Surface waves at the interface of superconductor media and nonlinear metamaterials

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We theoretically study the nonlinear surface wave propagation at the interface between superconductor media and nonlinear metamaterials. The dispersion equation is analytically derived and solved numerically. Moreover, we present the power for the propagating waves at the interface. The results display different behaviors of the propagating waves as the nonlinear term or temperature is tuned. These results indicate that this structure can have potential applications in superconductor waveguide devices and integrated optics.

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Metamaterials with negative permittivity and/or permeability have received much attention due to their extraordinary electromagnetic characteristics[1–4]. Presently, most of the metamaterials were realized by artificial metallic structures with metallic plasma resonance, such as using split-ring resonators (SRRs) to produce effective negative permeability. Nonlinear electromagnetic responses in metamaterials are established by the addition of metamaterials into nonlinear materials, for example, by inserting the metallic wires and SRRs in a Kerr-type dielectric[5]. Recently, with the development of the metamaterial manufacturing technique, surface waves, which propagate along the interface between different media and decay in the transverse directions, have drawn considerable attention in novel metamaterials[6], whose surface waves are useful for improving resolution and the amplification of evanescent modes by the excitation of surface waves (or surface polaritons) at both interfaces of the slab.

In the terahertz band, semiconductor is a suitable material for discussing active plasma photonics. The terahertz plasmonic switches controlled by temperature, electric field, and magnetic field have been widely studied in the past few years[7–9], but there are high losses. In order to solve this problem, using high-temperature superconductor instead of semiconductor has epoch-making significance in surface plasmon polaritons. It has been confirmed that superconductor can be used in plasma optical media, for example, Josephson plasma resonance effect exists between copper oxide layers of cuprate superconductors[10]. Tsiatmas *et al.*[11] studied surface plasmon polaritons transmission enhancement effect of high-temperature superconductor with periodic subwavelength hole on a sapphire substrate in the microwave, and obtained a smaller loss than that using the metal film. Zhang *et al.* experimentally studied yttrium–barium–copper oxide superconductor thin film[12–14], and discovered that the properties of the superconductor can be tuned in a very wide temperature range with low loss, and that it also has important applications in terahertz surface plasmon polaritons device.

According to the current theory, we can consider superconductors as the media with negative real dielectric constant and kinetic resistance. Therefore, the electromagnetic property of correlative structures falls into the sphere of plasmonics. In order to study the electrodynamics of a superconductor at nonzero temperature[15,16], Liang *et al.* employed the two-fluid model and found that the dielectric constant of a superconductor was strongly influenced by temperature, thus providing the potential possibility for realizing tunable optical devices[17].

In this letter, we conduct a theoretical investigation of nonlinear surface waves localized at the interface between superconductor media and nonlinear metamaterials. Based on the continuity of the electric and magnetic fields at the interface, we obtain the dispersion relation of the nonlinear surface waves. We further discuss the real and imaginary parts of the effective refractive index, field amplitude distribution, and the power flux of the nonlinear surface polaritons in detail.

We consider an interface between the nonlinear metamaterials in the $y < 0$ region and the superconductor media in the $y > 0$ region (Fig. 1). The permittivity

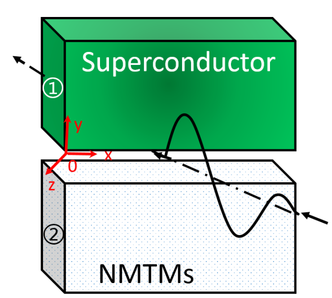

Fig. 1. Structure of the interface between two semi-infinite media of nonlinear metamaterial and high-superconductor media.

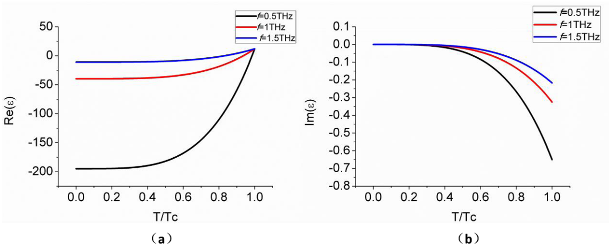

Fig. 2. Variations of the real and imaginary parts of the dielectric constant versus temperature T in three different frequencies f (black line $f = 0.5$ THz), (red line $f = 1$ THz), and (blue line $f = 1.5$ THz): (a) real part and (b) imaginary part of the dielectric constant of the superconductor.

