# Modal analysis of $1 \times 3$ reflective triangular gratings under normal incidence 

Jin Wang（王 津）$)^{1,2}$ ，Changhe Zhou（周常河）${ }^{1, *}$ ，Jianyong Ma（麻健勇）${ }^{1}$ ， Yonghong Zong（宗永红）$)^{1,2}$ ，and Wei Jia（贾 伟）${ }^{1}$<br>${ }^{1}$ Shanghai Institute of Optics and Fine Mechanics，Chinese Academy of Sciences，Shanghai 201800，China<br>${ }^{2}$ University of Chinese Academy of Sciences，Beijing 100049，China<br>＊Corresponding author：chazhou＠mail．shcnc．ac．cn

Received October 27，2016；accepted December 23，2016；posted online February 10， 2017


#### Abstract

Modal analysis of the $1 \times 3$ highly efficient reflective triangular grating operating in the 800 nm wavelength under normal incidence for TE polarization is presented in this Letter．The rigorous coupled wave analysis and simulated annealing algorithm are used to design this beam splitter．The reflective grating consists of a highly reflective mirror and a transmission grating on the top．The mechanism of the reflective triangular grating is clarified by the simplified modal method．Then，gratings are fabricated by direct laser writing lithography． OCIS codes：090．1950，050．0050，050．1950． doi：10．3788／COL201715．040902．


In a high－power laser system，reflective gratings with high efficiency are needed ${ }^{[1]}$ ．Reflective gratings are also used in various fields，such as beam combining，pulse compres－ sion ${ }^{[2]}$ ，infrared spectroscopy ${ }^{[3]}$ ，and femtosecond lasers ${ }^{[4]}$ ． Metal dielectric reflection gratings as three－channel beam splitters can be widely used in optical systems of hologra－ phy and interferometers ${ }^{[5]}$ ．The beam splitters are key el－ ements and have been widely researched and used ${ }^{[6,7]}$ ． However，conventional beam splitter gratings，such as Dammann gratings $s^{[8]}$ ，have some disadvantages，including low efficiency．The efficiency of traditional Dammann gra－ tings is about $75 \%{ }^{[9]}$ ．Such an efficiency is not high enough to meet the high－efficiency requirement of high－power la－ ser systems．Hence，subwavelength Dammann gratings with higher efficiencies should be taken into consideration．

The $1 \times 3$ Dammann grating is designed and optimized by the rigorous coupled wave analysis（RCWA）${ }^{[10]}$ and simulated annealing（SA）algorithm ${ }^{[11,12]}$ ．SA is a probabi－ listic technique that can approximate the global optimum of a given function．However，the RCWA is a purely numerical method，and it seems difficult to know the mechanism of triangular gratings by only using a numeri－ cal method．Therefore，for further understanding，we use the simplified modal method to explain the interference process in this grating．The modal method was first devel－ oped by Rytov and Collin $-\frac{[13,14]}{}$ ，and it was Botten et al．${ }^{[15]}$ who first applied it to dielectric gratings．The simplified modal method has been used to clarify the mechanisms of different gratings $\stackrel{[16-22]}{ }$ ．Zheng et al．analyzed the trans－ mission of triangular－groove gratings by the simplified mo－ dal method ${ }^{[17]}$ ．The expression of transmission the $1 \times 3$ Dammann grating under normal incidence has also been derived by the simplified modal method ${ }^{[18]}$ ．But this expression is not suitable for reflective gratings．Reflective gratings have only been analyzed in a Littrow moun－ ting ${ }^{[21,22]}$ with the simplified modal method by Hu et al．In addition，as we know，this method has already been
applied to analyze beam splitting gratings under the sec－ ond Bragg incidence ${ }^{[20]}$ ．However，a simplified modal analy－ sis of reflective triangular gratings under normal incidence has not been done before to the best of our knowledge．

In this Letter，the metal－mirror－based reflective grating consists of a highly reflective mirror and a transmission grating on the top．By using the grating modes and ignor－ ing the absorption of the mirror and the evanescent wave， the expressions for the diffraction efficiency of the reflec－ tive grating can be derived．The diffraction efficiency of the optimized grating is higher than $98 \%$ at the wave－ length of 800 nm ，and these Dammann gratings are fabricated by direct laser writing（DLW）lithography．

Figure 1 is the schematic of this $1 \times 3$ reflective triangu－ lar grating for TE polarization under normal incidence；it is composed of a grating layer（depth $h$ and index $n_{2}=1.65$ ），a thin gold layer（depth $d_{a}=200 \mathrm{~nm}$ and in－ dex $n_{a}$ ），and a fused－silica substrate（index $n_{s}$ ）．$T$ repre－ sents the period of the grating，and $n_{1}$ is the refractive index of the incident medium，which is $n_{1}=1$ in air．

