Experimental determination of the azimuthal and radial mode orders of a partially coherent LG_{pl} beam

(Invited Paper)

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It is known that one can determine the mode orders (i.e., the azimuthal order and radial order) of a partially coherent LG_{pl} beam (i.e., a partially coherent vortex beam) based on the measurement of the cross-correlation function (CCF) and the double correlation function (DCF) together. The technique for measuring the CCF is known. In this Letter, we propose a method for measuring the DCF. Based on the proposed method, the determination of the mode orders of a partially coherent LG_{pl} beam is demonstrated experimentally.

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In the past decades, vortex beams have been explored extensively and applied in various fields, such as optical manipulation[1], optical imaging[2], free-space data transmission[3], quantum information transfer[4], and the detection of a spinning object[5]. The phase term $\exp(il\varphi)$ (l and φ are the topological charge and azimuthal angle, respectively) in a vortex beam imposes orbital angular momentum (OAM) on the beam, and each photon of such a beam has an OAM of $l\hbar$[6]. Up to now, many methods based on the measurement of the intensity distribution, diffraction, or interference pattern of a vortex beam were proposed to measure the OAM or l[7–10].

Partially coherent vortex beams were introduced by Gori, Ponomarenko, and collaborators[11–13]. Partially coherent LG_{0l} beams and LG_{pl} beams are typical kinds of partially coherent vortex beams. A partially coherent vortex beam can be generated by imposing a vortex phase into a partially coherent beam through a spiral phase plate[14,15] or a spatial light modulator (SLM)[16]. More recently, it was shown in Ref. [17] that one can generate a partially coherent vortex beam with an arbitrary azimuthal index using only an SLM. Partially coherent vortex beams exhibit interesting properties, e.g., their cross-correlation function (CCF) displays correlation singularities (i.e., ring dislocations)[14]. The correlation singularity is defined as the point where the magnitude of the CCF is zero, but the phase is undefined. It was found in theory[18,19] and verified in an experiment[20] that the magnitude of the topological charge $|l|$ of a partially coherent LG_{0l} beam can be determined by measuring the far-field ring dislocations due to the fact that $|l|$ equals the number of ring dislocations. More recently, it was demonstrated in Ref. [21] that one can determine not only $|l|$ but also the sign of l of a partially coherent LG_{0l} beam through the measurement of the mutual correlation function (MCF) or CCF with the help of cylindrical lenses. For a partially coherent LG_{pl} beam[22], it was demonstrated both theoretically[23] and experimentally[24] that the far-field ring dislocations equals $2p+|l|$. It was demonstrated numerically that one can determine the azimuthal and radial mode orders (i.e., p and $|l|$) through measuring the CCF and the double correlation function (DCF) together[25]. The technique for measuring the MCF or CCF is known[20,21,24,26]. In this Letter, a method is proposed for measuring the DCF, and we demonstrate experimental determination of the mode orders of a partially coherent LG_{pl} beam based on the proposed method.

The MCF of a partially coherent LG_{pl} beam at $z=0$ (source plane) in a cylindrical coordinate system is expressed as[16,22,23]

$$\Gamma(\rho_1,\rho_2) = \left(\frac{\sqrt{2}\rho_1}{\omega_0}\right)^l \left(\frac{\sqrt{2}\rho_2}{\omega_0}\right)^l L_p^l\left(\frac{2\rho_1^2}{\omega_0^2}\right) L_p^l\left(\frac{2\rho_2^2}{\omega_0^2}\right)\exp[il(\varphi_1-\varphi_2)] \times \exp\left(-\frac{\rho_1^2+\rho_2^2}{\omega_0^2}\right)\exp\left[-\frac{\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\varphi_1-\varphi_2)}{2\delta_g^2}\right], \tag{1}$$

where ρ is the radial coordinate and φ is the azimuthal coordinate, ω_0 is the beam width and δ_g is the coherence width, p and l denote the radial and azimuthal orders, respectively. Usually, l is called the topological charge.

