# Five－port beam splitter of a single－groove grating 

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#### Abstract

A single－groove grating for five－port TE－polarization beam splitting under normal incidence at the wavelength of 1550 nm is presented．The transmitted diffraction efficiency of the gratings is over $94.5 \%$ with uniformity better than $2 \%$ ．A physical view of diffraction inside the grating is presented by the simplified modal method（SMM）． Initial parameters of the grating profiles are obtained by use of SMM and then optimized by employing rigorous coupled－wave analysis and the simulated annealing algorithm．


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Beam splitters are common optical components used in various optical systems，such as holography，interferome－ ters ${ }^{[1]}$ ，photonic crystal fabrication ${ }^{[2,3]}$ ，microlens array fabrication ${ }^{[4]}$ ，specific pattern generating ${ }^{[5]}$ ，and four－wave mixing ${ }^{[6]}$ ．A high density，low contrast binary grating as a beam splitter has advantages，such as high transmitted efficiency and easy fabrication．Several beam splitters based on high density，low contrast binary gratings have been proposed：Wang et al．${ }^{[7]}$ designed a wideband two－port beam splitter based on a binary fused－silica phase grating，and Feng et al．${ }^{[8]}$ designed three－port beam split－ ters．To achieve more splitting ports，some researchers chose double－groove gratings for more configurable param－ eters and designed five－port and seven－port ${ }^{[9,10]}$ beam split－ ters．However，to the best of our knowledge，no one has presented a single－groove grating for a five－port beam split－ ter，and it is easier to fabricate than double－groove gratings． In this Letter，a five－port beam splitter based on a single－ groove grating is presented．The diffraction efficiency of the beam splitter is over $94.5 \%$ ，and the uniformity of beam splitting is better than $2 \%$ when the fabrication errors of the grating period and groove depth are both below 10 nm ．

The modal method for dielectric lamellar gratings pro－ posed by Botten et al．${ }^{[11]}$ describes the progress of grating diffraction as a few grating modes propagating in the gra－ ting area and coupling with diffraction orders at the gra－ ting interface．Furthermore，a simplified modal method （SMM）based on the modal method approximately ne－ glects the effects of the non－propagating grating modes and the reflection at the grating interfaces ${ }^{[12]}$ ．With the help of these approximations，SMM provides a simple and clear physical view of grating diffraction and cuts down calculation expense．We employed SMM to deter－ mine an initial set of parameters of the profile of a five－port beam splitter grating．

Figure 1 presents a diffraction grating and the coordi－ nate system used in the discussion．The period of the
grating is $\Lambda$ ，the etched depth is $h$ ，and the widths of the grating groove and ridge are $g$ and $c$ ，with the refractive indices of the groove and ridge as $n_{g}$ and $n_{c}$ ，respectively． Duty cycle $f$ is defined by $f=c / \Lambda$ ．The origin of the co－ ordinate system is located at the midpoint of the top of a ridge．A TE－polarized monochromatic plane wave with a wavelength of $\lambda$ is normally incident on the grating and excites grating modes in the grating region．According to the modal method，the grating modes can be expressed as

$$
\begin{equation*}
E_{y q}(x)=u_{q}(x) \exp \left(i n_{q, \mathrm{eff}} z\right) \tag{1}
\end{equation*}
$$

where $u_{q}(x)$ is the unified mode profile of the $q$ th grating mode $(q=0,1,2, \ldots)$ ．The detailed expression of $u_{q}(x)$ can be found in Ref．［12］，and the unification is followed by

$$
\begin{equation*}
\frac{1}{\Lambda} \int_{0}^{\Lambda} u_{p}(x) u_{q}^{*}(x) \mathrm{d} x=\delta_{p q} \tag{2}
\end{equation*}
$$

$n_{q, \text { eff }}$ is the effective refractive index and determined by the dispersion equation ${ }^{[11]}$ ：

$$
\begin{equation*}
\cos (\beta g) \cos (\gamma c)-\frac{\beta^{2}+\gamma^{2}}{\beta^{2} \gamma^{2}} \sin (\beta g) \sin (\gamma c)=1 \tag{3}
\end{equation*}
$$

where $\beta=2 \pi / \lambda\left(n_{1}^{2}-n_{\text {eff }}^{2}\right)^{1 / 2}, \gamma=2 \pi / \lambda\left(n_{2}^{2}-n_{\text {eff }}^{2}\right)^{1 / 2}$ ．


