Entanglement-free determination of pretilt angles of twisted nematic liquid-crystal cells by phase measurement

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The pretilt angle of a twisted nematic (TN) liquid crystal display has a strong effect on viewing angles and response times of the display as well as shifting the light-leaking disclination lines into the black-matrix area within pixels of a thin-film-transistor driven TN panel for a high contrast ratio. In this Letter, we develop an optical method to determine the TN pretilt angle based on heterodyne measurements of the phase versus the incident angle of a thin TN cell subjected to out-of-plane rotations. We believe that our method of measuring phases has advantages of simpler setup and calculations, better stability, and less sensitive to ambient electromagnetic interference than the commonly used method of measuring intensities.

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Because of low cost, fast response time, and high light transmittance, thin-film-transistor (TFT) driven twisted nematic (TN) liquid-crystal displays (LCDs) have been widely used in calculators, computer screens, and cell phones. The pretilt angle of the TN medium within the TFT-TN panel affects not only its response times and viewing angles but also the light-leakage positions of fringed-field-induced disclination lines within pixels of the TFT-TN panel[1]. It is important to design and measure pretilt angles of thin TFT-TN panels accurately to optimize the display qualities. The optical methods to measure the pretilt angles of TN LCDs can be classified into two methods: in-plane rotation (IPR) and out-of-plane rotation (OPR) methods. The incident light is always perpendicular to the TN cell in the IPR method, which has been widely applied to determine pretilt angles, cell gaps, and twist angles of TN cells by curve-fitting methods[2,3]. However, in these methods, the derived pretilt angle and cell gap are entangled through the Mauguin parameter equation ($u = 2\pi\Delta nd/\lambda$), where d and λ are the cell gap and the wavelength of the incident light, respectively, and Δn is the liquid crystal (LC) birefringence with the pretilt angle embedded in it. Therefore, by using the IPR method, the pretilt angle and cell gap could not be measured independently. In 1976, Baur *et al.* published the OPR method[4,5] by measuring the transmission of a monochromatic incident light at 632.8 nm versus the OPR angle to determine the pretilt angle of a thick TN cell. They utilized a 4×4 matrix formulation to calculate the transmission of similar incident light versus the OPR angle to fit the experimental results for the determination of the pretilt angle. However, as pointed out by Yang[6], a single-wavelength transmission through a thin TN cell calculated by the 4 by 4 matrix method had strong Fabry–Perot effects caused by many optical layers of the TN cell with potential errors in the thickness and index of refraction of each layer making difficulties to compare with the experimental results. In 1980, Birecki and Kahn[7] used the OPR method to find the transmission extrema to determine the pretilt angle of a thick TN cell. The derived pretilt angle was independent of the TN cell gap. However, Birecki and Kahn indicated that there were large errors in the determined pretilt angles for thin TN cells when the cell gap was around the Gooch and Tarry[8] first-minimum in transmission where $d = 0.866\lambda/\Delta n$, the most commonly used cell gap since the appearance of TFT-TNs. For TFT-TN products, the first-minimum cell gap is essential to having fast response times and wide viewing angles. In 2013, Wang and Yang[9] published a paper by using intensity measurements based on a new configuration of the OPR method and analyzed data by extended the Jones matrix method to obtain accurate pretilt angles for thin (first-minimum) TN cells, avoiding the entanglements of cell gap and pretilt angle measured by the IPR methods.

In this Letter, we have replaced the intensity measurements by phase measurements in a similar OPR method[9] to determine the pretilt angles of thin TN cells. We show that the analyses using the extended Jones matrix to calculate the phase versus OPR angle to fit the corresponding experimental results to determine the TN pretilt angle are much simpler than analyzing the measured intensities.

In our phase measurements, we have used heterodyne mixing of two laser beams from a Zeeman laser system[10]. We believe that our method of measuring phases has the advantages of a simpler setup, better stability, simpler calculations, and less sensitivity to ambient electromagnetic interference (EMI) than the published method of measuring intensities[9].

