Generation of polarization vortex beams by segmented quarter-wave plates

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A spatially variable retardation device, an SQWP, is designed to generate polarization vortex beams. The transformation of Laguerre–Gaussian beams by the SQWP is further studied, and it is found that the SQWPs can also be used to generate helical beams and measure the topological charges of helical beams.

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Beams with singularity in phase or polarization have attracted much attention in recent years. One example is the helical beam, such as Laguerre–Gaussian (LG) beams, with a spiral phase ramp around the center and a doughnut-intensity distribution. A helical beam has a complex amplitude term, $\exp(il\varphi)$, where φ is the azimuthal angle and l is the topological charge. Helical beams have applications in quantum optics information processing[1,2], free-space communications[3–5], and optical micromanipulations[6,7]. Compared with helical beams, polarization vortex beams (PVBs), with undefined polarization at the center and a doughnut-intensity distribution, are another kind of beam with singularities[8]. The polarization orientation of a PVB field can be described with a Jones vector, $E_p = [\cos(p\varphi)\sin(p\varphi)]^T$, where p is the polarization order number (topological charge) of the light field. When $p = 1$, it represents a cylindrical vector beam, including radially polarized beams (RPBs) and azimuthally polarized beams (APBs). RPBs have been proposed for the laser cutting of metals[9], charged-particle acceleration[10], and surface-plasmon resonance excitation[11], and APBs have been proposed for atom trapping[12]. Meanwhile, higher-order PVBs have potential applications in exciting surface plasma[13].

The segmented half-wave plate (SHWP) is a spatially variable retardation device that has been used to generate PVBs[14] and helical beams[15] and to measure the topological charges of helical beams[16]. In this Letter, another spatially variable retardation device, segmented quarter-wave plates (SQWPs), is designed to generate PVBs, and the transformation of LG beams by SQWPs is investigated. It is found that an mth order SQWP will transform an incident LG beam into a left-handed circularly polarized beam with a topological charge of n and a right-handed circularly polarized beam with a topological charge of $2m + n$, which implies the SQWP can be used to generate helical beams and measure the topological charges of helical beams.

The SQWP is composed of M sectors of sub-QWPs (QWP: quarter-wave plate), whose fast axes are at angles of $2m(n-1)/M$ to the x-axis, where n is the serial number of the sub-QWP and m is defined as the order of the SQWP, as shown in Fig. 1. An mth order SQWP is designed to generate an mth order PVB in the following discussion. An mth order SQWP can be described as a Jones matrix:

$$J_m(\phi) = \sum_{n=1}^{M} \begin{bmatrix} A_n(\theta) & B_n(\theta) \\ C_n(\theta) & D_n(\theta) \end{bmatrix} \times \begin{cases} A_m(\theta) = \cos^2 m\phi_n(\theta) + i\sin^2 m\phi_n(\theta) \\ B_m(\theta) = \sin m\phi_n(\theta)\cos m\phi_n(\theta)(1-i) \\ C_m(\theta) = \sin m\phi_n(\theta)\cos m\phi_n(\theta)(1-i) \\ D_m(\theta) = \sin^2 m\phi_n(\theta) + i\cos^2 m\phi_n(\theta) \end{cases}, \tag{1}$$

where $\phi_n(\theta) = 2\pi(n-1)/M$, $(2n-3)/M < \theta < \pi(2n-1)/M$, $n = 1, 2, \ldots M$. As a stepped device, an SQWP can be

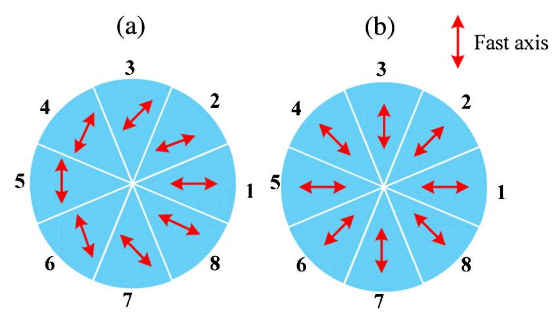

Fig. 1. Structures of SQWPs: (a) 1/2-order SQWP and (b) first-order SQWP.

considered as a "continuous" device when M is large enough. A "continuous" SQWP can be described as follows:

$$J_m(\theta)=\begin{bmatrix}\cos^2 m\theta+i\sin^2 m\theta & \sin m\theta\cos m\theta(1-i)\\ \sin m\theta\cos m\theta(1-i) & \sin^2 m\theta+i\cos^2 m\theta\end{bmatrix}. \tag{2}$$

