the SM and TM.

For the freeform surfaces system obtained in step (2), the positions and shapes of the SM and TM are maintained while the PM is degraded into a plane whose position is the same as that of the PM in the initial planar system. The plane PM is then rotated around the $X$ axis to increase the angle $\left|\theta_{1}\right|$.

Following rotation of the plane PM, the directions of the reflected rays from the PM are changed, and consequently, the biased-object fields are changed. That is, the FOV of the system remains unchanged, but the angles between the chief ray of each field and the $Z$ axis are changed. Suppose that each object field $\boldsymbol{\psi}_{m}=$ $\left(\varphi_{x}^{(m)}, \varphi_{y}^{(m)}\right)$ corresponding to image point $T_{m}$ becomes $\boldsymbol{\psi}_{m}^{\prime}=\left(\varphi_{x}^{(m) \prime}, \varphi_{y}^{(m) \prime}\right)$ following rotation of the PM. When the PM rotates an angle of $\gamma$ around the $X$ axis, the rays of the object field rotate by an angle of $2 \gamma$. The new object field $\boldsymbol{\psi}_{m}^{\prime}=\left(\varphi_{x}^{(m) \prime}, \varphi_{y}^{(m) \prime}\right)$ after the rotation can be calculated by Eq. (3):

$$
\begin{equation*}
\varphi_{x}^{(m) \prime}=\varphi_{x}^{(m)}, \varphi_{y}^{(m) \prime}=\varphi_{y}^{(m)} \pm 2 \gamma \quad m=0,1, \ldots, M-1 \tag{3}
\end{equation*}
$$

After each new object field $\boldsymbol{\psi}_{m}^{\prime}$ is obtained, we find the vector $\boldsymbol{r}_{1, m}^{(k)}$ along the direction of the reflected ray at data point $P_{1, m}^{(k)}$ on the PM following its rotation. The direction


Fig. 3. (a) Forward ray tracing; (b) reverse ray tracing.
of the vector $\boldsymbol{r}_{1, m}^{(k)}$ corresponds to that of the parallel rays of the new object field $\boldsymbol{\psi}_{m}^{\prime}$. The vector $\boldsymbol{r}_{1, m}^{(k)}$ along the direction of the incident ray at point $P_{1, m}^{(k)}$ is obtained through reverse ray tracing. After the vectors $\boldsymbol{r}_{1, m}^{(k) \prime}$ and $\boldsymbol{r}_{1, m}^{(k)}$ are obtained, the surface normal vector $\boldsymbol{N}_{1, m}^{(k)}$ at point $P_{1, m}^{(k)}$ can be calculated, and the PM is reconstructed using the C-3D method via reverse ray tracing.

The process of rotating the plane PM, changing the biased-object fields, and reconstructing the PM is repeated until the angle $\left|\theta_{1}\right|$ is greater than the angle $w_{1}$.

In step (4), the SM is rotated to ensure that the min $|\mathrm{AOI}|$ on it, $\left|\theta_{2}\right|$, is greater than the expected angle $w_{2}$. To keep the AOIs on the PM unchanged, the SM is then reconstructed using the C-3D method with forward ray tracing.

In the system obtained in step (3), only the SM is degraded into a plane whose position is the same as that of the SM in the initial planer system. The plane SM is then rotated around the $X$ axis to increase angle $\left|\theta_{2}\right|$. Taking the rotated planar SM as input, the freeform surface SM is finally reconstructed using the C-3D method via forward ray tracing. The process of rotating the plane SM and reconstructing the SM is repeated until the angle $\left|\theta_{2}\right|$ is greater than the expected angle $w_{2}$.

The change of the SM seriously affects the min $|\mathrm{AOI}|$ on the TM $\left|\theta_{3}\right|$. Therefore, in step (5), we need to increase $\left|\theta_{3}\right|$. The image points are first shifted along the image plane to ensure that $\left|\theta_{3}\right|$ is greater than the expected angle $w_{3}$. To keep the AOIs on the PM and SM unchanged, the TM is then reconstructed using the C-3D method with forward ray tracing.

Image points are first shifted along the image plane according to the difference between $\left|\theta_{3}\right|$ and the expected angle $w_{3}$. As shown in Fig. 4, the image plane is placed along the $+Y$ axis. The freeform surface TM is shown as a black curve.

