Polarization de-multiplexing using a modified Kalman filter in CO-OFDM transmissions

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We propose the modified Kalman filter (MKF) using the received signal for observation and constructing an inverse process of the conventional Kalman filter (CKF) for polarization de-multiplexing in coherent optical (CO) orthogonal frequency-division multiplexing (OFDM) transmissions. The MKF can avoid the convergence error problem in CKF without matrix inverse operation and has a faster converging speed and a much larger tolerance to the process and measurement noise covariance, about two orders of magnitude more than those of CKF. We experimentally demonstrate the 12 Gbaud OFDM signal transmission over 480 km standard single-mode fiber. The performance of MKF and CKF outperforms pilot-aided polarization de-multiplexing with better accuracy and nonlinearity tolerance.

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Polarization-division multiplexing (PDM) is a straightforward way to use the two orthogonal polarizations of a lightwave to double the spectrum efficiency of coherent optical (CO) systems[1]. Thanks to digital signal processing (DSP), polarization de-multiplexing and channel equalization can be implemented in the digital domain at the receiver. Several methods have been proposed for polarization de-multiplexing, such as the constant mode algorithm (CMA)[2], its variant of multiple modulus algorithm (MMA)[3], Stokes space[4], and Kalman filtering. However, CMA and MMA both suffer from the singularity problem, i.e., the output signals cannot converge or they converge to the same polarization tributary[5]. Due to the fast convergence speed and lower computational complexity, Kalman filtering has been proposed for polarization de-multiplexing[6]. The Kalman filter is an optimum adaptive filter and has already been applied successfully in other fields, like engineering control[7–9]. Moreover, it is a well-known recursive algorithm for signal estimation and tracking in time-varying systems[10]. Therefore, it has also been used for frequency offset (FO) estimation[11], phase noise (PN) compensation, and amplitude noise estimation[12]. However, there are also the convergence error problems in the conventional Kalman filter (CKF), which cannot converge or even converge to the wrong state in polarization de-multiplexing. Moreover, since Kalman filtering is system-dependent, the initialization of the process and measurement noise covariance Q and R is challenging, which needs much more time to adjust for convergence and affects the performance of Kalman filtering significantly. In general, it is difficult to have a common method to decide the optimum values of Q and R in different systems; then, it is desirable to increase the tolerance for process and measurement noise covariance of Kalman filtering[1].

In this Letter, we propose the modified Kalman filter (MKF) to use the received signal as the observation vector and construct the inverse process of the CKF for polarization de-multiplexing[13]. Since there is no matrix inversion in the estimation of the Jones matrix for the proposed MKF, it can avoid the convergence error problem in CKF and has a faster converging speed and a much larger tolerance than CKF to the process and measurement noise covariance. We first carry out quadrature amplitude modulation (16-QAM) transmissions in a simulation. We find that the CKF also has convergence error problems. On the other hand, the MKF can work well and converges with only 70 training symbols. Secondly, we also conduct numerical simulations and experiments in a CO orthogonal frequency-division multiplexing (OFDM) transmission system to compare the tolerance of MKF and CKF to the process and measurement noise covariance Q and R. We optimize the tuning parameters Q and R, i.e., the covariance matrix of the process and measurement noise, to obtain the optimized performance. The simulation results show good converging performance and a faster converging speed than both the CKF and the conventional pilot-aided polarization de-multiplexing[14]. The experiment results also verify that both the MKF and CKF have similar performance, which is superior to the conventional pilot-aided polarization de-multiplexing at different launch powers after 160 and 480 km transmission.

