Vector properties of a tunable random electromagnetic beam in non-Kolmogrov turbulence

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Received March 31, 2016; accepted June 14, 2016; posted online July 14, 2016

Analytical formulas for a class of tunable random electromagnetic beams propagating in a turbulent atmosphere through a complex optical system are derived with the help of a tensor method. One finds that the far field intensity distribution is tunable by modulating the source correlation structure function. The on-axis spectral degree of polarization monotonically increases to the same value for different values of order M in free space while it returns to the initial value after propagating a sufficient distance in turbulence. Furthermore, it is revealed that the state of polarization is closely determined by the initial correlation structure rather than by the turbulence parameters.

OCIS codes: 010.1300, 030.0030, 260.5430.

doi: 10.3788/COL201614.080101.

It is well known the spatial correlation structure of random beams significantly affects the propagation intensity distribution[1]. However, only a few correlation function models such as the Schell-model source, Bessel-correlated source, and the Lambertian source have been introduced over the past decades[1–3]. As a typical correlation source, the Schell-model source with a Gaussian correlation function has been extensively studied both in theory and in experiments over the past decades[4–9]. Since Gori proposed a sufficient condition for the generation of genuine correlation functions based on a non-negative definiteness[10], a variety of correlation functions have been proposed, both theoretically and experimentally. Beams generated by non-uniform correlation sources have found various unique properties in terms of self-accelerating, self-focusing and special beam profiles in terms of dark hollow, flat-topped and rectangular frame, etc.[11–19].

Coherence and polarization are two fundamentals of light fields both in classical and quantum optics. They had been studied separately in literature until James first reported that the spectral degree of polarization (DOP) generally changes on propagation induced by the source correlation property, even in free space[20]. Since Wolf developed a unified theory of coherence and polarization for random electromagnetic beams, it is widely used to determine the statistical properties of random electromagnetic beams in free space as well as in various media[21–28].

Optical communication exhibits various advantages in terms of high speed, high bandwidth, and anti-interference as compared with microwave communication. However, refractive index fluctuations caused by a turbulent atmosphere significantly limit the transmission of optical signals. As a general extension of Kolmogorov turbulence, non-Kolmogorov turbulence has been studied widely both in theory and in experiment in the past decades[29–37]. It is verified that random beams are found as a suitable way for reducing the disadvantages induced by a turbulent atmosphere[38–52]. Recently, random electromagnetic beams have been reported as a better optimization of scalar random beams for reducing turbulence-induced scintillation[53–56]. In some practical applications such as radar systems, imaging systems, and adaptive optics systems, propagation systems with one or more optical elements along the path are involved. The methods used for the line-of-sight propagation problem do not readily adapt and an extended Huygens–Fresnel integral principle has been developed for atmospheric paraxial beam propagation through a complex optical system characterized by an ABCD ray matrix.

In this Letter, analytical formulas for the 2×2 cross-spectral density (CSD) matrix of a class of tunable random electromagnetic beam propagation through the turbulent atmosphere are derived by using a tensor method. One finds that the intensity distribution in the far field is tunable by modulating the source coherence width and the beam order M. The effects of turbulence on the statistical properties of a tunable random electromagnetic beam such as intensity distribution, the spectral DOP, and the state of polarization (SOP) have been studied in detail.

According to the unified theory for random electromagnetic beams, the second-order spatial coherence properties of a fluctuating electromagnetic light beam are characterized by the 2×2 CSD matrix $\overleftrightarrow{W}_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2; \omega)$ with elements $W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2; \omega) = \langle E^*_\alpha(\mathbf{r}_1; \omega) E_\beta(\mathbf{r}_2; \omega)\rangle$, $(\alpha = x, y)$ at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ at frequency ω. The asterisk denotes the complex conjugate and the angular brackets stand for the ensemble average. Within the paraxial approximation, the propagation of a random electromagnetic beam through an isotropic turbulent atmosphere can be studied with the extended Huygens–Fresnel integral[23,24]