Assume that $\left|\theta_{3}\right|$ of the ray $\boldsymbol{R}_{n}^{(k)}$ on the TM is the smallest. Ray $\boldsymbol{R}_{n}^{(k)}$ intersects the image plane at its ideal image point $T_{n}$ after being reflected by point $P_{3, n}^{(k)}$ on the TM. When the ideal image point $T_{n}$ corresponding to the object field $\boldsymbol{\psi}_{n}^{\prime}$ moves down the image plane to point $T_{n}^{\prime}$, the directions of the reflected rays are changed, thus increasing the angle between the incident and reflected rays. Assume that angle $\left|\theta_{3}\right|$ is exactly equal to the expected angle $w_{3}$. In the triangle $P_{3, n}^{(k)} T_{n} T_{n}^{\prime}$, the moving distance $d$


Fig. 4. Shifting image points to change AOIs on the TM.
between the old point $T_{n}$ and the new point $T_{n}^{\prime}$ can be calculated according to the law of sines, as shown in Eq. (4):

$$
\begin{equation*}
d=T_{n} T_{n}^{\prime}=T_{n} P_{3, n}^{(k)} \times \frac{\sin \left(2 w_{3}-2 \theta_{3}\right)}{\sin \left(\beta+2 \theta_{3}-2 w_{3}\right)} \tag{4}
\end{equation*}
$$

Here, $\beta$ indicates the angle between the image plane and the reflected ray $P_{3, n}^{(k)} T_{n}$. As image point $T_{m}$ moves a distance $d$ along the image plane, we obtain new image point $T_{m}^{\prime}$ corresponding to object field $\boldsymbol{\psi}_{m}^{\prime}$. To ensure new objectimage relationships, the TM needs to be reconstructed. In the system obtained in step (4), the TM is degraded into a plane whose position is the same as that of the TM in the initial planar system. Taking the plane TM as input, the TM is reconstructed using the C-3D method with forward ray tracing. The process of shifting the image points and reconstructing the TM will be repeated until the angle $\left|\theta_{3}\right|$ is greater than the expected angle $w_{3}$.

The system designed using the above five steps is called the initial system, in which the min $|\mathrm{AOI}|$ on each surface is greater than the corresponding expected angle. The initial system can be directly optimized to achieve high image quality.

Two freeform off-axis three-mirror imaging systems with the same specifications (listed in Table 1) are designed; one by the method proposed in this Letter and one by the CI-3D method. Both design results are symmetrical about the $Y O Z$ plane, and the SM is the aperture stop. The $2 \times 5$ sample fields over a half-full FOV are employed in the construction process. For each field, 98 feature rays are sampled following the polar-ray grids.

The system designed by the method proposed in this Letter is called System 1. An initial planar system is first

Table 1. Optical System Specifications

| Parameter | Specification |
| :--- | :---: |
| Field of view (FOV) | $3^{\circ} \times 4^{\circ}$ |
| F-number | 1.5 |
| Focal length | 100 mm |
| Wavelength | $8-12 \mu \mathrm{~m}$ |

established according to the method introduced in step (1). The layout of the initial planar system is shown in Fig. 5(a), in which the expected angles $w_{i}$ on the PM, SM, and TM are $30^{\circ}, 25^{\circ}$, and $25^{\circ}$, respectively. The central field $\boldsymbol{\psi}_{0}$ of System 1 is $\left(0^{\circ},-30^{\circ}\right)$. The absolute values of tilt angles of these mirrors are calculated to be $2^{\circ}, 7^{\circ}$, and $7^{\circ}$, respectively, according to Eq. (2).

Taking the initial planar system shown in Fig. 5(a) as input to improve the image quality evaluated by the RMS deviation of the actual image points from the ideal image points, the freeform surfaces described by a fourth-order $X Y$ polynomial without odd coefficients of $x$ are constructed using the C-3D method. The layout of this system is shown in Fig. $\underline{5(\mathrm{~b})}$. The angles $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ are $28.3^{\circ}, 21.7^{\circ}$, and $19.3^{\circ}$, which are less than $30^{\circ}, 25^{\circ}$, and $25^{\circ}$, respectively. The min $|\mathrm{AOI}|$ value on the $\mathrm{PM}\left|\theta_{1}\right|$ is first increased. The PM is rotated clockwise at an angle of $1.6^{\circ}$ around the $X$ axis. The system FOV is unchanged, but the central field becomes $\boldsymbol{\psi}_{0}^{\prime}=\left(0^{\circ},-33.2^{\circ}\right)$ following the rotation of the PM. Then, the rotated PM is reconstructed into a freeform surface. The system layout is shown in Fig. $5(\mathrm{c})$, in which $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ are $30.4^{\circ}$, $22.5^{\circ}$, and $19.6^{\circ}$, respectively. Angle $\left|\theta_{2}\right|$ is the next to be increased to be greater than $25^{\circ}$. The SM is rotated clockwise at an angle of $3.5^{\circ}$ around the $X$ axis and is reconstructed. The system is shown in Fig. 5(d), in which $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ are $30.4^{\circ}, 25.5^{\circ}$, and $21.6^{\circ}$, respectively. Angle $\left|\theta_{3}\right|$ is less than $25^{\circ}$ and should be increased. Image points are first moved down a distance of 33.7 mm along the image plane according to Eq. (4). The TM is reconstructed. However, angle $\left|\theta_{3}\right|$ is $24.1^{\circ}$ and is still less than $25^{\circ}$. Image points are moved a distance of 7.7 mm , and the TM is reconstructed again. The system is shown in Fig. $5(\mathrm{e})$, with $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ being $30.4^{\circ}, 25.5^{\circ}$, and $25.6^{\circ}$, respectively.