There are various impairments in the optical transmission systems, such as chromatic dispersion (CD), FO, and PN[1]. Instead of the CMA algorithm or other adaptive filter algorithms, which calculate the tap coefficients of a digital finite impulse response filter, the Kalman filters can use a mathematical model with the transmission impairments to mitigate their effects[15]. They provide accurate estimation to the distortions by minimizing the variance[16]. Since we focus on the effect of MKF for polarization de-multiplexing, we employ a simplified transmission model of optical fiber transmission by assuming that the major channel impairments, such as CD, FO, and PN, have already been compensated perfectly after polarization-diversified coherent detection. Therefore, with the Jones matrix J, the recovered signals Z_c and received signals Z_o can be simplified as

$$Z_c(k) = J^{-1}(k) \cdot \begin{bmatrix} r_x(k) \\ r_y(k) \end{bmatrix} + n(k), \tag{1}$$

$$Z_o(k) = \begin{bmatrix} r_x(k) \\ r_y(k) \end{bmatrix} = J(k) \cdot \begin{bmatrix} t_x(k) \\ t_y(k) \end{bmatrix} + m(k), \tag{2}$$

$$J(k) = \begin{bmatrix} a(k) & b(k) \\ c(k) & d(k) \end{bmatrix}, \tag{3}$$

where k denotes the time index. Furthermore, r_x, r_y, t_x, and t_y are the received signals and transmitted signals in x and y polarizations, respectively. We need to estimate the polarization state parameters $[a, b, c, d]$ of the Jones matrix J, which determines the polarization rotation and the polarization state. Both n and m are the additive white Gaussian noise (AWGN) terms, mainly from amplified spontaneous emission (ASE) from erbium-doped fiber amplifiers (EDFAs) in both polarizations[11].

We employ the received signals Z_o as the observation vector in the MKF and construct the inverse process of CKF as Eq. (2), which is different from the CKF that uses the recovered signals Z_c in Eq. (1) as the observation vector to estimate the Jones matrix. Therefore, the MKF can estimate the Jones matrix, avoiding the convergence error problem in CKF because there is no inversion of the Jones matrix in the polarization tracking process with training sequences. Our target is to obtain an optimal estimation of the polarization state parameters $[a, b, c, d]$. Moreover, we utilize $H = [h_{11}, h_{12}, h_{21}, h_{22}]$ as the state vector in the modified Kalman filtering, which is the estimation of the Jones matrix J. The state and measurement equations of MKF can be expressed as Eqs. (4) and (5), respectively:

$$H(k) = H(k-1) + w(k), \tag{4}$$

$$Z_o(k) = \begin{bmatrix} r_x(k) \\ r_y(k) \end{bmatrix} = J(k) \cdot \begin{bmatrix} t_x(k) \\ t_y(k) \end{bmatrix} + v(k), \tag{5}$$

where w and v represent the process and measurement noises, both of which are AWGN. Then, we can update prediction vector H_p and the prior error covariance P_p by the time update as

$$H_p(k) = H_c(k-1), \tag{6}$$

$$P_p(k) = P_c(k-1) + Q, \tag{7}$$

where H_c, P_c, and Q are the correction vector, the posterior error covariance, and the process noise covariance. The state update of the MKF can be represented as

$$M(k) = \begin{bmatrix} Z_{\mathrm{in},x}(k) & Z_{\mathrm{in},y}(k) & 0 & 0 \\ 0 & 0 & Z_{\mathrm{in},x}(k) & Z_{\mathrm{in},y}(k) \end{bmatrix}, \tag{8}$$

$$\Delta e(k) = Z_o(k) - M(k)H_p(k), \tag{9}$$

$$K = P_p(k)M(k)^T[M(k)P_p(k)M(k)^T + R]^{-1}, \tag{10}$$

$$H_c(k) = H_p(k) + K\Delta e(k), \tag{11}$$

$$P_c(k) = P_p(k) - KM(k)P_c(k), \tag{12}$$

where M, R, Z_{in}, Δe, and K are the measurement matrix, the measurement noise covariance, the input of the Kalman filter, the innovation vector, and the Kalman gain, respectively.

Figure 1 shows the block diagram of the proposed MKF scheme. We first switch the input of the Kalman filter Z_{in} to the training sequences to construct the measurement matrix as Eq. (8) and realize pre-convergence as the solid framework shown in Fig. 1. After the acquisition of the Jones matrix for MKF to pre-converge using the training sequences, even if the polarization is time-varying, we can track it by switching Z_{in} to the decision module, as the dotted framework for tracking shown in Fig. 1. It is different from the CKF that uses the received signals as the input of the Kalman filtering. The Z_{out} is the output of the Kalman filter and also the signals after polarization de-multiplexing.