This kind of grating is similar to a gold grating．Hence， the threshold of it is about $0.6 \mathrm{~J} / \mathrm{cm}^{2}$ in a high－power laser


Fig．1．Schematic of grating．
system ${ }^{[23]}$. The RCWA in association with the SA algorithm is adopted to calculate and optimize this structure. The set $[T, h]$ is chosen as the optimization parameters. The merit function (MF) is

$$
\begin{gather*}
\mathrm{MF}=\frac{1}{M} \sum_{\lambda}\left[\frac{\sum_{p=-1}^{1}\left(\eta_{p}-\eta_{\mathrm{av}}\right)^{2}}{\sum_{p=-1}^{1} \eta_{p}}\right]^{1 / 2},  \tag{1}\\
\eta_{\mathrm{av}}=\frac{1}{3} \sum_{p=-1}^{1} \eta_{p}, \tag{2}
\end{gather*}
$$

where $\eta_{p}$ is the reflective efficiency of the $p$ th order calculated by the RCWA, and $\eta_{\mathrm{av}}$ is the average value of the efficiencies of the three orders. $M$ is the number of wavelength points from 780 to 820 nm .

The RCWA and SA algorithm are used to make the minimization subject to MF. The smaller the value of MF is, the closer the iteration point is to the optimal parameter. After optimization, we obtained the optimal value [1290, 558]. The unit is nanometer. The MF with optimal values may not be the global minimum. But the efficiency and uniformity of the designed gratings with the optimal values can meet our requirements. The efficiencies of the zeroth order and the first order are $32.24 \%$ and $33.21 \%$ in theory.

Figure $\underline{2}$ shows the efficiencies of the zeroth and first orders for TE polarization versus the wavelength under normal incidence. The efficiencies of the $\pm 1$ st are equal under normal incidence. From 780 to 820 nm , the efficiencies are all above $30 \%$. Especially around the wavelength of 800 nm , the efficiencies of the three orders are nearly equal, which demonstrates the good ability of the beam splitting.

The efficiency and uniformity will change if the groove depth or period deviates from the optimized values. Figure $\underline{3}$ shows the contour of the ratio of efficiency of the first order and zeroth order versus the period $T$ and depth $h$. From Fig. 3, it can be seen that the ratio can be nearly equal to 1 and the gratings can be done with


Fig. 2. Diffraction efficiencies of the zeroth and first orders.


Fig. 3. Contour of the efficiency ratio between the first and the zeroth diffractive orders versus the period and depth.
a period ranging from 1200 to 1300 nm and a depth ranging from 480 to 620 nm . Hence, around wavelength 800 nm , there is a large tolerance, which provides convenience for fabrication.
For further understanding, we use the simplified modal method to explain the interference process in this grating. According to the simplified modal method, when the period of a grating is of the order of the wavelength, the diffraction of the grating could be predicted by a few propagating grating modes. Since the contrast of the refractive indices is low $\left(n_{2} / n_{1}<2\right)$, the modes of the gratings can be regarded as propagating independently. For a triangular grating, it can be approximated by a stack of lamellar gratings. Compared to the Littrow incidence, the expression of efficiencies of zeroth and $\pm 1$ st orders are more complicated for TE polarization under normal incidence.
The effective indices of every lamellar grating can be found by solving the following eigenfunction equation for TE polarization:

$$
\begin{align*}
& \cos \left[k_{1}(1-f) T\right] \cos \left(k_{2} f T\right) \\
& -\frac{k_{1}^{2}+k_{2}^{2}}{2 k_{1} k_{2}} \sin \left[k_{1}(1-f) T\right] \sin \left(k_{2} f T\right)=\cos (\alpha T),  \tag{3}\\
& k_{i}=k_{0} \sqrt{n_{i}^{2}-n_{\text {eff }}^{2}}, \quad i=1,2, \tag{4}
\end{align*}
$$

with $\alpha=k_{0} \sin \theta$ and $\cos (\alpha T)=1$, with incidence angle $\theta=0^{\circ}$ and the wavenumber $k_{0}=2 \pi / \lambda$ in a vacuum. For this triangular-groove grating, the duty cycle $f$ goes from 0 (top) to 1 (bottom). The average mode indices of mode 0 , mode 1 , and mode 2 are shown as follows:

$$
\begin{align*}
& \bar{n}_{\text {oeff }}=\sum_{i} n_{i, \text { oeff }} h_{i} / h,  \tag{5}\\
& \bar{n}_{\text {eff }}=\sum_{i} n_{i, \text { leff }} h_{i} / h, \tag{6}
\end{align*}
$$