We used the extended Jones matrix[11] to calculate phase differences after the incident light passes through the TN cell at oblique incidences. The LC medium within a TN cell was configured into m layers of equal thickness of d/m, as shown in Fig. 1. The TN cell was assumed to have a Φ (twist angle) of 90°. The difference in the averaged twist angle of LC directors within any two adjacent layers can be approximated as $\Delta\Phi = 90°/m$. The parameters n_o and n_e are reflective indices of ordinary and extraordinary lights, respectively. We assume that alpha is the pretilt angle between the LC director and the x-y plane and Φ is the twist angle from −45° to 45°. The dielectric tensor of each layer is shown in

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}, \tag{1}$$

$$\varepsilon_{xx} = n_o^2 + (n_e^2 - n_o^2)\cos^2\theta\cos^2\Phi,$$
$$\varepsilon_{xy} = \varepsilon_{yx} = (n_e^2 - n_o^2)\cos^2\theta \sin\Phi \cos\Phi,$$
$$\varepsilon_{xz} = \varepsilon_{zx} = (n_e^2 - n_o^2)\sin\theta \cos\theta \cos\Phi,$$
$$\varepsilon_{yy} = n_o^2 + (n_e^2 - n_o^2)\cos^2\theta\sin^2\Phi,$$
$$\varepsilon_{yz} = \varepsilon_{zy} = (n_e^2 - n_o^2)\sin\theta \cos\theta \sin\Phi,$$
$$\varepsilon_{zz} = n_o^2 + (n_e^2 - n_o^2)\sin^2\theta. \tag{2}$$

Assume that the incident light lies in the x-z plane and the angle between y-z plane and the incident light is θ_k. The incident light can be described as

$$\vec{k} = k_0(\sin\theta_k, 0, \cos\theta_k), \tag{3}$$

$$k_0 = \frac{2\pi}{\lambda}, \tag{4}$$

where λ refers to the wavelength of the incident light, and $k_0(\sin\theta_k)$ is assigned as k_x, to be used below.

In Jones matrix formulations, the oblique incident polarized light propagating through a TN LC medium can be described by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix}_{out} = J_m J_{m-1}...J_2 J_1 \begin{bmatrix} E_x \\ E_y \end{bmatrix}_{in}, \tag{5}$$
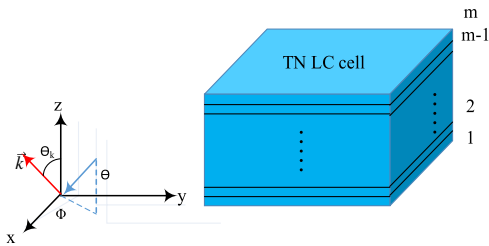

Fig. 1. Coordinates of the incident light and the schematic diagram of the TN medium within a TN cell.

$$J_m = (SGS^{-1})_m, \tag{6}$$

$$S = \begin{bmatrix} 1 & c_2 \\ c_1 & 1 \end{bmatrix}; \quad G = \begin{bmatrix} e^{ik_{z1}\frac{d}{m}} & 0 \\ 0 & e^{ik_{z2}\frac{d}{m}} \end{bmatrix}, \tag{7}$$

$$\frac{k_{z1}}{k_0} = \sqrt{[n_o^2 - (k_x/k_0)^2]}, \tag{8}$$

$$\frac{k_{z2}}{k_0} = -\frac{\varepsilon_{xz}}{\varepsilon_{zz}}\frac{k_x}{k_0}$$
$$+ \frac{n_o n_e}{\varepsilon_{zz}}\left[\varepsilon_{zz} - \left(1 - \frac{n_e^2 - n_o^2}{n_e^2}\cos^2\theta\sin^2\Phi\right)\left(\frac{k_x}{k_o}\right)^2\right]^{1/2}, \tag{9}$$

