# Measuring method for the vertex radius of curvature of an asphere with a laser tracker

**Ruigang Li (李锐钢)**

*Key Laboratory of Optical System Advanced Manufacturing Technology, Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, China*

*Corresponding author: Liruigang@sohu.com*

Received March 18, 2014; accepted July 16, 2014; posted online January 16, 2015

Aspheric elements are widely applied in optical systems and the vertex radius of curvature (VROC) is one of the important fundamental parameters of an asphere. We present a method for measuring the VROC of asphere. We use a portable laser tracker to measure the optical interval of the null testing path and then determine the VROC of the asphere through ray tracing. Based on this method, we carry out an accurate measurement. The accuracy can reach up to 0.056 mm on an asphere with VROC of approximately 2 m and the relative error is 0.003%.

*OCIS codes: 220.0220, 220.1250, 220.4840,120.0120.*

*doi: 10.3788/COL201513.S12201.*

Aspheric optical elements being used in optical systems features so many merits, for example, controlling the aberration more effectively, reducing the quantity of the elements, and decreasing the size and mass of optical system, etc.[1]. Therefore, optical designers tend to use aspheric optical elements in optical systems[2–4].

For an optical lens or mirror, radius of curvature (ROC) is one of the important parameters, which is closely related to its surface. The methods used to measure the ROC of spherical optical surfaces are well known and can be divided into two major categories. One is according to the center of curvature of the sphere, to measure the distance between the center of sphere and the spherical surface by mechanical or optical means[5,6]. The other is to measure the sag of the spherical surface and then calculate the ROC from the sag value. Since for an aspheric optical surface, different points on the surface have different ROCs, the vertex ROC (VROC) should be measured. There are several methods to measure the VROC of asphere[7–9]. According to the laws of reflection and the theory of aberration, one can calculate the VROC of asphere through ray tracing. By testing residual figure errors of the aspheric surface, the VROC can be deduced. Using a reference testing plate with a computer-generated hologram, it is possible to build a test system to measure the deviation of VROC.

In this letter, we present a method that is able to determine the VROC of aspheric surface using the portable laser tracker. Basically, there is a definite relationship between the optical interval and the VROC of the asphere under test among the interferential null testing setups. The principle and procedure are explained in detail, and the analysis of the systematic errors is given. Finally, an experiment of the VROC of an asphere using laser tracker was carried out, and showed that the accuracy was up to 0.056 mm, and the relative error was as low as 0.003%.

At present, the widely used testing method of asphere's figure error during polishing is interferential null testing with compensator[10]. A planar or spherical wavefront is emitted from the He–Ne laser of the interferometer, transmitting through a compensator designed specially and transforming to an aspheric wavefront, and then casting on the asphere under test and reflecting with the asphere's figure error information. After going through the compensator again, finally it returns to interferometer and interferes with the reference wavefront, and then we can acquire the figure error of the asphere under test by image processing.

Figure 1 shows the optical layout of the interferential null testing with compensator (transmission-type) for an asphere. Between the vertices of the last surface of the compensator and the asphere, there is a definite relationship with the VROC of the tested asphere. Thus, we can obtain the VROC indirectly from measuring the optical interval of the optical path using laser tracker.

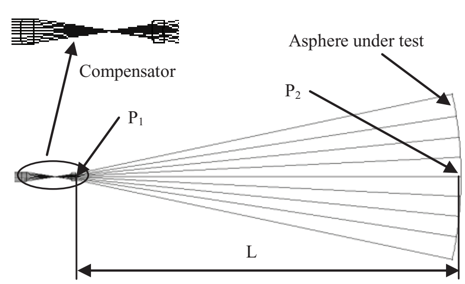


Fig. 1. Layout of interferential null testing of asphere.

 

The next component of the measurement is the laser tracker, which is a kind of portable coordinate measurement machine with high precision. It is a distance measuring interferometer (DMI) with self-tracking ability. A laser tracker uses a DMI to measure the distance, and two angular encoders to measure the rotation angles. A laser beam (He–Ne laser with emission wavelength of 633 nm) is directed to and reflected by a sphere-mounted retroreflector (SMR). When the SMR moves, a feedback sensor detects the motion, and the mechanic and electronic devices ensures the machine to keep tracking.

Due to many merits such as large measuring range, fast responding speed, and high precision, the laser tracker has been widely employed in the fields such as space telescopes, aircraft industry, auto making, heavy-duty machinery manufacturing, and shipbuilding industry[11–13]. In this letter, we use the laser tracker as a critical tool to measure the VROC of an aspheric surface.