With the help of the generalized Collins formula[27], we can obtain the detail expression for the MCF $\Gamma(\mathbf{r}_1,\mathbf{r}_2)$ of a partially coherent LG_{pl} beam after passing through a paraxial ABCD optical system (see Ref. [22]). The average intensity is obtained as $\langle I(\mathbf{r})\rangle = \Gamma(\mathbf{r},\mathbf{r})$, and the normalized MCF is obtained as[28]

$$\mu(\mathbf{r}_1,\mathbf{r}_2) = \frac{\Gamma(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{\langle I(\mathbf{r}_1)\rangle\langle I(\mathbf{r}_2)\rangle}}. \tag{2}$$

For the case of $\mathbf{r}_2 = -\mathbf{r}_1 = -\mathbf{r}$, the MCF $\Gamma(\mathbf{r},-\mathbf{r})$ usually is called the CCF[14]. For the case of $\mathbf{r}_1 = \mathbf{r}$, $\mathbf{r}_2 = 2\mathbf{r}$, the MCF $\Gamma(\mathbf{r},2\mathbf{r})$ is called the DCF[25]. As shown in Refs. [18,19], the MCF $\Gamma(\mathbf{r},0)$ and the CCF $\Gamma(\mathbf{r},-\mathbf{r})$ display similar distributions.

The distribution of the far-field normalized MCF is similar to that in the focal plane. We assume that the partially coherent LG_{pl} beam is focused by a thin lens with focal length f, which is located at $z = 0$, and the receiver plane is at $z = f$. Then, we obtain $A = 0$, $B = f$, $C = -1/f$, and $D = 1$. We calculate in Fig. 1 the normalized average intensity and modulus of the correlation functions of a partially coherent LG_{pl} beam with $f = 40$ cm, $p = 1$, $l = 1$, $\omega_0 = 0.57$ mm, and $\delta_g = 0.8$ mm in the focal plane. Figure 1(a) clearly shows that the average intensity of a partially coherent LG_{pl} beam does not reveal any information about p and l. Figures 1(b) and 1(c) show that the distributions of the normalized MCF and CCF display ring dislocations are as expected, and the number of dark rings is equal to $2p + |l|$, as expected[23,24]. Figure 1(d) shows that the DCF also displays ring dislocation, while only one dark ring exists in this case.

Figure 2 shows the distribution of the normalized modulus of the DCF of a partially coherent LG_{pl} beam for different l with $p = 1, \omega_0 = 0.57$ mm, and $\delta_g = 0.8$ mm in the focal plane. One sees from Fig. 2 that only one dark ring exists for different l, even for $l = 0$ (without the vortex phase), while the size of the dark ring decreases with the increase of l. Figure 2 indicates that the azimuthal model

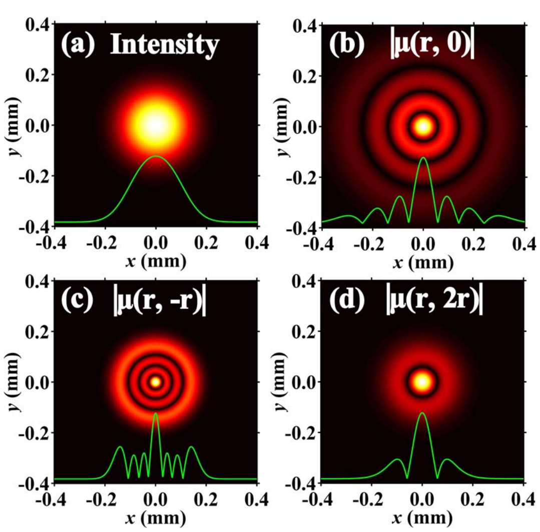

Fig. 1. Distributions of the normalized average intensity and modulus of the correlation functions of a partially coherent LG_{pl} beam with $p = 1$, $l = 1$, $\omega_0 = 0.57$ mm, and $\delta_g = 0.8$ mm in the focal plane. (a) $I(\mathbf{r})$, (b) $|\mu(\mathbf{r},0)|$, (c) $|\mu(\mathbf{r},-\mathbf{r})|$, and (d) $|\mu(\mathbf{r},2\mathbf{r})|$.