$$E_\alpha(\boldsymbol{\rho};\omega)=\frac{ik\exp(ikz)}{2\pi z}\iint\exp\left[-\frac{ik(\mathbf{r}-\boldsymbol{\rho})^2}{2z}\right]\times E_\alpha(\mathbf{r};\omega)\exp[\Psi_\alpha(\mathbf{r},\boldsymbol{\rho};\omega)]\mathrm{d}^2\mathbf{r},\qquad(\alpha=x,y),\tag{1}$$

where E_x and E_y represent the two mutually orthogonal components of the random electric vector with respect to the x and y directions, $E_\alpha(\mathbf{r};\omega)$ and $E_\alpha(\boldsymbol{\rho};\omega)$ denote the random electric vector in the source plane and the output plane, ω denotes the angular frequency, $k=2\pi/\lambda$ is the wave number with λ being the wavelength, and $\boldsymbol{\Psi}_\alpha$ is the Rytov complex random phase perturbation along the $\boldsymbol{\alpha}$ direction for characterizing the isotropic turbulence.

Based on the Rytov approximation and the weak fluctuation conditions, the ABCD method greatly simplifies the analysis as compared with other techniques[29]. The extended Huygens–Fresnel integral formula for light beam propagation through turbulence with optical elements placed along the path can be rewritten in the tensor form

$$W_{\alpha\beta}(\tilde{\boldsymbol{\rho}};\omega)=\frac{k^2}{4\pi^2[\det(\tilde{\mathbf{B}})]^{1/2}}\iint\iint W_{\alpha\beta}(\tilde{\mathbf{r}};\omega)\times\exp\left[-\frac{ik}{2}(\tilde{\mathbf{r}}^T\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{r}}-2\tilde{\mathbf{r}}^T\tilde{\mathbf{B}}^{-1}\tilde{\boldsymbol{\rho}}+\tilde{\boldsymbol{\rho}}^T\tilde{\mathbf{B}}^{-1}\tilde{\boldsymbol{\rho}})\right]\times\mathbf{K}_{\alpha\beta}(\tilde{\mathbf{r}},\tilde{\boldsymbol{\rho}};\omega)\mathrm{d}^4\tilde{\mathbf{r}},\tag{2}$$

$$\mathbf{K}_{\alpha\beta}(\tilde{\mathbf{r}},\tilde{\boldsymbol{\rho}};\omega)\approx\exp\left[-\frac{ik}{2}\tilde{\mathbf{r}}^T\tilde{\mathbf{Q}}_{\alpha\beta}\tilde{\mathbf{r}}\right],\tag{3}$$

$$\tilde{\mathbf{B}}=\begin{pmatrix}z\cdot\mathbf{I}&0\cdot\mathbf{I}\\0\cdot\mathbf{I}&-z\cdot\mathbf{I}\end{pmatrix},\qquad\tilde{\mathbf{Q}}_{\alpha\beta}=\frac{2}{ik\rho_{0\alpha\beta}^2}\begin{pmatrix}\mathbf{I}&-\mathbf{I}\\-\mathbf{I}&\mathbf{I}\end{pmatrix},\tag{4}$$

where $\tilde{\mathbf{r}}^T=(\mathbf{r}_1^T\quad\mathbf{r}_2^T)$ and $\tilde{\boldsymbol{\rho}}^T=(\boldsymbol{\rho}_1^T\quad\boldsymbol{\rho}_2^T)$, $\tilde{\mathbf{B}}$ denotes the 4×4 ray transfer matrix element corresponding to the entire propagation path, $\mathbf{I}$ is the 2×2 unit matrix, and $\mathbf{K}_{\alpha\beta}$ represents the spherical wave structure function (SWSF). ρ_0 is the spatial coherence width of the spherical wave in turbulence under the second quadratic approximation. In this Letter, we employ the non-Kolmogrov spectrum, which is[30]

$$\rho_{0\alpha\beta}=\left[\frac{\xi\tilde{C}_n^2k^2\Gamma(\xi-1)\mathrm{Cos}(\pi\xi/2)}{2^\xi(\xi-1)[\det(\tilde{\mathbf{B}}^{(r)})]^{-1/4}}\right]^{\frac{-1}{\xi-2}},\tag{5}$$