Fig．1．Illustration of a single－groove grating for five－port beam splitting．

Since the reflection at the grating interfaces is neglected, the coupling between grating modes and diffraction orders can be simplified as

$$
\begin{gather*}
\exp \left(i k_{x 0} x\right)=\sum_{q} J_{q 0} u_{q}(x),  \tag{4}\\
\sum_{q} \exp \left(i k_{0} n_{q, \operatorname{eff}} h\right) J_{q 0} u_{q}(x) \\
=\sum_{p, q} J_{q p}^{*} \exp \left(i k_{0} n_{q, e f f} h\right) J_{q 0} \exp \left(i k_{x p} x\right),  \tag{5}\\
J_{q m}=\frac{1}{\Lambda} \int_{0}^{\Lambda} \exp \left(i k_{x m} x\right) u_{q}^{*}(x) \mathrm{d} x,  \tag{6}\\
J_{q m}^{*}=\frac{1}{\Lambda} \int_{0}^{\Lambda} u_{q}(x) \exp \left(-i k_{x m} x\right) \mathrm{d} x, \tag{7}
\end{gather*}
$$

where $p$ stands for the $p$ th diffraction order, $k_{x p}=k_{x 0}+2 p \pi / \Lambda ; " * "$ stands for the complex conjugation. Equation (4) represents the coupling between the incident wave and the grating modes, while Eq. (5) represents the coupling between the grating modes and the transmitted orders.

Usually, for a grating that has five or more propagating transmitted orders, there are five or more propagating modes in the grating area, which brings complexity for calculation and the physical view. However, modes in a grating under a normal incidence can be classified into two symmetric categories (see Fig. 2). Table $\underline{1}$ shows $\left|J_{q 0}\right|^{2}$ of six modes, which are equivalent to the overlap integral between the modes and incident wave ${ }^{[12]}$ and represent the amount of energy these modes received from the incident wave. Only even symmetric modes can be excited by the normal incident wave, since the coupling


Fig. 2. Unified profiles of all of the propagating modes of a grating under a normal incidence with five transmitted orders. The shaded region is meant to be the ridge of the grating region. The parameters of the grating are $f=0.5, \Lambda=3800 \mathrm{~nm}$, wavelength of incidence is $\lambda=1550 \mathrm{~nm}$, the refractive indices of the groove and ridge are $n_{g}=1, n_{c}=1.5007$. Modes $0,1,4$ are odd symmetric, while modes $2,3,5$ are even symmetric.

Table 1. Coupling Coefficients and Effective Indices

| Mode | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $n_{\text {eff }}$ | 2.1431 | 1.8245 | 1.3393 | 0.8252 | 0.7955 | 0.1228 |
| $\left\|J_{q 0}\right\|^{2}$ | 0 | 0 | 0.5454 | 0.1792 | 0 | 0.2689 |

coefficient $\left|J_{q 0}\right|^{2}$ of an odd symmetric mode and the normal incident wave is zero. Therefore, the number of excited propagating modes is reduced to three. The effective refractive index of these even symmetric modes can be obtained by solving a factorization of Eq. (3) ${ }^{133}$ :

$$
\begin{equation*}
\beta \tan \left(\frac{\beta c}{2}\right)+\gamma \tan \left(\frac{\gamma g}{2}\right)=0 . \tag{8}
\end{equation*}
$$

Denoting these three modes as modes $q_{a}, q_{b}, q_{c}$, the diffraction efficiency of all five transmission orders can be determined by

$$
\begin{gather*}
\eta_{0}=\left|\sum_{q \in Q} J_{q 0}^{*} \exp \left(i k_{0} n_{q, \text { eff }} h\right) J_{q 0}\right|^{2},  \tag{9}\\
\eta_{1}=\eta_{-1}=\left|\sum_{q \in Q} J_{q 1}^{*} \exp \left(i k_{0} n_{q, \text { eff }} h\right) J_{q 0}\right|^{2},  \tag{10}\\
\eta_{2}=\eta_{-2}=\left|\sum_{q \in Q} J_{q 2}^{*} \exp \left(i k_{0} n_{q, \text { eff }} h\right) J_{q 0}\right|^{2}, \tag{11}
\end{gather*}
$$

where $Q=\left\{q_{a}, q_{b}, q_{c}\right\}$.
The main objective of designing a beam splitter is to minimize the uniformity $U$,

$$
\begin{equation*}
U=\frac{\eta_{\max }-\eta_{\min }}{\eta_{\max }+\eta_{\min }} \tag{12}
\end{equation*}
$$

and keep a high total transmitted efficiency. We chose BK-7 glass (whose refractive index at $\lambda=1550 \mathrm{~nm}$ is 1.5007 ) as the grating material. By applying Eqs. (9)(11), we calculated the uniformity of the -2 nd to +2 nd orders of the grating, whose parameters are $\{f, h, \Lambda \mid f=0.5$, $h \in[500 \mathrm{~nm}, 3000 \mathrm{~nm}], \quad \Lambda \in[3200 \mathrm{~nm}, 4600 \mathrm{~nm}]\} \quad$ (see Fig. 3). The grating with $h=2170 \mathrm{~nm}$, and $\Lambda=$ 3310 nm has the minimum of uniformity of $4.51 \%$.