The optical elements of the compensator are mounted into a mechanical cylinder according to optical design. The compensator's optical eccentricity, deflection, and interval between the elements are adjusted within certain tolerance. The mechanical exterior cylindrical surface and backplane of compensator are the two reference surfaces that are frequently used for aligning and testing, and which are always fabricated up to high precision.

As shown in Fig. 1, using laser tracker to measure the exterior cylindrical surface and the backplane of the compensator, we can get the vertex's coordinates $P_1(x_1, y_1, z_1)$ of the last surface of the compensator's optical elements through coordinates processing, and measure the asphere's best fitting sphere (BFS). We can obtain the vertex's coordinates $P_2(x_2, y_2, z_2)$ of the asphere through coordinates processing. After that, the optical interval of the compensator is calculated as

$$L = \overline{P_1 P_2}. \quad (1)$$

We use the optical design software for ray tracing optical intervals $L_i$ and $R_i$ with a certain sampling density. Finally, we acquire the actual VROC by interpolating or inquiring the ray tracing table.

Figure 2 illustrates the measuring procedure of VROC in flow diagram. The procedure includes, firstly, setting up the interferometer, the compensator, and the asphere under test according to the optical designed data. Secondly, we measure the reference surface of the compensator and the asphere's body, and construct the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ by coordinate processing such as intersecting and translating. Lastly, we calculate the optical interval between points $P_1$ and $P_2$ via

$$L = \left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2\right]^{1/2}. \quad (2)$$

We can then find the actual VROC by ray tracing.

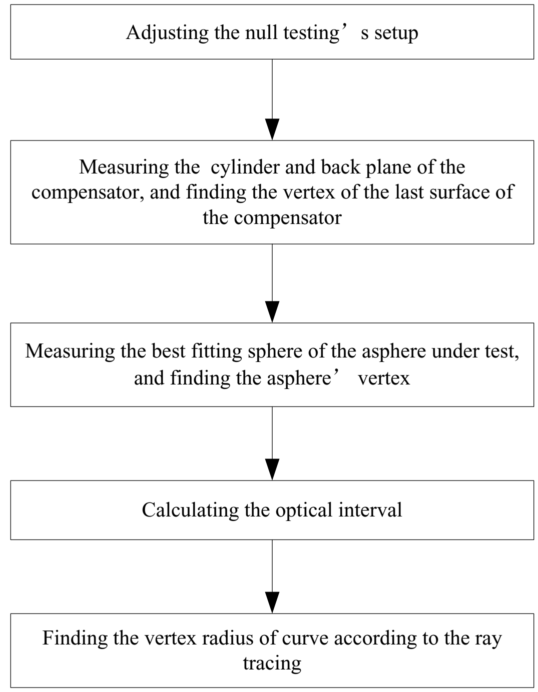


Fig. 2. Flow diagram of VROC measurement using laser tracker.

As it is an indirect method to measure the optical interval $L$ using laser tracker, the accuracy of the measurement depends mainly on the accuracy of the measurement of $L$. Moreover the accuracy of measured $L$ is greatly affected by the measurement accuracy of the two points involved, that is, the vertex of the compensator's last surface, $P_1$ and the vertex of the asphere under test, $P_2$.

There are three types of systematic errors:

1. Errors that have direct influence on the measurement of interval $L$. For example, the flatness of the backplane of the compensator brings errors to the point $P_1$, which affect the interval's measuring.
2. Errors that make the transverse position (perpendicular to optical axis) of point $P_1$ or point $P_2$ varying a distance of d$s$, so that the longitudinal distance (along the optical axis) $L$ is changed by d$z$. Because point $P_1$ or point $P_2$ is positioned on spherical surface, we set $s^2 = x^2+y^2$. Meanwhile, we solve the spherical equation

$$x^2+y^2+(z-r)^2 = r^2, \quad (3)$$

where $r$ is the radius of the sphere and we get the derivation $\frac{\mathrm{d}z}{\mathrm{d}s}$. Then we know when $|s| < \frac{r}{\sqrt{2}}$, $\left|\frac{\mathrm{d}z}{\mathrm{d}s}\right| < 1$, namely $|\mathrm{d}z| < |\mathrm{d}s|$.

Errors that effect the transverse position of point $P_1$ or point $P_2$ are eccentricity and deflection that appear in the compensator's assembling, and $|s| << \frac{r}{\sqrt{2}}$, which has an effect on optical interval $|\mathrm{d}z| << |\mathrm{d}s|$.