The Jones matrix of a "continuous" SQWP enables us to study the transformation process using a mathematical analysis. To simplify the mathematical expression, normalized Jones vectors and a Jones matrix are used to represent the incident beams and transformation devices. When a left-handed circularly polarized beam is transformed by an mth order SQWP, the emerging beam can be written as follows:

$$E_{\text{out}}(\phi)=J_m(\theta)\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}=\begin{bmatrix}\cos(m\theta-\pi/4)\\ \sin(m\theta-\pi/4)\end{bmatrix}\exp(im\theta). \tag{3}$$

From Eq. (3), we find that the emerging beam can be expressed as a product of an extended Jones vector and a helical phase term. The extended Jones vector is usually used to describe the polarization distribution of a PVB and the helical phase term is always used to describe the phase distribution of a helical beam. To generate a PVB, the incident beam can be replaced by a helical beam to eliminate the helical phase of the Eq. (3). This process can be described as follows:

$$\begin{aligned}E_{\text{out}}(\phi)&=J_m(\phi)\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}\exp(-im\phi)\\ &=\begin{bmatrix}\cos(m\phi-\pi/4)\\ \sin(m\phi-\pi/4)\end{bmatrix}.\end{aligned} \tag{4}$$

Equation (4) is the principle to generate PVBs by SQWPs, and an mth order PVB can be obtained from the transformation of a helical beam with a topological charge of $-m$ by an mth order SQWP. Figure 2 shows the generation of a first-order PVB by a first-order SQWP. It is found that the initial polarization orientation of the generated PVB is −45°. To obtain a RPB or APB, two

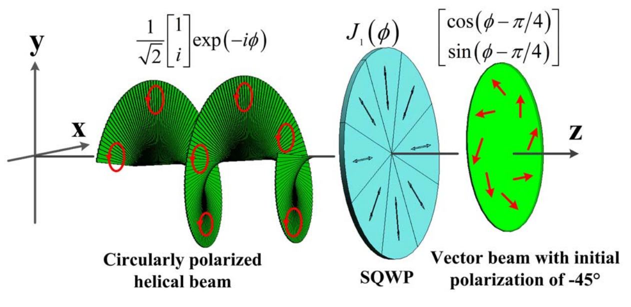

Fig. 2. Schematic diagram of the generation of a first-order PVB by a first-order SQWP.

half-wave plates can be used to rotate the initial polarization orientation[17].

The generation of PVBs by the SQWP is only a special case of transformation of LG mode beams by an SQWP. In the following section, a general discussion is carried out. The process of an LG beam with a topological charge of n incident to an mth order SQWP can be written as

$$\begin{aligned}\vec{E}_{\text{out}}&=J_m(\phi)\exp(\text{in}\phi)\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ i\end{bmatrix}\\ &=\frac{1}{2}\left(\begin{bmatrix}1\\ i\end{bmatrix}\exp(i(n\phi+\pi/4))\right.\\ &\quad\left.+\begin{bmatrix}1\\ -i\end{bmatrix}\exp(i((2m+n)\phi-\pi/4))\right).\end{aligned} \tag{5}$$

From Eq. (5) we can find that an mth order SQWP will transform the incident LG beam into a left-handed circularly polarized beam with topological charge of n and a right-handed circularly polarized beam with topological charge of $2m+n$. If the two beams are spatially separated, some potential applications, such as the generation of helical beams and identification of the topological charges of the helical beams, will also be found.

The first one is for $n=0$: in this case, a Gaussian beam was transformed into a left circularly polarized Gaussian beam and a right circularly polarized helical beam with a topological charge of $2m$. So we can obtain a helical beam by spatially separating the two beams, but the conversion efficiency is less than 50%.

The second one is for $n=-2m$: in this case, the incident beam is transformed into a left-handed circularly polarized helical beam with a topological charge of n and a right-handed circularly polarized Gaussian beam. So we can use the separating method to identify the topological charges of helical beams by observing the profile of the right-handed circularly polarized beam. If the profile of the right-handed circularly polarized beam is a spot (a Gaussian beam without a topological charge) other than a doughnut, we can determine the topological charge of the incident helical to be n.