and permeability of the nonlinear metamaterial are $\varepsilon_2^{\mathrm{NL}}(\omega) = \varepsilon_2(\omega) + \alpha\left|E\right|^2$ and $\mu_2(\omega)$, respectively, where $\varepsilon_2(\omega)$, is the linear part of metamaterial and the nonlinear parameter α describes Kerr-type nonlinearity. The nonlinear parameter may be positive (or negative), describing self-defocusing (or self-focusing) property, $\alpha = 3\times10^{-9}$. It has been shown by Essadqui *et al.*[5] that the linear part is given in $\varepsilon_2(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}$ and $\mu_2(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}$, where ω is the angular frequency of the field, ω_0 and ω_{p} are the effective magnetic and electric plasma frequencies, ω_0, F and ω_{p} are known constants, which are the functions of the size, the geometry (shape), and the spacing of the structure.

Based on the two-fluid model, the superconductor complex electrical conductivity σ is the sum of conductivities of the unpaired normal state electrons σ_{n} and the paired superconducting state electrons σ_{s} in the presence of a time-harmonic electromagnetic field. It has been shown by Li *et al.*[18] that the complex conductivity of the superconductor can be improved to

$$\sigma(\omega) = \left[\frac{\varepsilon_0 c^2}{\lambda_{\mathrm{L}}^2(0)}\right](\tau - i\tau^2\omega)\left(\frac{T}{T_{\mathrm{c}}}\right)^4 - i\left[\frac{\varepsilon_0 c^2}{\lambda_{\mathrm{L}}^2(0)}\right]\left[1 - \left(\frac{T}{T_{\mathrm{c}}}\right)^4\right]/\omega,$$

where T and T_{c} are temperatures of sample and phase transformation, respectively and $\lambda_L(0)$ is the London penetration depth at absolute zero temperature. By applying the equation $\varepsilon_{\mathrm{c}} = \varepsilon'(1 - \mathrm{i}\sigma/\omega)$[19] to the model, where ε' is the static dielectric constant, the common dielectric response of superconductor can be investigated. There is a typical superconductor (high temperature) $Bi_{1.85}Pb_{0.35}Sr_2Ca_2Cu_{3.1}O_y$, where $T_{\mathrm{c}} = 107$ K, $\lambda_{\mathrm{L}}(0) \approx 23\ \mu$m, and $\varepsilon' = 12$[17]. Figure 2 shows the variation of the real and imaginary parts of the dielectric constant versus the temperature, and it displays the temperature-dependent properties of superconductor complex dielectric constant ε.

In this letter, we only consider transverse electric field. Applying the fields into Maxwell' equations, the following wave equations can be constructed in the two regions:

$$\frac{\partial^2 E_x(y)}{\partial y^2} + \left(\kappa_0^2\varepsilon_1\mu_1 - \beta^2\right)E_x(y) = 0,\ \ y > 0, \tag{1}$$

$$\frac{\partial^2 E_x(y)}{\partial y^2} + \left[k_0^2\left(\varepsilon_2\mu_2 + \mu_2\alpha\left|E_x(y)\right|^2\right) - \beta^2\right] E_x(y) = 0,\ \ y < 0, \tag{2}$$

where $\beta = n_{\mathrm{eff}}\kappa_0$, n_{eff} is the effective index and κ_0 is the propagation constant in free space. In this letter, we only consider a self-focusing nonlinearity with $\alpha > 0$. The solutions of Eqs. (1) and (2) are given by

$$E_x(y) = E_0 e^{-\kappa_1 y},\ \ y > 0, \tag{3}$$

$$E_x(y) = \frac{\kappa_2}{\kappa_0}\sqrt{\frac{2}{\alpha\mu_2}}\mathrm{sech}\left[\kappa_2\left(y - y_0\right)\right],\ \ y < 0, \tag{4}$$

where $\kappa_1 = \sqrt{\beta^2 - \kappa_0^2\varepsilon_1\mu_1}$ is the transverse decay index in superconductor media, $\kappa_2 = \sqrt{\beta^2 - \kappa_0^2\varepsilon_2\mu_2}$ is the transverse decay index in nonlinear metamaterials, and y_0 is the position of the magnitude maximum of the nonlinear surface polaritons.