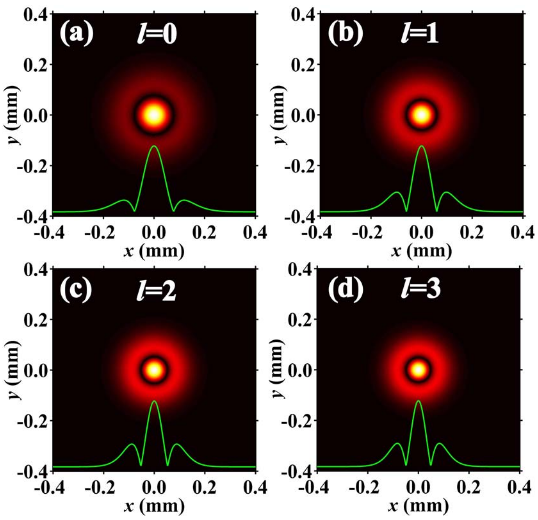

Fig. 2. Distribution of the normalized modulus of the DCF of a partially coherent LG_{pl} beam for different l with $p = 1$. (a) $l = 0$, (b) $l = 1$, (c) $l = 2$, and (d) $l = 3$.

order l does not affect the number of the dark rings in the normalized DCF. Figure 3 shows the distribution of the normalized modulus of the DCF of a partially coherent LG_{pl} beam for different p with $l = 6$, $\omega_0 = 0.57$ mm, and $\delta_g = 0.8$ mm in the focal plane. One finds from Fig. 3 that the number of dark rings in the distribution of the normalized DCF increases as p increases, and its value equals p. Thus, as indicated in Ref. [25], one can determine the value of p of a partially coherent LG_{pl} beam through measuring its DCF in the focal plane. Then, one can determine $|l|$ through measuring the MCF or CCF in the focal plane.

Now, we demonstrate the determination of the azimuthal and radial mode orders of a partially coherent LG_{pl} beam experimentally. The experimental setup for generating a partially coherent LG_{pl} beam and measuring the

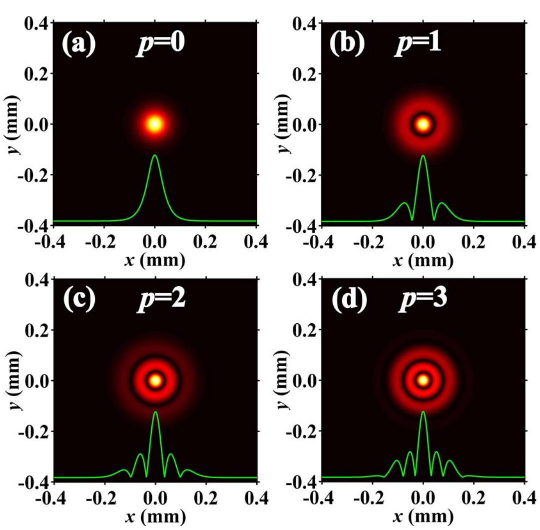

Fig. 3. Distribution of the normalized modulus of the DCF of a partially coherent LG_{pl} beam for different p with $l = 6$ in the focal plane. (a) $p = 0$, (b) $p = 1$, (c) $p = 2$, and (d) $p = 3$.