$$
\begin{equation*}
\bar{n}_{2 \mathrm{eff}}=\sum_{i} n_{i, 2 \mathrm{eff}} h_{i} / h \tag{7}
\end{equation*}
$$

where $h_{i}$ is the depth of the $i$ th lamellar grating, and $n_{i, 0 \text { eff }}$, $n_{i, \text { eff }}$, and $n_{i, 2 \text { eff }}$ are the effective indices of mode 0 , mode 1 , and mode 2 of the corresponding layer. Hence, we can obtain the average mode indices of mode 0 , mode 1 , and mode 2 with a period of 1290 nm and a wavelength of 800 nm , and they are equal to $1.48733,1.24690$, and 1.08297, respectively.

A reflective grating consists of a transmission grating and a highly reflective mirror. The propagating and diffraction in the grating are shown in Fig. 4. When the reflective grating is illuminated under normal incidence, firstly, three transmission diffractive orders occur. $T_{A}$, $T_{B}$, and $T_{C}$ represent the diffractive orders of the first diffraction.

According to the diffraction under normal incidence ${ }^{[18]}$, the complex amplitudes of three orders can be derived as

$$
\begin{align*}
e_{0} & =\frac{1}{T} \int_{0}^{T}\left[t_{0} u_{0}(x) e^{-i k_{z} \bar{n}_{\text {eff }} h}+t_{2} u_{2}(x) e^{-i k_{z} \bar{n}_{2 \text { eff }} h}\right] \mathrm{d} x  \tag{8}\\
e_{ \pm 1} & =\frac{1}{T} \int_{0}^{T}\left[t_{0} u_{0}(x) e^{-i k_{z} \bar{n}_{\text {eff }} h}+t_{2} u_{2}(x) e^{-i k_{z} \bar{n}_{2 \text { eff }} h}\right] \mathrm{d} x \tag{9}
\end{align*}
$$

where $t_{m}$ can be derived from the overlap integral, and $u_{m}(x)$ is the electric field of mode $m(m=0,2)$, with $k_{z}=$ $2 \pi / \lambda$ and $k_{x}=2 \pi / T$.

Then, the diffractive orders are reflected back by the Au layer and diffracted again by the grating with nine diffraction waves denoted by 1 to 9 . According to the grating equation, the zeroth order diffracts again under normal incidence, while the $\pm 1$ st orders diffract again under the second Bragg incidence. Hence, the expressions of the complex amplitudes of the second diffraction under normal incidence are the same as Eqs. (́) and (ㅂ), and the others are derived from the diffraction under the second Bragg incidence. The modal method analysis has been done under the second Bragg incidence ${ }^{[20]}$; therefore, the complex amplitudes of the three orders can be written as

$$
\begin{equation*}
e e_{-1}=\frac{1}{T} \int_{0}^{T}\left[u_{0}(x) e^{-i k_{z}\left(\bar{n}_{\text {eff }}-\bar{n}_{2 \text { eff }}\right) h}+u_{2}(x)\right] \mathrm{d} x \tag{10}
\end{equation*}
$$



Fig. 4. Diffraction process of the metal-mirror-based reflective grating.

$$
\begin{align*}
e e_{0}= & \frac{1}{T} \int_{0}^{T}\left[\left(u_{0}(x) e^{-i k_{z}\left(\bar{n}_{\text {eff }}-\bar{n}_{\text {eff }}\right) h}+u_{2}(x)\right) \cos \left(k_{x} x\right)\right. \\
& \left.+i u_{1}(x) e^{-i k_{z}\left(\bar{n}_{\text {leff }}-\bar{n}_{2 \text { eff }}\right) h} \sin \left(k_{x} x\right)\right] \mathrm{d} x  \tag{11}\\
e e_{-2}= & \frac{1}{T} \int_{0}^{T}\left[\left(u_{0}(x) e^{-i k_{z}\left(\bar{n}_{\text {oeff }}-\bar{n}_{\text {eff }}\right) h}+u_{2}(x)\right) \cos \left(k_{x} x\right)\right. \\
& \left.-i u_{1}(x) e^{-i k_{z}\left(\bar{n}_{\text {eeff }}-\bar{n}_{\text {eff }}\right) h} \sin \left(k_{x} x\right)\right] \mathrm{d} x . \tag{12}
\end{align*}
$$

So, considering twice diffraction, the complex amplitudes $R_{j}$ of the nine diffraction waves can be obtained, where $j$ is from 1 to 9 . The expressions are shown as follows: $R_{1}=e_{1} * e e_{0} * \exp (i a), R_{2}=e_{1} * e e_{-1} * \exp (i a)$, $R_{3}=e_{1} * e e_{-2} * \exp (i a), R_{4}=e_{0} * e_{1} * \exp (i a), R_{5}=e_{0}^{2} *$ $\exp (i a), \quad R_{6}=e_{0} * e_{1} * \exp (i a), R_{7}=e_{1} * e e_{-2} * \exp (i a)$, $R_{8}=e_{1} * e e_{-1} * \exp (i a), \quad$ and $\quad R_{9}=e_{1} * e e_{0} * \exp (i a)$, where $a$ is the phase difference.