where Γ stands for the gamma function and $\tilde{\mathbf{B}}^{(r)}$ is a matrix element that arises from reciprocal propagation from the output plane. For horizontal propagation paths, it is customary to treat the generalized refractive-index structure parameter $\tilde{C}_n^2$ as a constant with units $m^{3-\xi}$, and ξ is the non-Kolmogorov slope restricted to the interval $3<\xi<4$. When $\xi=11/3$, the generalized power spectrum reduces to the conventional Kolmogorov spectrum. The limit value of $\xi=3$ leads to no turbulence causing the power spectrum invariant. Also, the value $\xi=4$ is excluded for leading an infinite discontinuity in some of the statistical quantities.

The CSD of a tunable random electromagnetic beam at the source plane can be alternatively rewritten in the tensor form

$$W_{\alpha\beta}(\tilde{\mathbf{r}};\omega)=\frac{A_\alpha A_\beta B_{\alpha\beta}}{C_0}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{(-1)^{m+n-2}}{\sqrt{mn}}\times\binom{M}{m}\binom{N}{n}\exp\left[-\frac{ik}{2}\tilde{\mathbf{r}}^T\mathbf{M}_{0\alpha\beta}^{-1}\tilde{\mathbf{r}}\right],\tag{6}$$

where C_0 stands for the normalization factor, $\binom{M}{m}$ and $\binom{N}{n}$ are the binomial coefficients. A_α and $B_{\alpha\beta}=|B_{\alpha\beta}|\exp(i\varphi_{\alpha\beta})=B_{\beta\alpha}^*$ represent the amplitude of the electric field component along the $\boldsymbol{\alpha}$ direction and the complex correlation coefficient between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ components of the electric field, respectively. $\mathbf{M}_{0\alpha\beta}^{-1}$ is the 4×4 complex curvature tensor given by

$$\mathbf{M}_{0\alpha\beta}^{-1}=\begin{pmatrix}\frac{(\boldsymbol{\sigma}_\alpha^2)^{-1}}{2ik}+\frac{(\boldsymbol{\delta}_{\alpha\beta}^2)^{-1}}{ik}&\frac{i(\boldsymbol{\delta}_{\alpha\beta}^2)^{-1}}{k}\\\frac{i(\boldsymbol{\delta}_{\alpha\beta}^2)^{-1}}{k}&\frac{(\boldsymbol{\sigma}_\beta^2)^{-1}}{2ik}+\frac{(\boldsymbol{\delta}_{\alpha\beta}^2)^{-1}}{ik}\end{pmatrix},\tag{7}$$

$$\begin{aligned}(\boldsymbol{\sigma}_\alpha^2)^{-1}&=\sigma_\alpha^{-2}\cdot\mathbf{I},\\(\boldsymbol{\sigma}_\beta^2)^{-1}&=\sigma_\beta^{-2}\cdot\mathbf{I},\end{aligned}\qquad(\boldsymbol{\delta}_{\alpha\beta}^2)^{-1}=\begin{pmatrix}m\delta_{\alpha\beta x}^2&0\\0&n\delta_{\alpha\beta y}^2\end{pmatrix}^{-1},\tag{8}$$

where $\boldsymbol{\sigma}_\alpha^2$ and $\boldsymbol{\sigma}_\beta^2$ stand for the 2×2 transverse square beam width matrix and $\boldsymbol{\delta}_{\alpha\beta}^2$ is a transverse square coherence width matrix and elements need to satisfy the realizability condition[26].