correlation functions is shown in Fig. 4. A light beam from a diode-pump solid-state laser ($\lambda = 532$ nm) first propagates through a neutral density filter and a beam expander and then is reflected by a reflecting mirror (RM). The reflected beam from the RM is focused by a thin lens L_1 onto a rotating ground-glass disk (RGGD), producing incoherent light. The incoherent light from the RGGD propagates through a thin lens L_2 and a Gaussian amplitude filter, then becomes a partially coherent beam with a Gaussian beam profile[29]. The generated partially coherent Gaussian beam goes towards an SLM. The SLM plays the role of a grating with a fork pattern, which is calculated through computer-generated holograms. A circular aperture is used to select the first-order diffraction pattern of the light field from the SLM, and then, a partially coherent LG_{pl} beam is obtained[16,24]. The generated partially coherent LG_{pl} beam is focused onto a charge-coupled device (CCD) by a thin lens L_3 with a focal length $f_3 = 40$ cm. The CCD is used to measure the average intensity, MCF, CCF, and DCF. The coherence width of the partially coherent LG_{pl} beam is controlled by varying the focused beam's spot size on the RGGD[18,29].

With the help of the Gaussian moment theorem[28], the normalized MCF of a partially coherent beam is related with the normalized fourth-order correlation function by the following expression,

$$g^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{\langle I(\mathbf{r}_1)I(\mathbf{r}_2)\rangle}{\langle I(\mathbf{r}_1)\rangle\langle I(\mathbf{r}_2)\rangle} = 1 + |\mu(\mathbf{r}_1, \mathbf{r}_2)|^2, \quad (3)$$

where $I(\mathbf{r})$ and $\langle I(\mathbf{r})\rangle$ denote the instantaneous intensity and the average intensity at position $\mathbf{r}$, respectively.

In our experiments, a CCD with a pixel size $4.4\,\mu\text{m} \times 4.4\,\mu\text{m}$ captures 4000 frames in total. Each frame represents one realization of the beam cross section and is read out as an intensity matrix $I_n(x, y)$ by Matlab software. Here, n is the sequence number of the frames ranging from 1 to 4000, and x and y are the pixel spatial coordinates of the frame. We set the coordinate origin (0,0) of each frame at position $(2m + 1, 2m + 1)$. Then, we have $-2m \le x \le 2m$ and $-2m \le y \le 2m$, with $m \ge 0$ being a non-negative integer.

The average intensity can be obtained from the following expression,

$$\langle I(\mathbf{r})\rangle = \sum_n^N I(\mathbf{r})/N, \quad (4)$$

where N is the total number of captured frames. The term $\langle I(\boldsymbol{r}_1)I(\boldsymbol{r}_2)\rangle$ can be obtained by the following expression,

$$\langle I(\mathbf{r}_1)I(\mathbf{r}_2)\rangle = \sum_n^N I_n(\mathbf{r}_1)I_n(\mathbf{r}_2)/N. \quad (5)$$

Then, we can obtain following expression for the square of the normalized modulus of the MCF,

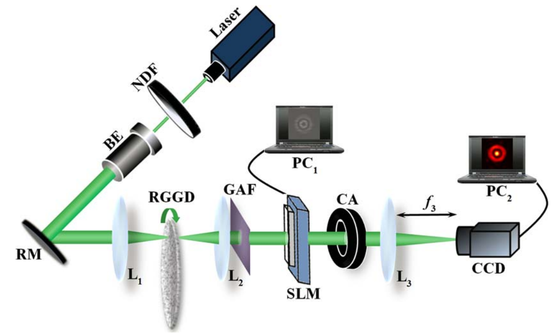

Fig. 4. Experimental setup for generating a partially coherent LG_{pl} beam and measuring the correlation functions. NDF, neutral density filter; BE, beam expander; L_1, L_2, L_3, thin lenses; GAF, Gaussian amplitude filter; CA, circular aperture; PC_1, PC_2, personal computers.

$$|\mu(\mathbf{r}_1, \mathbf{r}_2)|^2 = \frac{N\sum_n^N I_n(\mathbf{r}_1)I_n(\mathbf{r}_2)}{\sum_n^N I_n(\mathbf{r}_1)\sum_n^N I_n(\mathbf{r}_2)} - 1. \quad (6)$$