Solving the boundary condition problem at $y = 0$ will result in the dispersion equation

$$\tanh\left(\kappa_2 y_0\right) = \frac{\mu_2\kappa_1}{\mu_1\kappa_2}. \tag{5}$$

The power fluxes in superconductor and nonlinear metamaterials are described by the Poynting vector

$$\mathbf{P} = P^s + P^{\mathrm{NL}} = \frac{1}{2}\mathrm{Re}\int_0^{+\infty}\left(\boldsymbol{E}\times\boldsymbol{H}^*\right)_z \mathrm{d}x + \frac{1}{2}\mathrm{Re}\int_{-\infty}^{0}\left(\boldsymbol{E}\times\boldsymbol{H}^*\right)_z \mathrm{d}x. \tag{6}$$

The fields (Eqs. (3) and (4)), dispersion equation (Eq. (5)), and total (single) power (Eq. (6)) are solved numerically. However in the numerical results presented here, the analytical scheme is only used, as it is not based on approximations.

In numerical analysis, the main idea is organized around the effective refractive index, field amplitude distribution, and the power fluxes of the nonlinear surface polaritons. In order to exhibit the tunability of effective refractive index of the surface wave, we plot the real and imaginary parts of the effective refractive index in Fig. 3, where $F = 0.82$, $\omega_{\mathrm{p}} = 10$ GHz, $\omega_0 = 3$ GHz, and $\alpha = 3\times10^{-9}$. Figures 3(a), (d), (g), and (i) show the real parts of the effective refractive indices, Figs. 3(b), (e), (h), and (j) show the imaginary parts of the effective refractive indices, and Figs. 3(c) and (f) show the partial enlarged views. In Fig. 3, Figs. 3(a) and (b), (d) and (e), (g) and (h), (i) and (j) are drowned in differently when the values of nonlinear term $\alpha E_0^2/2$ are 0.3, 0.8, 1.3, 3. By comparison, we find that the real part of the effective refractive index increases when nonlinear term increases, but the imaginary part of the effective refractive index decreases. The results may be