We first carry out a simulation of a dual-polarization 16-QAM system on MATLAB to verify that the proposed MKF scheme is effective for polarization de-multiplexing.

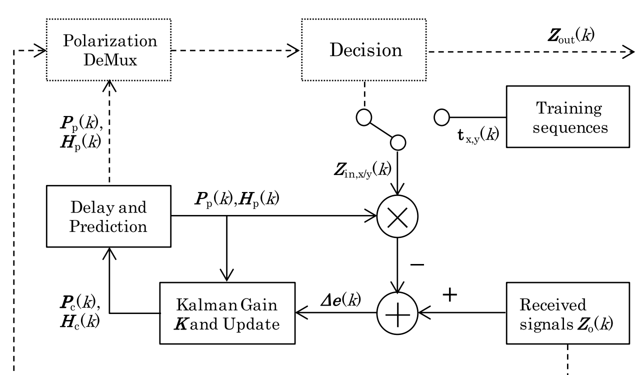

Fig. 1. Block diagram of the proposed polarization de-multiplexing with the MKF scheme.

We intentionally set a matrix that is expressed as a vector in Kalman filtering to verify the MKF, avoiding the convergence error problem in CKF. Figure 2 depicts the converging performance of CKF and MKF under the optimized tuning parameters, respectively. While CKF cannot converge and even converges to the wrong state, the MKF can converge rapidly with only 70 symbols, which verifies the feasibility of the MKF that can be used for polarization de-multiplexing while avoiding the convergence error problem in CKF. We then use a unitary matrix H to compare the converging speed. Figure 3 shows the comparison of mean squared error (MSE) between the CKF and the MKF. The converging speed for the MKF is apparently faster than that of the CKF. Due to the innovation vector, Δe represents the convergence criterion for the Kalman filter. Since we employ the received signals as the observation vector to obtain the innovation vector, a more explicit target for convergence is supplied and results in a faster converging speed.

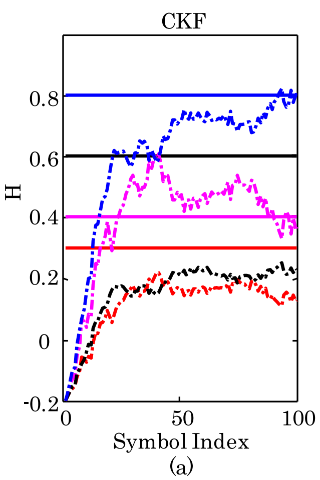

Fig. 2. Jones matrix estimation in simulation: (a) CKF; (b) MKF; dotted line, estimated H.

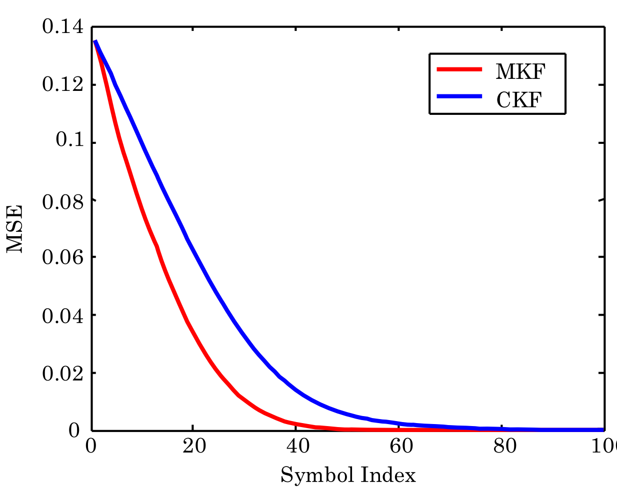

Fig. 3. Converging performance of the MKF and CKF.