$$c_1 = \frac{[(k_x/k_0)^2 - \varepsilon_{zz}]\varepsilon_{yx} + [(k_x/k_0)(k_{z1}/k_0) + \varepsilon_{zx}]\varepsilon_{yz}}{[(k_{z1}/k_0)^2 + (k_x/k_0)^2 - \varepsilon_{yy}][(k_x/k_0)^2 - \varepsilon_{zz}] - \varepsilon_{yz}\varepsilon_{zy}}, \tag{10}$$

$$c_2 = \{[(k_x/k_0)^2 - \varepsilon_{zz}]\varepsilon_{xy}$$
$$+ [(k_x/k_0)(k_{z2}/k_0) + \varepsilon_{xz}]\varepsilon_{xz}\}/\{[(k_{z2}/k_0)^2$$
$$- \varepsilon_{xx}][(k_x/k_0)^2 - \varepsilon_{zz}]$$
$$- [(k_x/k_0)(k_{z2}/k_0) + \varepsilon_{zx}][(k_x/k_0)(k_{z2}/k_0) + \varepsilon_{xz}]\}. \tag{11}$$

Based on Eqs. (1)–(11), we have used MATLAB (mathematics calculation software) to calculate and derive the total phase of any oblique incident light passing through the TN medium and compared it with the measured results.

The parameters of the TN LC cell used in the simulation are shown in Table 1. The TN LC medium[12] is Merck MLC-9200-100, which has equations of dispersions of refractive indices given by Eq. (12) for the calculations of n_o and n_e at the incident-light wavelength of 633 nm,

$$n_i = A_i + B_i\lambda^{-2} + C_i\lambda^{-4}, \tag{12}$$

where i can be e-ray or o-ray ($A_e = 1.572$, $B_e = 0.0111$, $C_e = 4.00 \times 10^{-5}$), ($A_o = 1.4740$, $B_o = 0.0063$, $C_o = 6.04 \times 10^{-18}$).

By configuring the TN medium into 180 layers of equal thickness (0.18 μm), the average difference in the twist angles between two adjacent layers is $\Delta\Phi = 0.5°$. Within each layer, the birefringent effect is much stronger than

Table 1. TN LC Cell Parameters

Refractive Index		Cell gap (μm)	Twist angle (deg.)
n_e	1.600	3.3	90 (−45–45)
n_o	1.490		
Wavelength λ (nm)			633

the LC-director twist-angle effect to cause a phase change for an incident light so that the latter can be neglected. In this case, the Jones matrix G in Eq. (7) was a very good approximation and it took only a few minutes to compute a phase-versus-incident-angle curve of the TN medium.

We assumed that the incident angle θ varies from 30° to −30° and utilized Eq. (5) to calculate the relationship between the incident angle and the phase difference. Figure 2 shows the calculated phase difference versus incident angle using the pretilt angle as a parameter. The simulated results showed that different pretilt angles were obtained from the positions of minimum retardations occurring at corresponding extreme angles. We further plotted the extreme angle versus the pretilt angle as shown in Fig. 3, indicating a linear relationship between the two. We can obtain the sensitivity $S = d\theta_{\text{Extremun}}/d\theta_{\text{Pretilt}} = -2.02$ from our method. It illustrates that the extremum angle is sensitive to a changing pretilt angle. From this relationship, the TN LC pretilt angle can be determined.

We used a heterodyne interferometer with a Zeeman laser system, Agilent 5519A[10], to measure the phase versus incidence angle of our TN cells. The experimental setup is shown in Fig. 4. The OPR method had a rotation axis bisecting the angle between two orthogonal rubbing directions on the polyimide alignment films of the TN cell. The central wavelength of the Zeeman laser was at 633 nm with an orthogonally polarized collinear sideband having a wavelength that deviated by 2.4 MHz. Our TN samples were fabricated in a standard TFT LCD production line by Chimei-Innolux Corporation.

Figure 5 shows the measured results of the phase versus incident angle by the OPR method. The experimental results show that an extreme value of the phase difference appears at a sample rotation angle of −9.8°. Phase values of 4.975 and 4.825 rad were measured at rotation angles of 30° and −30°, respectively.