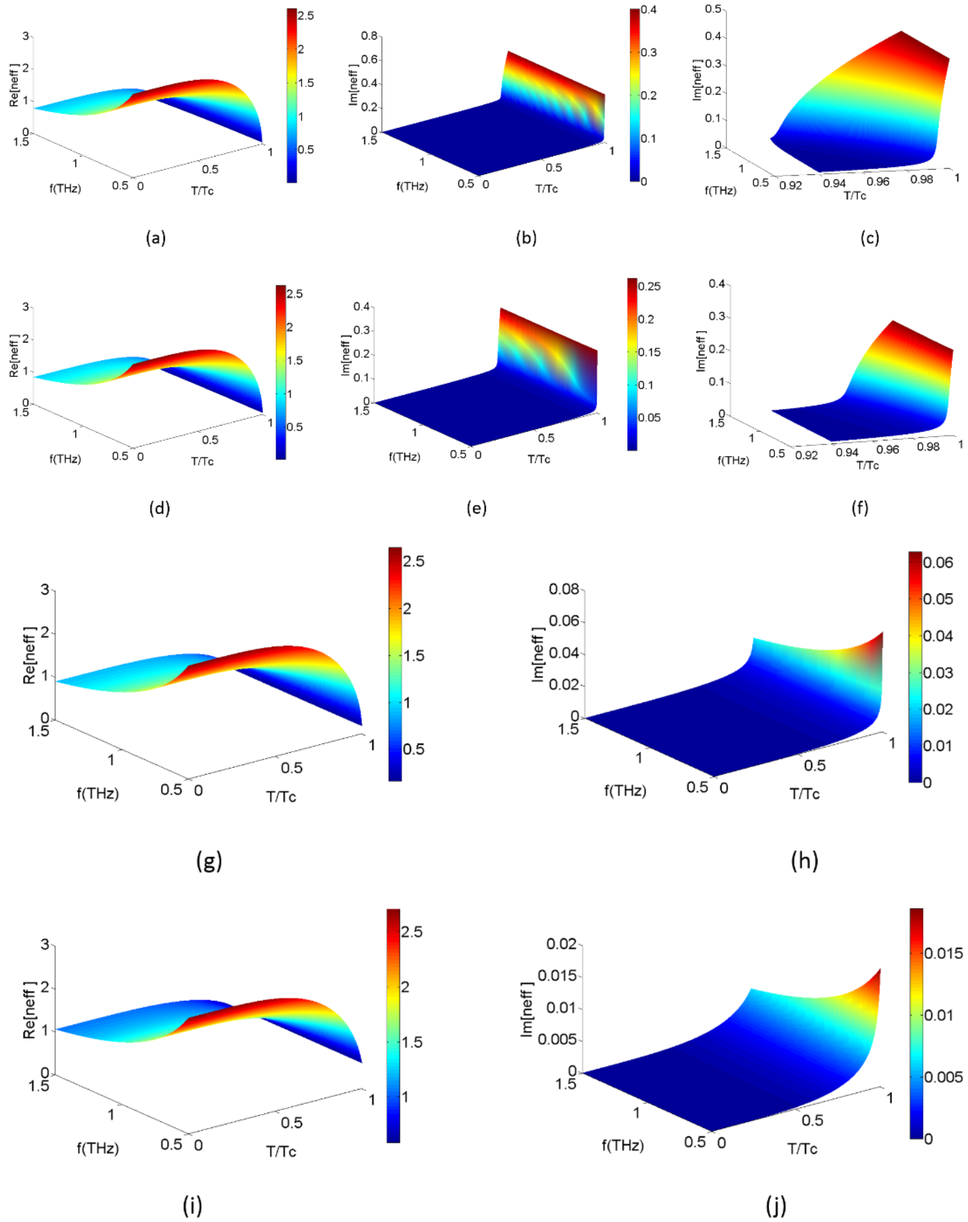

Fig. 3. Effective refractive indices of nonlinear surface polaritons versus temperature in different frequencies: (a) and (b) $\alpha E_0^2/2 = 0.3$, (c) partial view of (b), (d) and (e) $\alpha E_0^2/2 = 0.8$, (f) partial view of (e), (g) and (h) $\alpha E_0^2/2 = 1.3$, and (i) and (j) $\alpha E_0^2/2 = 3$.

explained reasonably by the effective medium theory. Because the permittivity and refractive index of the nonlinear metamaterial increase with the nonlinear term, therefore, the real part of the effective refractive index increases when the nonlinear term increases. Since we only consider the loss of the superconductor that results from imaginary parts of the dielectric constant, the imaginary part of the effective refractive index and the loss of surface wave come from the superconductor. As shown in Fig. 2(b), the imaginary part of the superconductor is almost zero in low temperature, and it increases slowly with temperature when the temperature is close to the phase transition temperature. Moreover, the transverse energy of surface wave is mainly distributed in nonlinear metamaterials

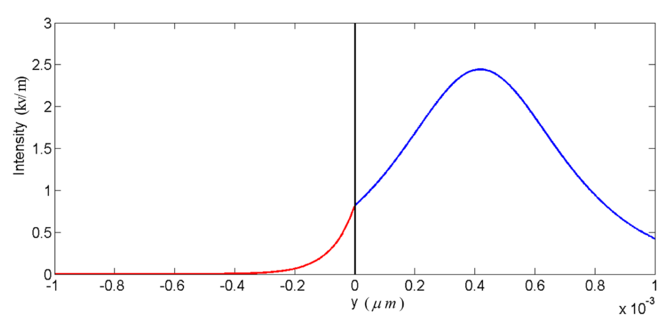

Fig. 4. Spatial profile of the field amplitude distribution when $T = 0.98\ T_c$, $f = 0.5$ THz, and $\alpha E_0^2/2 = 0.1$. The red line represents the characteristics of the electric field in superconductor media and the blue line represents the characteristics of the electric field in nonlinear metamaterial.

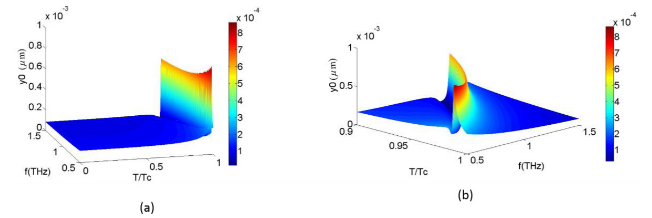

Fig. 5. Positions of the magnitude maximum of the nonlinear surface polaritons versus temperature in different frequencies: (a) $\alpha E_0^2/2 = 0.3$ and (b) partial view of (a).

without propagation loss as can be seen from Fig. 4 discussed in the following. Therefore, the imaginary part of the effective refractive index approaches zero in wide range of temperature, and would be very small even though the temperature is near the phase transition temperature. While the nonlinear term becomes large in the nonlinear metamaterial, the energy of surface wave will concentrate into the nonlinear metamaterial, thus the imaginary part of the effective refractive index and the loss will decrease with the nonlinear term (Figs. 2(b), (e), (h), and (j)). We also find an interesting result by numerical calculations, all of the imaginary parts of the effective refractive indices almost tend to zero when the nonlinear term $\alpha E_0^2/2 = 1.2$. This indicates that it has potential application prospect in dealing with the loss of the superconductor plasmon polaritons waveguide.