The interference between diffraction waves $R_{1}, R_{4}$, and $R_{7}$ will determine the diffraction efficiency of the positive first order of this reflective grating, and that between diffraction waves $R_{2}, R_{5}$, and $R_{8}$ determines the diffraction efficiency of the zeroth order. The negative first order is decided by $R_{3}, R_{6}$, and $R_{9}$. Hence, the total complex amplitudes of this reflective grating are $E_{+1}=R_{1}+R_{4}+R_{7}$, $E_{0}=R_{2}+R_{5}+R_{8}$, and $E_{-1}=R_{3}+R_{6}+R_{9}$. The efficiency can be obtained using

$$
\begin{align*}
\eta_{ \pm 1} & =\left|E_{+1}\right|^{2}=\left|E_{-1}\right|^{2}=\left|e_{1}\left(e e_{0}+e e_{-2}+e_{0}\right) \exp (i \alpha)\right|^{2}  \tag{13}\\
\eta_{0} & =\left|E_{0}\right|^{2}=\left|\left(2 e_{1} e e_{-1}+e_{0}^{2}\right) \exp (i \alpha)\right|^{2} \tag{14}
\end{align*}
$$

In this Letter, a $1 \times 3$ beam splitter is designed, where $\eta_{0}=\eta_{+1}=\eta_{-1}$ is required. Therefore, for a $1 \times 3$ splitting triangular grating with period of 1290 nm operating in the 800 nm wavelength under normal incidence, the depth can be obtained using Eqs. (8) to (14) and is equal to 533 nm approximately when neglecting the evanescent wave.

The gratings can be made on a photoresist by the lithography technique $\stackrel{[24,25]}{ }$. We have already built a DLW lithography system based on a Dammann grating ${ }^{[26-30]}$; it has the advantages of being low cost, having low requirements for the environment, and having a high flexibility. A DLW system based on a rotating Dammann grating is shown in Fig. 5.

The method of the rotating Dammann grating is a more flexible way and can improve the precision of the period. The period of designed grating is

$$
\begin{equation*}
T=d_{0} \cos \varphi \tag{15}
\end{equation*}
$$

where $d_{0}$ is the initial period based on the Dammann grating, and $\varphi$ is the rotation angle. In this setup, a $1 \times 6$ Dammann grating is applied. According to the design of whole system, the initial period $d_{0}$ is 1861.68 nm . Therefore, the rotation angle $\varphi$ is about $46.01^{\circ}$.

The $1 \times 3$ triangular grating is written directly on the photoresist mask. Figure $\underline{6}$ is the angular spectrum that


Fig. 5. Schematic of laser writing system.
shows the theoretical efficiency with optimal parameters and experimental values of the $\pm 1$ st order and the zeroth order for different incident angles at the wavelength of 800 nm for TE polarization.

The experimental values basically agree with the theoretical values. Figure $\underline{7}$ is the atomic force microscopy (AFM) results of the fabricated grating. From the AFM results, the measured values of the period and depth are 1295 and 529 nm . The structure is similar to a triangle, which is consistent with the theoretical design. The efficiency of this fabricated beam splitter can reach approximately $88 \%$.

There are differences between the measured results and the optimal values. The uncertainties associated with the experimental results are the measurement errors, the writing lithography errors, and the processing technology. Another important factor is that the groove structure approximates to the triangular structure because the distribution of the energy of the laser spot is a Gaussian distribution. Hence, the deviations of the grating period and the groove depth from the optimal parameters as well as the deviation of the grating profile from the


Fig. 6. Angular spectrum.

(b)


Fig. 7. AFM results: (a) AFM image, and (b) the profile along the line in (a).
standard triangular groove can all influence the efficiency. In addition, the slight roughness of the grating surface may also decrease the efficiency. Most uncertainties are not quantified. The difference between the about $98 \%$ design efficiency and the $88 \%$ experimental efficiency falls within the experimental uncertainty.