On substituting Eqs. (3)–(8) into Eq. (2), we obtain the analytical formulas for the CSD matrix elements of a tunable random electromagnetic beam in turbulence in the observation plane with the help of a tensor operation

$$W_{\alpha\beta}(\tilde{\boldsymbol{\rho}};\omega)=\frac{A_\alpha A_\beta B_{\alpha\beta}}{C_0}\sum_{m=1}^{M}\sum_{n=1}^{N}\frac{(-1)^{m+n-2}}{\sqrt{mn}}\times\binom{M}{m}\binom{N}{n}[\det(\tilde{\mathbf{I}}+\tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{B}}\,\tilde{\mathbf{Q}})]^{-1/2}\times\exp\left\{-\frac{ik}{2}\tilde{\boldsymbol{\rho}}^T[(\mathbf{M}_{0\alpha\beta}^{-1}+\tilde{\mathbf{Q}})^{-1}+\tilde{\mathbf{B}}]^{-1}\tilde{\boldsymbol{\rho}}\right\},\tag{9}$$

where $\tilde{\mathbf{I}}$ represents a 4×4 unit matrix.

The average spectral intensity and the spectral DOP of an electromagnetic beam are defined as[1,21]

$$I(\boldsymbol{\rho};\omega)=W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)+W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega),\tag{10}$$

$$P(\boldsymbol{\rho};\omega)=\sqrt{1-\frac{4\,\mathrm{Det}[\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)]}{\left\{\mathrm{Tr}[\overleftrightarrow{W}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)]\right\}^2}}.\tag{11}$$

where Tr and Det denote the trace and the determinant of a matrix.

The SOP of an electromagnetic beam can be studied by the polarization ellipse[38]. The CSD matrix can be represented as a sum of a completely polarized beam and a completely unpolarized beam. The polarization ellipse is a parameter characterizing the fully polarized portion of the beam. The orientation angle θ is given by the formula

$$\theta(\boldsymbol{\rho};\omega)=\frac{1}{2}\arctan\left[\frac{2\,\mathrm{Re}\,W_{xy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)}{W_{xx}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)-W_{yy}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)}\right],\quad(-\pi/2\le\theta\le\pi/2). \tag{12}$$

The major and minor semiaxes of the polarization ellipse take the form

$$A_{\pm}(\boldsymbol{\rho};\omega)=\frac{1}{\sqrt{2}}\left\{\sqrt{(W_{xx}-W_{yy})^2+4|W_{xy}|^2}\pm\sqrt{(W_{xx}-W_{yy})^2+4[\mathrm{Re}\,W_{xy}]^2}\right\}^{1/2}. \tag{13}$$

The degree of ellipticity ε characterizing the shape of the polarization ellipse of an electromagnetic beam is defined by

$$\varepsilon=A_{-}(\boldsymbol{\rho},\boldsymbol{\rho};\omega)/A_{+}(\boldsymbol{\rho},\boldsymbol{\rho};\omega),\qquad 0\le\varepsilon\le 1. \tag{14}$$

It is seen from Eq. (14) that the minimum value 0 denotes linear polarization and the maximum value 1 is for circular polarization, and otherwise denotes ellipse polarization. Using Eqs. (13) and (14) one can find that the degree of ellipticity remains invariant on propagation under certain conditions.

Figure 1 shows the normalized intensity distribution of a tunable random electromagnetic beam at different propagation distances in free space (i.e., $\tilde{C}_n^2=0$). The global parameters are set as: $A_x=A_y=1$, $|B_{xy}|=0.0625$, $\varphi=\pi/3$, $M=N$, $\lambda=632.8$ nm, $\sigma_x=\sigma_y=5$ cm, $\delta_{xyx}=\delta_{xyy}=1.2$ cm, $\tilde{C}_n^2=5\times10^{-14}$ $\mathrm{m}^{-0.5}$, $\xi=3.5$. It is illustrated that the intensity distribution in the far field is tunable in terms of elliptical, rectangular, star shape, and cross shape by modulating the order M and the initial coherence width, though the source beam profiles are all the same Gaussian distribution. The reason is that the beam distribution is closely determined by the coherence structure of the source beam. Since the coherence structures of common light sources usually satisfy Shell-model (Gaussian functions) distributions, various non-Gaussian shaped beams generated by these sources evolve into Gaussian distributions in the far field. Based on this principle, it is reasonable and possible for us to produce a variety of specially designed beams by modulating the initial coherence structure function. In general, the coherence structure can be synthesized either with the help of the liquid crystal spatial light modulator (SLM) or by a method reported recently[16].