The principle of generating PVBs by a "continuous" SQWP has been discussed above. Before the experimental study, a numerical simulation based on the Collins diffraction integral is carried out. Figure 3 shows the simulation results of different-order PVBs generation by SQWPs with sectors of 8, 12, and 16. It is found that the quality of the generated PVBs is degenerated with the increase of the order of the SQWP for the same number of sectors, so the sectors of the SQWP should be increased if higher-order PVBs with good quality must be generated.

Figure 4 demonstrates the simulation results when LG mode beams are incident to a 1/2-order and a first-order SQWP. The first column shows the modes of the incident beams, the second column shows the transformed patterns when a 1/2-order SQWP is used, and the third column shows the transformed patterns when a first-order SQWP

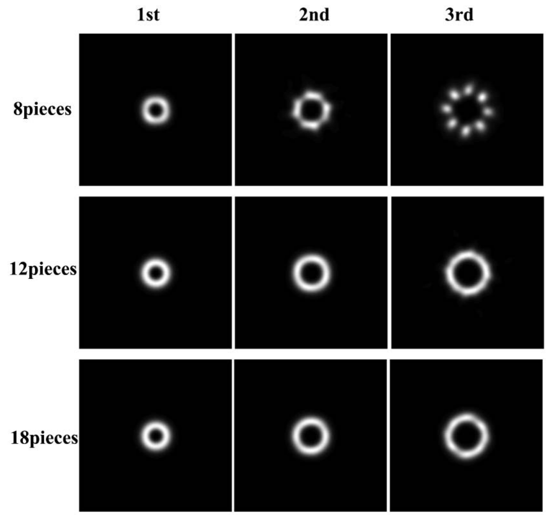

Fig. 3. Simulation results of generating different-order PVBs by SQWPs with sectors of 8, 12, and 16.

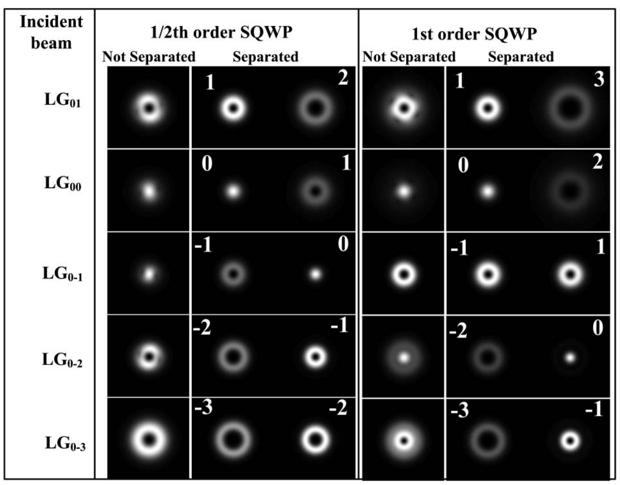

Fig. 4. Simulation results of LG mode beam transformed by 1/2-order and first-order SQWPs. Those numbers labeled near the separated beams represent their topological charges.

is used. The number represents the topological charge of the separated beam. The simulation results agree with the analysis from Eq. (5).

SQWPs can be fabricated into fractional orders. It also can be seen that a fractional order SQWP can be used to generate a helical beam and measure the topological charge of the helical beam but cannot be used to generate PVBs.

In our experiments, the first-order SQWP with 12 pieces of sub-QWPs was manufactured by splitting a commercial QWP and re-arranging the sub-QWPs into one optical element. The QWP was cut by the dicing saw first and then fixed onto a BK7 window by silicon rubber after high-temperature curing (the silicon rubber was scribbled at the edge). The experimental setup is shown in Fig. 5. A horizontally polarized He–Ne laser beam is collimated first by a beam expander and then illuminates a spatial light

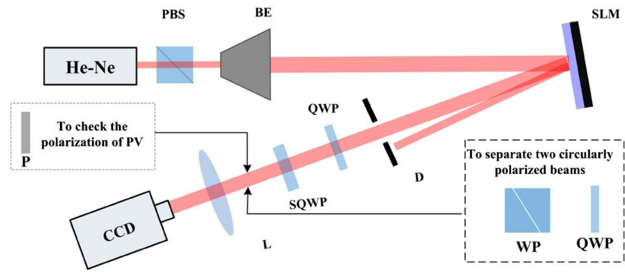

Fig. 5. Experimental setup to study the transformation of LG beams by SQWPs. PBS: polarizing beam splitter, BE: beam expander, WP: Wollaston prism, L: lens.