Table $\underline{2}$ shows the diffraction efficiencies of the -2 nd to +2 nd transmitted orders, and the beam splitting uniformity of the grating with the initial parameters obtained by the SMM. These results are calculated by using rigorous coupled-wave analysis (RCWA) ${ }^{[14,15]}$, which is a more accurate numeric method compared to SMM. Since the uniformity is not quite perfect, which is expected due to the approximation introduced by SMM, the parameters need to be optimized. We set $h=2170 \mathrm{~nm}, \Lambda=3310 \mathrm{~nm}$ as the initial data and employ RCWA and the simulated annealing (SA) algorithm for optimization ${ }^{[16,17]}$. For fiveport beam splitters, the cost function is ${ }^{[18]}$


Fig. 3. Contour plot of uniformity (\%) calculated by SMM. The grating material is BK-7 glass, and its duty cycle is $f=0.5$.

$$
\begin{equation*}
\phi(\Lambda, h)=\frac{\sum_{i=-2}^{2}\left(\eta_{i}-\bar{\eta}\right)}{\sum_{i=-2}^{2} \eta_{i}} \tag{13}
\end{equation*}
$$

where $\eta_{i}$ is the diffraction efficiency of the $i$ th order, and $\bar{\eta}$ is the average value of $\eta_{i}$.

The parameters of the optimized grating, which has the minimum value of $\phi(\Lambda, h)$, are $f=0.5, h=2167 \mathrm{~nm}$, and $\Lambda=3204 \mathrm{~nm}$. Table $\underline{2}$ shows the diffraction efficiencies of five transmitted orders and the beam split uniformity of the grating.

We successfully designed a five-port beam splitter based on a single-groove grating: the grating material is BK-7 glass, and the grating parameters are $f=0.5$, $h=2167 \mathrm{~nm}$, and $\Lambda=3205 \mathrm{~nm}$. It is also important to know how the fabrication errors affect the performance of the beam splitting grating. Figure $\underline{4}$ presents the total transmitted efficiency and beam split uniformity (calculated by RCWA) when the grating has some fabrication errors. Such a grating that is deep etched on BK-7 glass is usually realized by dry etching ${ }^{[19]}$, and 10 nm fabrication errors are common for dry etching ${ }^{[20]}$. The rectangle in Fig. $\underline{4}$ includes a range of $h=2167 \pm 10 \mathrm{~nm}, \Lambda=$ $3205 \pm 10 \mathrm{~nm}$. In this range, the total transmitted diffraction efficiency is over $94.65 \%$, and uniformity is better than $2 \%$.

In conclusion, we proposed a five-port beam splitter based on a single-groove grating. The splitter works under TE-polarized normal incidence at the wavelength of 1550 nm . The diffraction efficiency of the beam splitter is over $94.5 \%$, and the uniformity of beam splitting is


Fig. 4. (a) Total transmitted diffraction efficiency and (b) uniformity of $1 \times 5$ beam splitting gratings with some fabrication errors.
better than $2 \%$ when the fabrication errors of the grating period and groove depth are both below 10 nm . In the classical Fourier optics field, a single-groove grating can function as two-port or three-port beam splitting ${ }^{[18]}$. A single-groove five-port beam splitting grating cannot be explained by using the classical Fourier optics theory. It is the first time, to the best of our knowledge, that a single-groove grating for five-port beam splitting is found by using SMM. The result is verified and optimized by RCWA. The modal analysis shows that there are three modes excited inside the grating region. With a certain set of parameters, a special balance of these three modes could produce five equal-intensity transmitted orders.

Table 2. Diffraction Efficiency and Uniformity Before and After Optimization

|  | Diffraction efficiency (\%) |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Parameter | +2 nd | +1 st | 0 th | -1 st | -2 nd | Total | Uniformity (\%) |
| Before | 18.37 | 18.69 | 19.43 | 18.69 | 18.37 | 93.5 | 2.83 |
| After | 18.96 | 18.98 | 18.96 | 18.98 | 18.96 | 94.8 | 0.39 |

Since the binary-phase structure should be more easily fabricated than multiple-groove gratings, this singlegroove grating as a five-port splitter should be a useful optical element for practical applications.

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