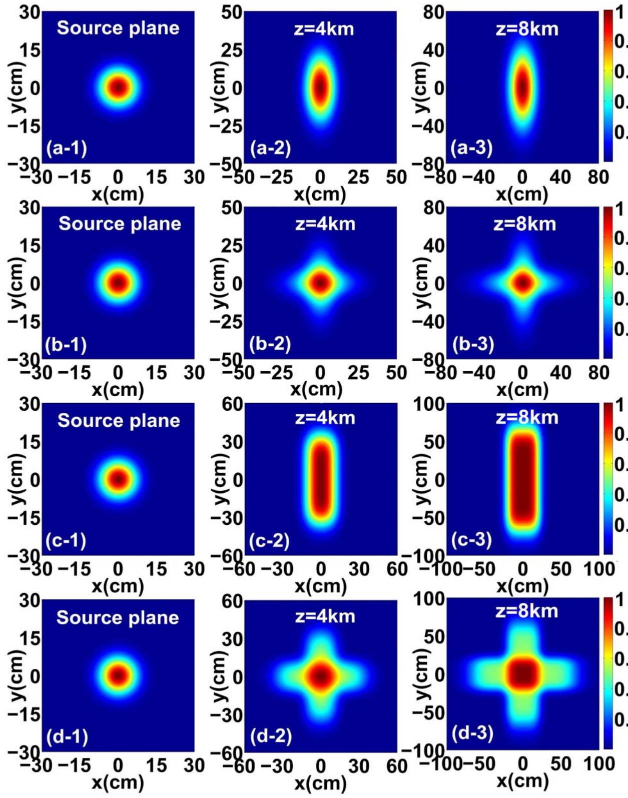

Fig. 1. Normalized intensity distribution of a tunable random electromagnetic beam at different propagation distances in free space with parameters set as: (a) and (c) $\delta_{xxx}=\delta_{yyx}=1$ cm, $\delta_{xxy}=\delta_{yyy}=0.3$ cm, (b) and (d) $\delta_{xxx}=\delta_{yyy}=1$ cm, $\delta_{xxy}=\delta_{yyx}=0.3$ cm. (a) and (b) $M=1$; (c) and (d) $M=15$.

For a comparison with Figs. 1(c) and 1(d), Fig. 2 shows the normalized intensity distribution of a tunable random electromagnetic beam at different propagation distances in turbulence with the same values of parameter corresponding to Figs. 1(c) and 1(d). It is clearly seen in Fig. 2 that the beam profile obviously blurs on propagation and a significant beam spreading appears. The

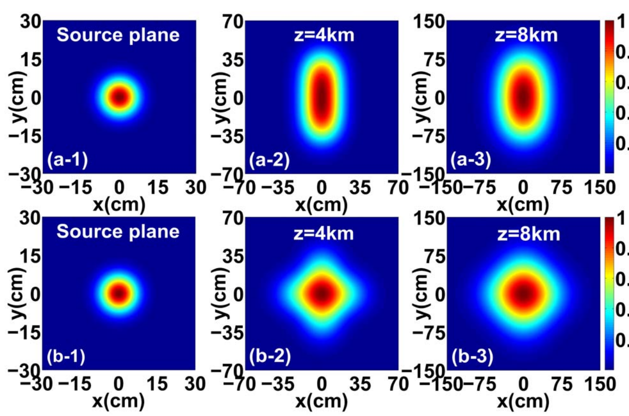

Fig. 2. Normalized intensity distribution of a tunable random electromagnetic beam at different propagation distances in turbulent atmosphere with $M=15$. (a) $\delta_{xxx}=\delta_{yyx}=1$ cm, $\delta_{xxy}=\delta_{yyy}=0.3$ cm, (b) $\delta_{xxx}=\delta_{yyy}=1$ cm, $\delta_{xxy}=\delta_{yyx}=0.3$ cm.

physical explanation is that the fluctuations of the atmospheric refractive index induced by the turbulence are statistically uniform, which lead to a reduction of the spatial coherence in light beams. In general, a spatial broadening of the beam spot is caused by the reduction of the spatial coherence. Therefore, the beam spot gradually evolves into a Gaussian shape due to the effects of turbulence.