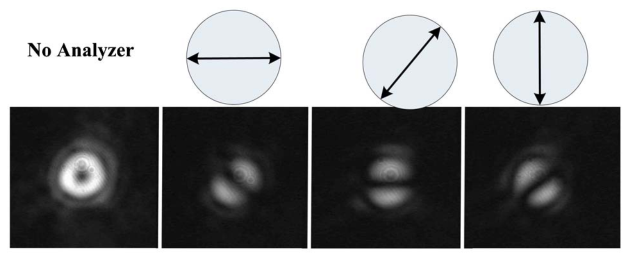

Fig. 6. Experiment results of first-order SQWP with sectors of 12.

modulator (SLM) to generate an LG beam. The horizontally polarized LG_{0-1} mode beam is transformed into a circularly polarized beam by the QWP. After the transformation of the SQWP, a first-order PVB is formed. The far-field intensity of the generated PVB is observed by a CCD camera at the focal plane. To analyze the polarization distribution of the generated PVBs, a rotating polarizer is required. The experimental results are shown in Fig. 6, and the generated first-order PVB is a doughnut beam. When the beam is checked by a rotating polarizer as an analyzer, it is divided into 2 lobes and the rotation speed of lobes is the same as that of the analyzer. This further shows that the initial polarization orientation of the generated first-order PVB is $-45°$.

The transformation of LG beams by first-order SQWPs was also experimentally studied. Several kinds of LG mode beams (LG_{01}, LG_{00}, LG_{0-1}, LG_{0-2}, and LG_{0-3} mode beams) were generated by the SLM first. After the transformation by SQWPs, two coaxially propagated circularly polarized beams are generated. To separate the two coaxially propagated beams, a QWP and a Wollaston prism are required. A 45°arranged QWP will transform the left-handed and right-handed circularly polarized beams into horizontally and vertically polarized beams, respectively. After the polarization splitting by the Wollaston prism, the two circularly polarized beams are spatially separated and the topological charges of the two beams remain. The coaxially propagated beams and the spatially separated two beams are both recorded by the CCD camera, which can be seen in Fig. 7(a). The experimental results agree

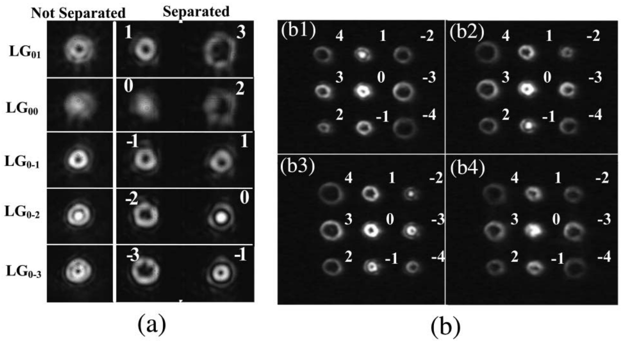

Fig. 7. Experimental results of LG mode beams transformed by the first-order SQWP. (a) The patterns of the transformed beams and the separated beams where the numbers labeled in (a) indicate the topological charges of those beams. (b) The measurement results of the topological charges of the separated beams where the numbers labeled in (b) indicate the diffractive orders of the computer-generated hologram.

with the simulation results in Fig. 4. To investigate the topological charge of the separated beams, a computer-generated hologram is used. If the diffractive order number and the topological charge of the incident beam are matched, the incident beam will be degraded into a simple Gaussian beam that is bright at the center. Parts of the experiments results are shown in Fig. 7(b). Figures 7(b1) and 7(b2) show the measurement results of two separated beams with topological charges of -1 and 1, which are obtained from the transformation of a LG_{0-1} mode beam by the first-order SQWP. Figures 7(b3) and 7(b4) show the measurement results of two separated beams with topological charges of -2 and 0, which are obtained from the transformation of a LG_{0-2} mode beam by the first-order SQWP. The experimental results are consistent with the theoretical predictions.

In this Letter, a spatially variable retardation device, an SQWP, is designed to generate PVBs. In addition, the transformation of LG beams by the SQWPs is also studied. Simulation and experimental studies are carried out. The experimental results agree with the theoretical analysis. Our study results not only give a way to generate PVBs, but also give a way to generate helical beams and measure the topological charges of helical beams.

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