To analyze the experimental data, we used the extended Jones matrix method programmed in MATLAB to calculate the phase information corresponding to experimental schemes to fit measured curves of phase versus incident angle, as shown in Fig. 6. To reduce the errors in measured pretilt angles and increase the simulation accuracy, three points were chosen to fit the curve: the phase differences at rotation angles of 30° and −30° as well as the extreme angle on the curve of the phase versus rotation angle.

Wang and Yang[9] published intensity measurements to determine pretilt angles of thin TN cells. Three different OPR schemes used to derive pretilt angles of the thin TN
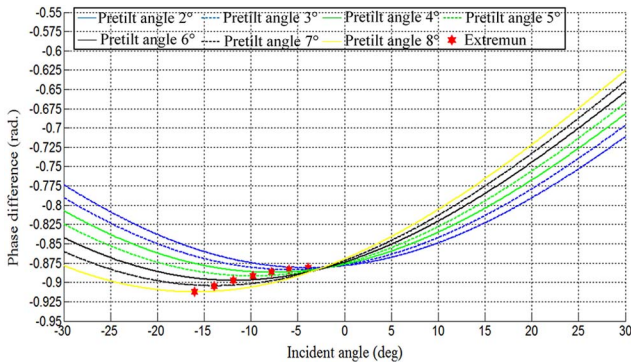

Fig. 2. (Color online) Phase difference of the TN cell versus the incident angle by varying the pretilt angle as a parameter.
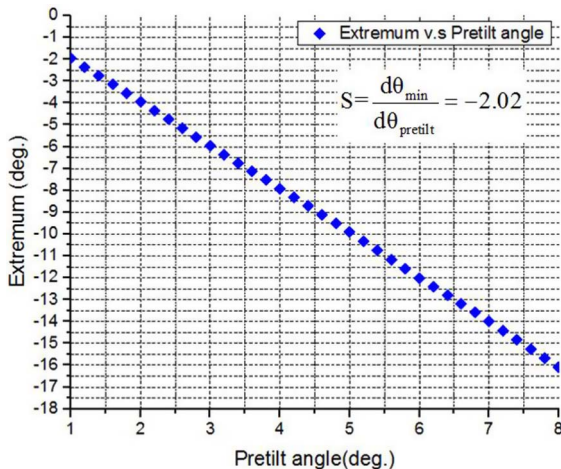

Fig. 4. Configuration of the out-of-plane crystal rotation for the measurements.

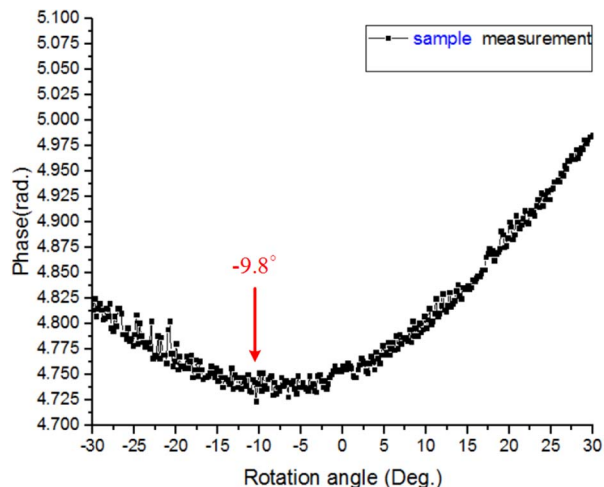Fig. 3 Data diagram of the pretilt angle versus the extreme angle.

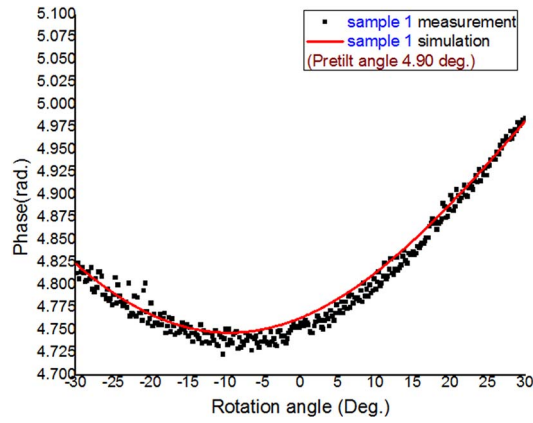Fig. 5. Experimental phase versus the incident angle curves.