Figures 3 and 4 show that the spectral DOP of a tunable random electromagnetic beam in free space and the turbulence with the same values of parameters in Fig. 2. It is seen from Fig. 3 that the on-axis spectral DOP monotonically increases to the same value for different values of the source beam parameter after a sufficiently long propagation distance in free space. However, the spectral DOP will return to the initial value in the source plane after propagating a sufficient distance in turbulence. Our further study shows that this trend is independent of the turbulent statistics that are in agreement with the previous results[24]. In addition, there is an obvious broadening of the effective transverse spectral DOP in turbulence compared with that in free space. The physical interpretation is similar to the beam spot in turbulence. Figure 4 implies that the profile of spectral DOP is closely determined by the initial beam parameters rather than the turbulent parameters.

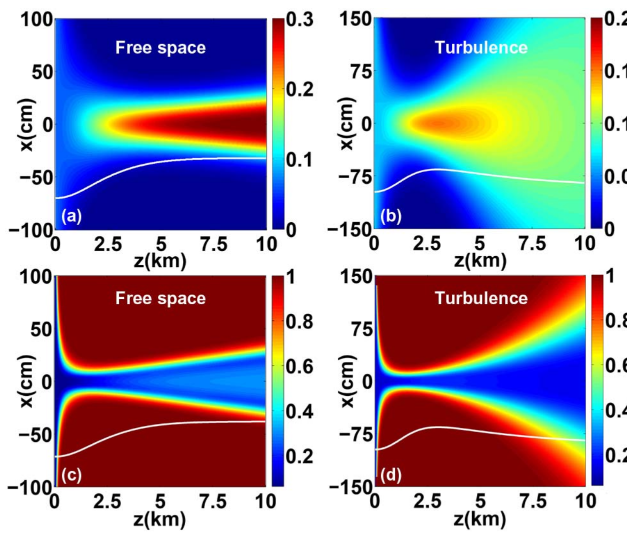

Fig. 3. DOP of a tunable random electromagnetic beam in free space and the turbulence.

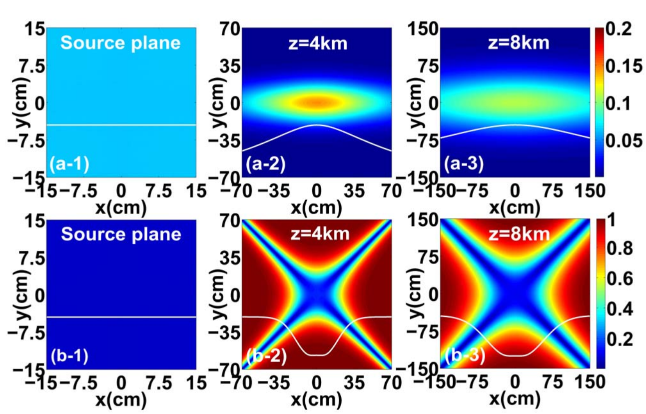

Fig. 4. DOP of a tunable random electromagnetic beam in turbulence at different propagation distances.

To learn more about the polarization properties, Fig. 5 plots the SOP at different propagation distances for different correlation structures with the same values of parameters in Fig. 2. One finds from Figs. 5(a–1)–5(a–3) that the SOP of a rectangular correlated electromagnetic beam remains invariant during propagation. However, there is a significant change in the SOP of a cross-correlated electromagnetic beam. The reason is that the far field spectral intensity components W_{xx} and W_{yy} are nonuniformly affected by the asymmetric correlation function of the source beam that leads to a significant modulation on the SOP. Except for the diagonal, off-diagonal, and a small area around the beam center that remains invariant, the SOP in the rest of the beam quickly evolves into being linearly polarized.