Based on Eq. (6), by setting $\mathbf{r}_1 = \mathbf{r}$ and fixing $\mathbf{r}_2$ at the coordinate origin, one can obtain the square of the normalized modulus of MCF $|\mu(\mathbf{r}, 0)|^2$[26]. By measuring the average intensity $\langle I(\boldsymbol{\rho})\rangle$ and $|\mu(\boldsymbol{\rho}, 0)|^2$ in the source plane, one can obtain the values of the beam width and coherence width of the generated partially coherent LG_{pl} beam, respectively, and in our experiment, they are $\omega_0 = 0.57$ and $\delta_g = 0.8$ mm.

One can obtain the new matrix $I_n(-\mathbf{r})$ by rotating the intensity matrix $I_n(\mathbf{r})$ 180 deg. Based on Eq. (6), we can obtain the square of the normalized modulus of the CCF $|\mu(\mathbf{r}, -\mathbf{r})|^2$. More information about measurements of the MCF and CCF can be found in Refs. [24,26], respectively.

Now we introduce a method for measuring the DCF. In order to measure the DCF, we first extract a sub-matrix $I'_n(x', y')$ with dimensions $(2m + 1) \times (2m + 1)$ from the average intensity matrix $I_n(x, y)$, and the sub-matrix elements are composed by the elements from line $m + 1$ to $3m$ and row $m + 1$ to $3m$ of $I_n(x, y)$, e.g., $-m \le x' \le m$, $-m \le y' \le m$,

$$I'_n(x', y') = \begin{pmatrix} (x'_{-m}, y'_m) & (x'_{-m+1}, y'_m) & \cdots & (x'_m, y'_m) \\ (x'_{-m}, y'_{m-1}) & (x'_{-m+1}, y'_{m-1}) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ (x'_{-m}, y'_{-m}) & \cdots & \cdots & (x'_m, y'_{-m}) \end{pmatrix}$$
$$= \begin{pmatrix} (x_{-m}, y_m) & (x_{-m+1}, y_m) & \cdots & (x_m, y_m) \\ (x_{-m}, y_{m-1}) & (x_{-m+1}, y_{m-1}) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ (x_{-m}, y_{-m}) & \cdots & \cdots & (x_m, y_{-m}) \end{pmatrix}. \quad (7)$$

The sub-matrix $I'_n(2x', 2y')$ with dimensiona $(2m + 1) \times (2m + 1)$ is composed from the matrix elements $I_n(x, y)$ of the odd line and row, which can be expressed as

$$
\begin{aligned}
&I'_n(2x', 2y') \\
&= I''_n(x'', y'') \\
&= \begin{pmatrix} (x''_{-m}, y''_m) & (x''_{-m+1}, y''_m) & \cdots & (x''_m, y''_m) \\ (x''_{-m}, y''_{m-1}) & (x''_{-m+1}, y''_{m-1}) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ (x''_{-m}, y''_{-m}) & \cdots & \cdots & (x''_m, y''_{-m}) \end{pmatrix} \\
&= \begin{pmatrix} (x_{-2m}, y_{2m}) & (x_{-2m+2}, y_{2m}) & \cdots & (x_{2m}, y_{2m}) \\ (x_{-2m}, y_{2m-2}) & (x_{-2m+2}, y_{2m-2}) & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ (x_{-2m}, y_{-2m}) & \cdots & \cdots & (x_{2m}, y_{-2m}) \end{pmatrix},
\end{aligned} \tag{8}
$$

where $x'' = 2x'$ and $y'' = 2y'$. In order to show the matrices clearly, we plot a graphical scheme of the transformations of the intensity matrix in Fig. 5 (a 9×9 matrix as an example). The matrices with green, yellow, and red elements denote the intensity matrix $I_n(x, y)$, the first sub-matrix $I'_n(x', y')$, and the second sub-matrix $I'_n(2x', 2y')$, respectively. We note that some elements of the matrices overlap in Fig. 5.