In conclusion, we analyze and fabricate a $1 \times 3$ reflective triangular Dammann grating operating in the 800 nm wavelength under normal incidence for TE polarization with a high efficiency of more than $98 \%$. Based on the former simplified modal analysis, the mechanism of the $1 \times 3$ reflective triangular Dammann grating is presented in this Letter. The simplified modal method gives a brief explanation from the aspect of physics mainly. We offer the guidelines for the design of a $1 \times 3$ beam splitter grating. The experimental efficiency of this beam splitter is about $88 \%$.

This work was supported by the National Natural Science Foundation of China (NSFC) (Nos. 61307064 and 61405214) and the Ministry of Science and Technology of China (MOST) (No. 2012YQ170004).

## References

1. M. Jiang, P. Zhou, H. Xiao, and P. Ma, High Power Laser Sci. Eng. 3, e25 (2015).
2. J. R. Leger, G. J. Swanson, and W. B. Veldkamp, Appl. Opt. 26, 4391 (1987).
3. P. M. Vandenbe, Appl. Sci. Res. 24, 261 (1971).
4. G. Li, C. Zhou, and E. Dai, J. Opt. Soc. Am. A-Opt. Image Sci. Vision 22, 767 (2005).
5. R. Schnabel, A. Bunkowski, O. Burmeister, and K. Danzmann, Opt. Lett. 31, 658 (2006).
6. S. Duan, Y. Chen, G. Li, C. Zhu, and X. Chen, Chin. Opt. Lett. 14, 042301 (2016).
7. J. Yang, J. Zhang, S. Xu, and S. Chang, Chin. Opt. Lett. 13, S12501 (2015).
8. H. Dammann and E. Klotz, Opt. Acta 24, 505 (1977).
9. C. Zhou and L. Liu, Appl. Opt 34, 5961 (1995).
10. M. G. Moharam, E. B. Grann, D. A. Pommet, and T. K. Gaylord, J. Opt. Soc. Am. A-Opt. Image Sci. Vision 12, 1068 (1995).
11. C.-R. Hwang, Acta Appl. Math. 12, 108 (1988).
12. S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, Science 220, 671 (1983).
13. S. M. Rytov, Sov. Phys. JETP 2, 466 (1956).
14. R. E. Collin, Can. J. Phys. 34, 398 (1956).
15. L. C. Botten, M. S. Craig, R. C. Mcphedran, J. L. Adams, and J. R. Andrewartha, Optica Acta 28, 1087 (1981).
16. S. Li, C. Zhou, S. Wang, H. Cao, W. Jia, and J. Wu, in Digital Holography \& 3-D Imaging Meeting, OSA Technical Digest (2015), paper DW5A.6.
17. J. Zheng, C. Zhou, J. Feng, and B. Wang, Opt. Lett. 33, 1554 (2008).
18. J. Feng, C. Zhou, B. Wang, J. Zheng, W. Jia, H. Cao, and P. Lv, Appl. Opt. 47, 6638 (2008).
19. S. Li, C. Zhou, H. Cao, and J. Wu, Opt. Lett. 39, 781 (2014).
20. Z. Sun, C. Zhou, H. Cao, and J. Wu, J. Opt. Soc. Am. A 32, 1952 (2015).
21. A. Hu, C. Zhou, H. Cao, J. Wu, J. Yu, and W. Jia, Appl. Opt. 51, 4902 (2012).
22. A. Hu, C. Zhou, H. Cao, J. Wu, J. Yu, and W. Jia, J. Opt. 14, 055705 (2012).
23. B. C. Stuart, M. D. Feit, S. Herman, A. M. Rubenchik, B. W. Shore, and M. D. Perry, J. Opt. Soc. Am. B 13, 459 (1996).
24. J. Fischer and M. Wegener, Opt. Mater. Express 1, 614 (2011).
25. T. Wang, W. Yu, D. Zhang, C. Li, H. Zhang, W. Xu, Z. Xu, H. Liu, Q. Sun, and Z. Lu, Opt. Express 18, 25102 (2010).
26. F. Zhu, J. Ma, W. Huang, J. Wang, and C. Zhou, Chin. Opt. Lett. 12, 080501 (2014).
27. J. Wang, C. Zhou, J. Ma, Y. Zong, and W. Jia, Appl. Opt. 55, 5203 (2016).
28. J. Ma, F. Zhu, and C. Zhou, IEEE Photon. Technol. Lett. 26, 1364 (2014).
29. W. Huang, J. Ma, F. Zhu, J. Wang, and C. Zhou, Chin. Opt. Lett. 12, 070501 (2014).
30. F. Zhu and J. Ma, Chin. Phys. Lett. 31, 185 (2014).