In conclusion, we investigate the statistical properties of a tunable random electromagnetic beam on propagation through a turbulent atmosphere, with the help of a tensor method. By modulating the source coherence structure and the order M, a variety of novel beam profiles such as elliptical, rectangular, star shape, and cross shape are derived in the far field. Since the correlation structure of a random electromagnetic beam is reformed by turbulence on propagation, the beam distribution gradually becomes uniform. It is interesting to see that the on-axis DOP monotonically increases to the same value for different orders M after a sufficiently long propagation distance in free space, while it returns to the initial value in turbulence, which is independent of the turbulent statistics. Furthermore, the SOP is greatly affected by the source correlation function, and the change in the turbulent statistics induces a relatively small effect, which may be useful for polarization sensing.

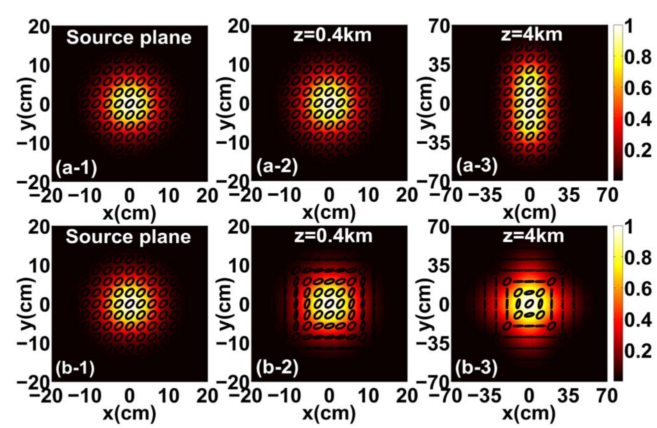

Fig. 5. SOP of a tunable random electromagnetic beam in turbulence at different propagation distances for different correlation structures.

This work was supported by the National Natural Science Foundation of China (No. 11504172), the Natural Science Foundation of Jiangsu Province (No. BK20150763), and the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