By inserting Eqs. (7) and (8) into Eq. (6), one obtains,

$$
\begin{aligned}
|\mu(\mathbf{r}', 2\mathbf{r}')|^2 &= |\mu(x', y'; 2x', 2y')|^2 \\
&= \frac{N \sum_n^N I_n(x', y') I_n(2x', 2y')}{\sum_n^N I_n(x', y') \sum_n^N I_n(2x', 2y')} - 1.
\end{aligned} \tag{9}
$$

Equation (9) indicates that one can measure the square of the normalized modulus of the DCF $|\mu(\mathbf{r}', 2\mathbf{r}')|^2$ by the experimental setup shown in Fig. 4. The Matlab software is used for matrix disposal.

The experimental results of the average intensity and square of the normalized modulus of different correlation functions of the generated partially coherent LG_{pl} beam with $p = 1$, $l = 1$, $\omega_0 = 0.57$ mm, and $\delta_g = 0.8$ mm in the focal plane are shown in Fig. 6. The experimental results of the square of the normalized modulus of the DCF of the generated partially coherent LG_{pl} beam with $p = 1$, $\omega_0 = 0.57$ mm, and $\delta_g = 0.8$ mm for different l in the focal plane are shown in Fig. 7. The experimental results of the square of the normalized modulus of the DCF of the generated partially coherent LG_{pl} beam with $l = 6$, $\omega_0 = 0.57$ mm and $\delta_g = 0.8$ mm for different p in the focal plane are shown in Fig. 8. We see that the experimental results shown in Figs. 6–8 are consistent with the numerical results shown in Figs. 1–3. As expected, the measured intensity has a solid beam profile [see Fig. 6(a)], the number of the dark rings in the measured MCF or CCF equals $2p + |l|$ [see Figs. 6(b) and 6(c)], and its value in the

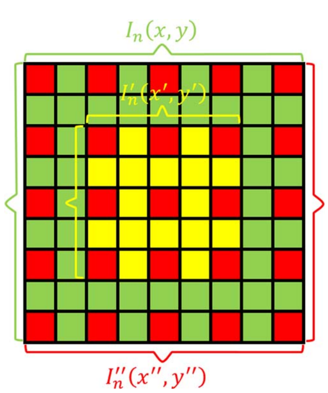

Fig. 5. Schematic for the transformations of the intensity matrix. The matrices with green, yellow, and red elements denote the intensity matrix $I_n(x, y)$, the first sub-matrix $I'_n(x', y')$, and the second sub-matrix $I'_n(2x', 2y')$, respectively.

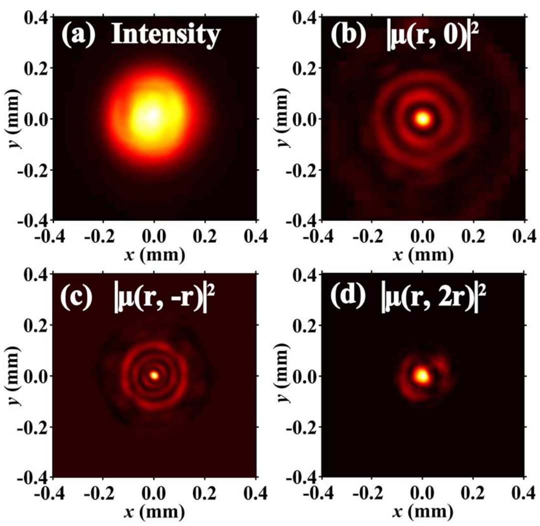

Fig. 6. Experimental results of the average intensity and square of the normalized modulus of different correlation functions of the generated partially coherent LG_{pl} beam in the focal plane with $p = 1$ and $l = 1$. (a) $I(\mathbf{r})$, (b) $|\mu(\mathbf{r}, 0)|^2$, (c) $|\mu(\mathbf{r}, -\mathbf{r})|^2$, and (d) $|\mu(\mathbf{r}, 2\mathbf{r})|^2$.