We finally conduct an experiment to compare the performance with the CKF, the MKF, and pilot-aided polarization de-multiplexing in a CO-OFDM transmission. Figure 4 is the experimental setup. At the transmitter side, we transmit 221 OFDM symbols, in which 20 are training symbols, and one is a null symbol. The OFDM symbols are transferred to the time domain by an inverse fast Fourier transform (IFFT) of the size 256, followed by 1/32 cyclic prefix (CP) insertion. The number of effective subcarriers is 212. Then, the OFDM signal is generated by the arbitrary waveform generator (AWG) operated at 12 GS/s. The polarization multiplexed signal is generated by a polarization emulator that consists of a polarization beam splitter and combiner and a delay line at a length of one OFDM symbol. The transmission spans N are chosen as 2 or 6. At the receiver side, after a polarization-diversified coherent detection, the signals are digitized by a digital phosphor oscilloscope (DPO) operating at 50 GS/s. Then, the received electrical signal is processed off-line to compensate for the impairments, such as CD and FO. After that, the MKF, CKF, and pilot-aided polarization de-multiplexing are used for polarization de-multiplexing before phase compensation, respectively.

We first discuss the tolerance to the initial process and measurement noise covariance parameters Q and R for the MKF and CKF. The process and measurement noise covariance parameters Q, R can be expressed as $Q = qI_4$ and $R = rI_2$. q and r are two constants, and hence, we need to optimize for both the MKF and CKF. I_k represents the unit matrix with the kth order. The MKF and CKF are tested under different launch powers and transmission distances. Figure 5 shows the bit error rates (BERs) versus q and r for MKF and CKF at different launch powers and transmission distances with colored contour maps. The tolerance to noise covariance Q and R of the MKF is about two orders of magnitude larger than that of the CKF, when the transmission distance is 160 km, and the launch power is −3 and 1 dBm. We change the transmission distance to 480 km and compare the noise covariance tolerance for MKF and CKF in Figs. 5(e) and 5(f). Similar to that

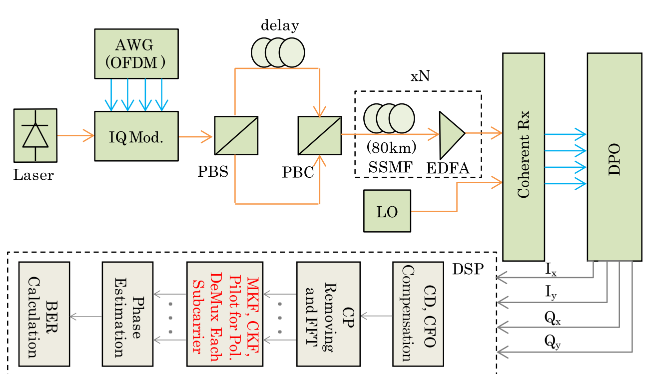

Fig. 4. Experimental setup of a polarization multiplexed CO-OFDM transmission. PB, polarization beam splitter; PBC, polarization beam combiner; SSMF, standard single-mode fiber; CD, chromatic dispersion; CFO, carrier frequency offset.