Fig. 6. Solid curves are the results of the calculation by the extended Jones matrix method.

Table 2. Experimental Results: Pretilt Angle Measured by OPR Methods

Authors	Pretilt angle (deg.)
Birecki and Kahn [7]	5.70 ± 0.1
Wang and Yang [9]	4.95 ± 0.1
This work	4.90 ± 0.1

cells are listed in Table 2 for comparison. Our measured pretilt angle by phase measurements was in good agreement with the corresponding results of Wang and Yang by intensity measurements.

All prior publications based on the optical IPR method could measure the pretilt angles of TN cells by curve-fitting methods without disengaging the entanglements of the cell gap and pretilt angle. The derived pretilt angle and cell gap were entangled in the equation of the Mauguin parameter so that the measurement errors in the pretilt angle and cell gap were interrelated. If a stringent accuracy is required, the OPR method by Birecki and Kahn[7] is only suitable to measure the pretilt angles of thick TN cells but unsuitable for thin TN cells. For the current products of TFT-TNs with thin cell gaps, accurate pretilt angles can be independently determined by using the OPR method of Wang and Yang[9] with intensity measurements. However, the calculations of the phase versus

OPR angle of the TN cell are much simpler than the calculations of the transmitting intensities by the extended Jones matrix method. The latter[9,13] required four multiplying Jones matrices upon the input Jones vector representing the transmissions through four interfaces in between two consecutively adjacent media of air, glass substrate, ITO electrode, PI alignment layer, and LC mixture, and four similar multiplying matrices in reverse order at the end of the Jones matrix stack. The accuracy of each matrix element within the interfacial matrixes depends on the indices of refraction of the two adjacent media forming the interface. Substantial errors may occur in calculating the interfacial transmissions because it is difficult to measure accurate indices of refraction of the glass substrate, ITO electrode, and PI alignment film within the TN cell.

Based on the extended Jones matrix and the OPR method, we believe that our method of measuring phases has the advantages of simpler setup and calculations, and a higher signal-to-noise ratio due to a lower pickup of ambient EMI than the commonly used method of measuring intensities.

References
1. A. Lien and R. J. John, IBM J. Res. Dev. **36**, 51 (1992).
2. C. C. Tsai, C. Chou, C. Y. Han, C. H. Hsieh, K. Y. Liao, and Y. F. Chao, Appl. Opt. **44**, 7509 (2005).
3. S. T. Tang and H. S. Kwok, J. Appl. Phys. **89**, 80 (2000).
4. G. Baur, V. Wittwer, and D. W. Berreman, Phys. Lett. **56**, 142 (1976).
5. D. W. Berreman, J. Opt. Soc. Am. **62**, 502 (1972).
6. K. H. Yang, J. Appl. Phys. **68**, 1550 (1990).
7. H. Birecki and F.-J. Kahn, in *The Physics and Chemistry of Liquid Crystal Devices*, G. J. Sprokel, ed. (Plenum, 1980), p. 115.
8. C. H. Gooch and H. A. Tarry, J. Phys. D **8**, 1575 (1975).
9. S.-Y. Wang and K.-H. Yang, Appl. Opt. **52**, 219 (2013).
10. S. J. Hwang, J. Disp. Technol. **1**, 77 (2005).
11. A. Lien, Liq. Cryst. **22**, 171 1997.
12. C. H. Wen, S. Gauza, R. Lu, and S. T. Wu, J. Disp. Technol. **1**, 51 (2005).
13. The interfacial transmission matrices between ITO to polyimide-film and polyimide-film to liquid crystal medium have been omitted in reference 9.