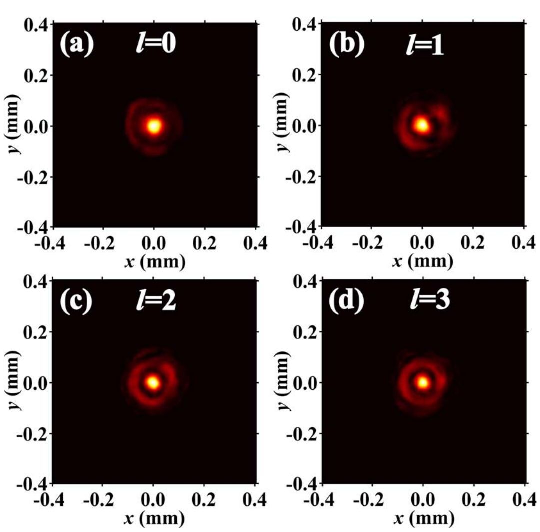

Fig. 7. Experimental results of the square of the normalized modulus of the DCF of the generated partially coherent LG_{pl} beam with $p = 1$ for different l in the focal plane. (a) $l = 0$, (b) $l = 1$, (c) $l = 2$, and (d) $l = 3$.

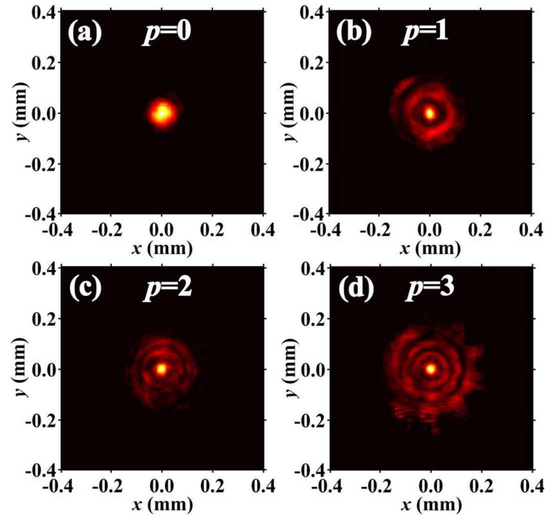

Fig. 8. Experimental results of the square of the normalized modulus of the DCF of the generated partially coherent LG_{pl} beam with $l = 6$ for different p in the focal plane. (a) $p = 0$, (b) $p = 1$, (c) $p = 2$, and (d) $p = 3$.

measured DCF equals p (see Fig. 8) and is independent of l (see Fig. 7). As shown in Ref. [21], the sign of l of a partially coherent LG_{pl} beam also can be determined through measuring the MCF or CCF of such a beam after passing through a couple of cylindrical lenses.

The numerical results in this Letter show the normalized modulus of the correlation functions, but the experimental results can only show the square of the normalized modulus of the correlation functions, which is the main reason why the numerical results and experimental results show some discrepancies. We may improve the experimental results by measuring the imaginary and real parts of the correlation functions by the method developed just recently[30], and we leave this for future study.

In conclusion, a method is proposed to measure the DCF of a partially coherent beam, and experimental measurements of the MCF, CCF, and DCF of a partially coherent LG_{pl} beam are demonstrated. Our experimental results confirm the numerical prediction that one can determine the radial mode order of a partially coherent LG_{pl} beam through measuring the DCF in the focal plane and then determine its azimuthal mode order l through measuring the MCF or CCF in the focal plane. The correlation functions carry rich information about the vortex phase, and our results may be useful for information transfer and recovery in turbulent media, since a coherent vortex beam will become a partially coherent vortex beam upon propagation in turbulent media.

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