The system in Fig. 5(e) is an initial system with low surface-figure-error sensitivity and can be directly optimized by commercial optical design software to achieve good image quality. Some constraints are employed to eliminate obscuration and prevent the min $|\mathrm{AOI}|$ on each surface from decreasing. The layout of the optimized system is shown in Fig. 5(f), in which the angles $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ are $43.6^{\circ}, 51.8^{\circ}$, and $26.3^{\circ}$, respectively. The field map of the RMSWFE is shown in Fig. 5(g), whose average value is $0.008 \lambda$ at $10 \mu \mathrm{~m}$.


Fig. 5. (a) Initial planar system; (b) initial freeform surfaces system; (c) freeform surfaces system with $\left|\theta_{1}\right|$ greater than $w_{1}$; (d) freeform surfaces system, in which $\left|\theta_{1}\right|,\left|\theta_{2}\right|$ are greater than $w_{1}, w_{2}$, respectively; (e) initial system with low surface-figure-error sensitivity, in which $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ are greater than $w_{1}, w_{2}$, and $w_{3}$, respectively; (f) optimized system; (g) field map of the RMSWFE.

For all six systems shown in Fig. $\mathbf{5}$, the absolute values of the minimum AOIs on each mirror are shown in Table 2 .

A contrast system called System 2 with the same system specifications as System 1 is designed using the CI-3D method. The central field $\boldsymbol{\psi}_{0}$ is $\left(0^{\circ},-9^{\circ}\right)$. The layout of the initial planar system is shown in Fig. 6(a), in which the $\min |\mathrm{AOI}|$ on the $\mathrm{PM}, \mathrm{SM}$, and TM are $9^{\circ}, 14^{\circ}$, and $12^{\circ}$, respectively. Taking the system in Fig. 6(a) as input, the freeform surfaces are constructed using the CI-3D method. The layout of the initial system is shown in Fig. 6(b), in which the min $\mid$ AOI $\mid$ values on each surface are $7.8^{\circ}, 10.8^{\circ}$, and $8.7^{\circ}$, respectively. The angle becomes smaller after construction. The system in Fig. 6(b) is optimized to improve image quality. The optimized system is shown in Fig. $\underline{6(\mathrm{c})}$, in which the angles $\left|\theta_{1}\right|,\left|\theta_{2}\right|$, and $\left|\theta_{3}\right|$ are $5.9^{\circ}, 8.6^{\circ}$, and $4.7^{\circ}$, respectively. The field map of the RMSWFE is shown in Fig. 6(d), with average values of $0.011 \lambda$ at $10 \mu \mathrm{~m}$.

Then, the surface-figure-error sensitivities of the two systems are analyzed to verify the effectiveness of our proposed method. Sensitivity analysis can generally be divided into three steps: (1) generation of surface figure errors to simulate realistic manufactured figure errors; (2) application of these errors to the freeform surface; (3) analysis of the degradation in the image quality of the system caused by these errors.

There are two approaches to create surface figure errors. One is to construct specified Zernike terms such as the Zernike coma and astigmatism ${ }^{[24,25]}$. The other is to create random surface errors based on the combinations of Zernike terms ${ }^{[2]}$.

Table 2. Absolute AOIs on Each Surface of the Systems in Fig. ${ }^{a}$

| System | AOI | PM | SM | TM |
| :--- | :---: | :---: | :---: | :---: |
| (a) | Min | 30 | 25 | 25 |
| (b) | Min | 28.34 | 21.66 | 19.25 |
| (c) | Min | 30.36 | 22.49 | 19.64 |
| (d) | Min | 30.36 | 25.46 | 21.60 |
| (e) | Min | 30.36 | 25.46 | 25.62 |
| (f) | Min | 43.58 | 51.75 | 26.27 |

${ }^{a}$ Data are all in degree $\left({ }^{\circ}\right)$.