We numerically solve Eqs. (3) and (4) and the field amplitude distribution is presented as in Fig. 4. As shown in Fig. 4, in the superconductor region, the intensity exhibits exponential decay starting from the interface, and in another region, there is a soliton, which has a maximum near the interface. The positions of the magnitude maximum of the nonlinear surface polaritons versus temperature in different frequencies are shown in Fig. 5. In Fig. 5, we chose the nonlinear parameter $\alpha E_0^2/2 = 0.3$, Fig. 5(b) is the partial view of Fig. 5(a). As shown in Fig. 5(b), it has a similar cosine function jump when the temperature changes from $0.9T_c$ to T_c, it illustrates the position of the maximum field amplitude

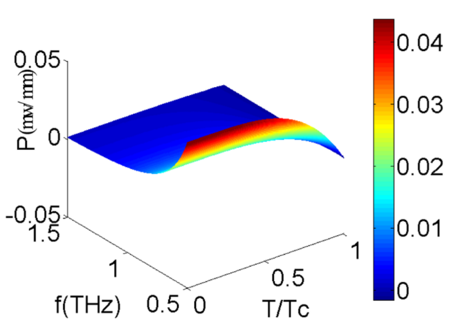

Fig. 6. Dependence of the total power flow at the nonlinear metamaterials/superconductor interface on temperature and frequency when $\alpha E_0^2/2 = 0.3$.

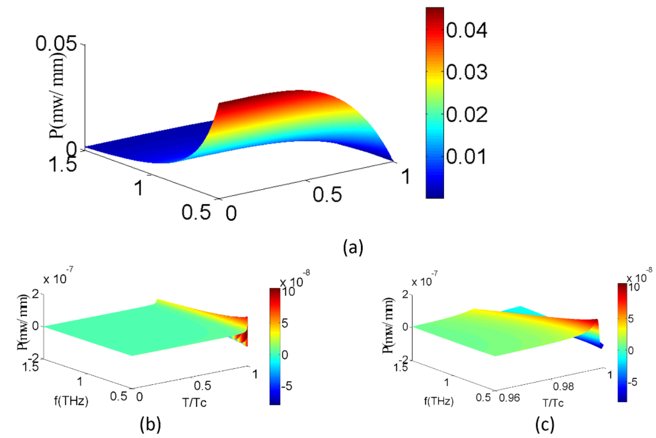

Fig. 7. Profiles of power in different materials: (a) characteristics of power in nonlinear metamaterials, (b) characteristics of power in superconductor, and (c) partial view of (b).

of the nonlinear surface polaritons exist swing under special conditions. We consider it can be used in designing some sensors.

The total power flow at the nonlinear metamaterials/superconductor interface on temperature and frequency is displayed in Fig. 6. We show the variation of the power flow when the nonlinear term $\alpha E_0^2/2 = 0.3$.

Through the numerical calculation, we find that when the nonlinear term $\alpha E_0^2/2 > 1.884$ there are no negative values of the total power. It explains that there are no backward waves and only forward waves. We also find that the power decreases when the nonlinear term increases. In addition, we plot the variation of single power flow versus temperature and nonlinear term in Fig. 7. Figure 7(a) displays the characteristics of total power in nonlinear metamaterials versus temperature and frequency when the nonlinear term $\alpha E_0^2/2 = 1.884$ and Fig. 7(b) displays the characteristics of total power in superconductor media accordingly.

In conclusion, we theoretically investigate the nonlinear surface waves localized at the single interface separating superconductor media and nonlinear metamaterials. Based on the continuity of the electric and magnetic fields at the interface, we discuss the effective refractive indices, the power fluxes, the positions of the magnitude maximum of the surface polaritons, and the variations of forward and backward waves. We find that the effective refractive index can be tuned by temperature, the position of the magnitude maximum of the nonlinear surface polaritons exist swing under special conditions and the forward and backward waves can be changed by nonlinear parameters or frequency. The results suggest that it can have potential applications in superconductor waveguide devices and integrated optics.

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