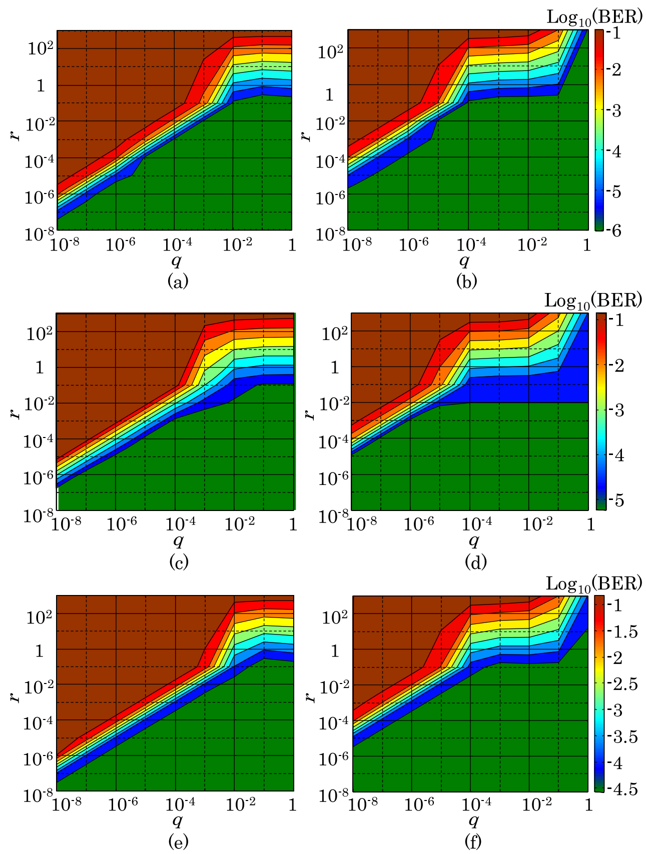

Fig. 5. q, r versus $\log_{10}$ (BER) with launch powers, fiber distances, and a Kalman filter being (a) −3 dBm, 160 km, and CKF; (b) −3 dBm, 160 km, and MKF; (c) 1 dBm, 160 km, and CKF; (d) 1 dBm, 160 km, and MKF; (e) −3 dBm, 480 km, and CKF; (f) −3 dBm, 480 km, and MKF.

in Figs. 5(a)–5(d), the MKF performs better than the CKF. In short, we find that the process and measurement noise tolerance of the MKF is better than that of the CKF. It is because the MKF has a faster converging speed than the CKF under the same parameters Q and R.

Figure 6 illustrates the BERs versus launch power after 160 and 480 km transmission for the CO-OFDM system, respectively. The tuning parameters Q and R for both Kalman filters have been optimized according to the results in Fig. 5. Since the Kalman gain, expressed as Eq. (9), is asked to be enough large for the Kalman filter to converge and track stably, q and r have to be limited in an appropriate range. Here, we use the optimized Q and R with the q, r in an appropriate range, as shown in Fig. 5 for MKF and CKF, which makes both MKF and CKF converge well. Therefore, the MKF has similar performance to the CKF besides a faster converging speed. Both of them perform better than the conventional pilot-aided polarization de-multiplexing at all launch powers at different transmission distances, which shows that the two Kalman filter schemes both have better accuracy and nonlinearity tolerance in addition to the fast converging feature.

Thanks to the flexibility of the Kalman filter, the proposed modified Kalman scheme can also be applied to

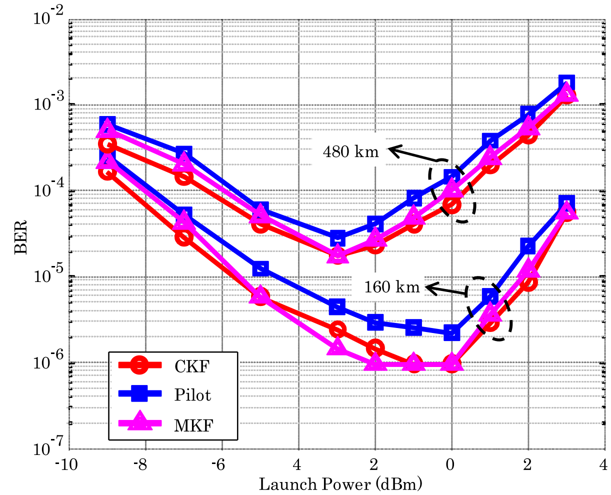

Fig. 6. BER versus launch power for three polarization de-multiplexing algorithms.

other modulation formats, e.g., quadrature phase-shift keying (QPSK) and M-QAM, where we do not need to change any hardware or framework except for the modulation format.

In this Letter, we have proposed the MKF that uses the received signal as the observation vector for the measurement equation and constructs an inverse process of the CKF for polarization de-multiplexing in CO-OFDM transmissions. The MKF can avoid the convergence error problem in the CKF for polarization de-multiplexing, which will disable the polarization de-multiplexing or deteriorate the performance. Furthermore, we have demonstrated that the MKF has a faster converging speed and a much larger tolerance to noise covariance Q and R, about two orders of magnitude larger compared with CKF by dual-polarization 16-QAM simulations and 12 Gbaud OFDM experiments. Since we have used the received signals as the observation vector in MKF, a more explicit target for convergence is given, resulting in a faster converging speed than CKF with only 70 training symbols used for convergence. The performances of MKF and CKF are similar after we used the optimized Q and R in the same circumstance of the process and measurement noise, and they both outperform pilot-aided polarization de-multiplexing.

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