The RMSWFE is used to evaluate the image quality in this Letter. Many random surface figure errors with specified RMS values are generated to model the realistic manufactured figure errors. Each figure error is separately added onto a certain surface to calculate the degradation in the system's RMSWFE utilizing commercial optical design software.
Assume that there are $n$ surfaces in the system. A total of $m$ fields are sampled, evenly spaced over the whole FOV. For each surface, a group of $k$ random surface figure errors with specified RMS values are generated. For the $s$ th surface $(s=1,2, \ldots, n)$, the corresponding $k$ random surface figure errors are separately applied to this surface in turn, and the RMSWFE change of each field, $\Delta W_{i, s, f}(i=1,2, \ldots, k, s=1,2, \ldots, n, f=1,2, \ldots, m)$, is calculated. $\Delta W_{s, f}$ is the RMS of the sequence of $\Delta W_{1, s, f}, \ldots, \Delta W_{k, s, f}$. Here, the subscripts $s$ and $f$ indicate the serial numbers of the surface and the field, respectively. We use $\Delta W_{s, f}$ to evaluate the sensitivity of the image quality of the $f$ th field to the surface figure errors on the sth surface. $\Delta W_{f}$ is the root sum square (RSS) of the sequence of $\Delta W_{1, f}, \ldots, \Delta W_{n, f}$, which is used to evaluate the sensitivity of the image quality of the $f$ th field to the surface figure errors on all surfaces simultaneously. $\Delta W$ is the average value of the sequence of $\Delta W_{1}, \ldots, \Delta W_{m}$, which is used to evaluate the system sensitivity to figure errors on all surfaces simultaneously. $\Delta W_{s, f}, \Delta W_{f}$, and $\Delta W$ are calculated by Eq. (5):
$\left\{\begin{array}{l}\Delta W_{s, f}=\sqrt{\sum_{i=1}^{k} \Delta W_{i, s, f}^{2} / k}, \quad s=1,2, \ldots, n, f=1,2, \ldots, m \\ \Delta W_{f}=\sqrt{\sum_{s=1}^{n} \Delta W_{s, f}^{2}}, f=1,2, \ldots, m \\ \Delta W=\sum_{f=1}^{m} \Delta W_{f} / m\end{array}\right.$.

For both systems, nine fields are selected for surface-figure-error sensitivity analysis. The 10th-order standard Zernike polynomials composed of totally 66 Zernike items are used to generate three groups of errors of $1 / 30 \lambda$ RMS and are applied to three mirrors. Each group includes 500 random surface figure errors. The sensitivities of the two systems are plotted in Fig. 7.
In System 1, designed by the proposed method in this Letter, when the PM, SM, and TM are perturbed with figure errors of $1 / 30 \lambda$ RMS, respectively, the maximum


Fig. 6. (a) Initial planar system; (b) initial system designed by CI-3D method; (c) optimized system; (d) field map of the RMSWFE.


Fig. 7. Sensitivity curve of the two systems to surface figure errors. (a) PM; (b) SM; (c) TM.
values of $\Delta W_{1, f}, \Delta W_{2, f}$, and $\Delta W_{3, f}$ are $0.0211 \lambda, 0.0246 \lambda$, and $0.0327 \lambda$, respectively. In System 2 , designed by the CI3 D method, the maximum values of $\Delta W_{1, f}, \Delta W_{2, f}$, and $\Delta W_{3, f}$ are $0.0381 \lambda, 0.0491 \lambda$, and $0.0398 \lambda$, respectively. $\Delta W_{1, f}, \Delta W_{2, f}$, and $\Delta W_{3, f}$ of System 1 are less than those of System 2. As shown in Fig. 7, System 1 is less susceptible to figure errors on each surface compared with System 2. When all surfaces are perturbed with figure errors of $1 / 30 \lambda$ RMS, simultaneously, the $\Delta W$ of Systems 1 and 2 are $0.0399 \lambda$ and $0.0647 \lambda$, respectively. These data indicate that the sensitivity of System 1 decreases by $38.33 \%$ compared with that of System 2.

To sum up, a design method for freeform reflectiveimaging systems with low surface-figure-error sensitivity is proposed in this Letter. Two freeform off-axis threemirror imaging systems are designed-one using the proposed method and one using the CI-3D method. The surface-figure-error sensitivities of the two systems are analyzed, and results indicate the superiority of the proposed method in this Letter. Moreover, this design method can be developed for the design of other types of systems with low surface-figure-error sensitivity, such as off-axis aspheric imaging systems and mixed-surface-type imaging systems. In our next research stage, a design method that considers reducing both surface-figure-error sensitivity
and assembly sensitivity while solving initial systems will be discussed.

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