References

1. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, 1995).
2. A. C. Schell, *The Multiple Plate Antenna* (Massachusetts Institute of Technology, 1961).
3. F. Gori, G. Guattari, and C. Padovani, Opt. Commun. **64**, 311 (1987).
4. R. Simon, E. C. G. Sudarshan, and N. Mukunda, Phys. Rev. A **31**, 2419 (1985).
5. Q. Lin and Y. Cai, Opt. Lett. **27**, 216 (2002).
6. S. Zhu, L. Liu, Y. Chen, and Y. Cai, J. Opt. Soc. Am. A **30**, 2306 (2013).
7. S. Zhu, Y. Chen, and Y. Cai, J. Opt. Soc. Am. A **30**, 171 (2013).
8. Y. Cai and S. Zhu, Opt. Lett. **39**, 1968 (2014).
9. S. Zhu and Z. Li, Appl. Phys. B **118**, 418 (2015).
10. F. Gori and M. Santarsiero, Opt. Lett. **32**, 3531 (2007).
11. H. Lajunen and T. Saastamoinen, Opt. Lett. **36**, 4104 (2011).
12. S. Sahin and O. Korotkova, Opt. Lett. **37**, 2970 (2012).
13. Z. Mei and O. Korotkova, Opt. Lett. **38**, 91 (2013).
14. Z. Mei, O. Korotkova, and Y. Mao, Opt. Lett. **39**, 6879 (2014).
15. C. Ding, O. Korotkova, Y. Zhang, and L. Pan, Opt. Express **22**, 931 (2014).
16. Y. Chen, F. Wang, L. Liu, C. Zhao, Y. Cai, and O. Korotkova, Phys. Rev. A **89**, 013801 (2014).
17. F. Wang, C. Liang, Y. Yuan, and Y. Cai, Opt. Express **22**, 23456 (2014).
18. X. Liu and D. Zhao, Opt. Express **23**, 16702 (2015).
19. S. Zhu, Y. Chen, J. Wang, H. Wang, Z. Li, and Y. Cai, Opt. Express **23**, 33099 (2015).
20. D. F. V. James, J. Opt. Soc. Am. A **11**, 1641 (1994).
21. E. Wolf, Phys. Lett. A **312**, 263 (2003).
22. T. Setälä, M. Kaivola, and A. T. Friberg, Phys. Rev. Lett. **88**, 123902 (2002).
23. Y. Cai and S. He, Appl. Phys. Lett. **89**, 041117 (2006).
24. X. Du, D. Zhao, and O. Korotkova, Opt. Express **15**, 16909 (2007).
25. R. Rao, Chin. Opt. Lett. **6**, 547 (2008).
26. S. Zhu and Y. Cai, Appl. Phys. B **99**, 317 (2010).
27. S. Zhu, F. Wang, Y. Chen, Z. Li, and Y. Cai, Opt. Express **22**, 28697 (2014).
28. X. Yao, X. Liu, W. Yu, and G. Zhai, Chin. Opt. Lett. **13**, 010301 (2015).
29. L. C. Andrews and R. L. Phillips, *Laser Beam Propagation in Random Media* (SPIE, 2005).
30. I. Toselli, B. Agrawal, and S. Restaino, J. Opt. Soc. Am. A **28**, 483 (2011).
31. H. Tang, B. Ou, B. Luo, H. Guo, and A. Dang, J. Opt. Soc. Am. A **28**, 1016 (2011).
32. B. Xue, L. Cui, W. Xue, X. Bai, and F. Zhou, J. Opt. Soc. Am. A **28**, 912 (2011).
33. H. Xu, Z. Cui, and J. Qu, Opt. Express **19**, 21163 (2011).
34. Y. Wang, C. Si, Y. Zhang, J. Wang, and J. Jia, Opt. Laser Eng. **49**, 1060 (2011).
35. P. Zhou, Y. Ma, X. Wang, H. Zhao, and Z. Liu, Opt. Lett. **35**, 1043 (2010).
36. R. Tao, L. Si, Y. Ma, P. Zhou, and Z. Liu, Appl. Opt. **51**, 5609 (2012).
37. Y. Zhou, Y. Yuan, J. Qu, and W. Huang, Opt. Express **24**, 10682 (2016).
38. O. Korotkova and E. Wolf, Opt. Commun. **246**, 35 (2005).
39. X. Ji, X. Chen, and B. Lü, J. Opt. Soc. Am. A **25**, 21 (2008).
40. X. Chu and G. Zhou, Opt. Express **15**, 7697 (2007).
41. G. Zhou, Y. Cai, and X. Chu, Opt. Express **20**, 9897 (2012).
42. F. Wang, Y. Cai, and O. Korotkova, Opt. Express **17**, 22366 (2009).
43. Y. Dan and B. Zhang, Opt. Lett. **34**, 563 (2009).
44. S. Zhu, Y. Cai, and O. Korotkova, Opt. Express **18**, 12587 (2010).
45. G. Wu, H. Zhou, and T. Zhao, J. Korean Phys. Soc. **64**, 826 (2014).
46. T. Cao, P. Wang, L. Guo, B. Yang, J. Li, and Y. Yang, Chin. Opt. Lett. **13**, 080101 (2015).
47. F. Wang and O. Korotkova, Opt. Lett. **41**, 1546 (2016).
48. G. Gbur, J. Opt. Soc. Am. A **31**, 2038 (2014).
49. R. Chen, L. Liu, S. Zhu, G. Wu, F. Wang, and Y. Cai, Opt. Express **22**, 1871 (2014).
50. S. Cui, Z. Chen, and J. Pu, Opt. Express **22**, 18278 (2014).
51. J. Yu, Y. Chen, L. Liu, X. Liu, and Y. Cai, Opt. Express **23**, 13467 (2015).
52. J. Wang, S. Zhu, H. Wang, Y. Cai, and Z. Li, Opt. Express **24**, 11626 (2016).
53. Y. Gu, O. Korotkova, and G. Gbur, Opt. Lett. **34**, 2261 (2009).
54. Y. Baykal and H. Gerçekcioğlu, Opt. Lett. **36**, 4554 (2011).
55. F. Wang, X. Liu, L. Liu, Y. Yuan, and Y. Cai, Appl. Phys. Lett. **103**, 091102 (2013).
56. Y. Baykal, J. Opt. Soc. Am. A **31**, 2198 (2014).