3. Errors of the laser tracker testing system, which can be looked up in the user's manual.

The analysis of systematic errors of the VROC's measurement is given in Table 1. We set length of the null testing optical path as $D$, the cylindricity of the exterior surface of the compensator as $C$, and the flatness of the backplane of compensator as $F$. And after assembling, the mechanical axis has an eccentricity $E$ and a deflection angle $A$ with respect to optical axis.

We carried out a measuring experiment using an off-axis aspheric mirror via the method and procedure as mentioned above. The physical dimension and reference surface of the compensator and the asphere under test are measured with Zeiss PRISMO Navigator, its accuracy is found to be 0.9 $\mu$m. cylindricity.

The parameters of the compensator are as follows: cylindricity is better than 10 $\mu$m, flatness is smaller than 10 $\mu$m, eccentricity is smaller than 10 $\mu$m, deflection is smaller than 5″, the distance between the backplane and the vertex of the last surface is 14.713 mm, and the optical interval $L$ between the vertices of the last surface and the asphere under test is 1901.530 mm.

The mirror under test is a concave and off-axis asphere, and the shape of the blank is rectangle with four circular corners. The parameters of the asphere are as follows: length is 661.032 mm, width is 216.048 mm, radius of the BFS is 1967.644 mm, deviation at the vertex from the BFS to the asphere is −2.704 $\mu$m, and designed value of the VROC of the asphere is 1968.196 mm.

The interferential null testing of asphere is illustrated in Fig. 1. We can obtain the relationship between the optical interval $L$ and the VROC of asphere $R$ by ray tracing using the optical design software ZEMAX. Table 2 summarizes the simulation results of the cited example, and the sampling interval is 0.1 mm. Figure 3 reveals the relationship between $L$ and $R$. Based on Table 2 and Fig. 3, we can clearly see that there is a good linear relationship between $\Delta R$ and $\Delta L$, and the slope is close to 1.

**Table 1.** Systematic Errors Analysis of Measuring Method of VROC of Asphere

| Item | Error | Remark |
|---|---|---|
| 1. Flatness of backplane | $e_1<F$ | $P_1$ |
| 2. Eccentricity | $e_2<<2*E$ | $P_1$ and $P_2$ |
| 3. Deflection | $e_3<< D*\tan(A)$ | $P_2$ |
| 4. Laser tracker | Distance accuracy: $e_4$=2 $\mu$m+$D$*0.4 $\mu$m/m Angle accuracy: $e_5$=10 $\mu$m+$D$*2.5 $\mu$m/m | FARO (ION), IFM (interferometer), distance accuracy and angle accuracy are all typical values |
| Total | $e=\sqrt{\sum_{i=1}^{4} e_i^2}$ | Root sum squared of individual errors |

Figure 4 shows the experimental setup and the measuring steps are described as follows:

1. In the measuring software of laser tracker, we choose "cylinder" to measure the compensator's exterior cylinder, the number of sampling points should be more than 4. The optical axis can be represented by the axis of this cylinder.
2. In the measuring software of laser tracker, we choose "plane" to measure the compensator's backplane, the number of sampling points should be more than 3. Let this "plane" intersect with the "cylinder" mentioned in step 1, build a point, then translate a distance of 14.713 mm, toward the last surface of the compensator to get $P_1(x_1, y_1, z_1)$.
3. In the measuring software of laser tracker, we choose "sphere" to measure the BFS of the asphere under test, the number of sampling points should be more than 4. And then we intersect it with the "cylinder" mentioned in step 1, build the vertex of BFS, then translate a distance of −2.704 $\mu$m toward the vertex of the asphere under test to get $P_2(x_2, y_2, z_2)$.
4. After we find the optical interval by evaluating Eq. (1), we then obtain the VROC of asphere under test through ray tracing.
5. To analyze the systematic errors of the method, and the random errors, we deal with them by averaging several measuring results.

We carried out independent measurement of the optical interval $L$ for five times. The results are listed as follows: 1901.734, 1901.722, 1901.736, 1901.735, and 1901.729 mm.

The average is 1901.731 mm. Through ray tracing, we get the VROC of the asphere as 1968.396 mm. For the systematic error of this measurement, we analyze it according to the method shown in Table 1 using the parameters given above. Finally, the accuracy is 0.056 mm and the relative error is 0.003%.

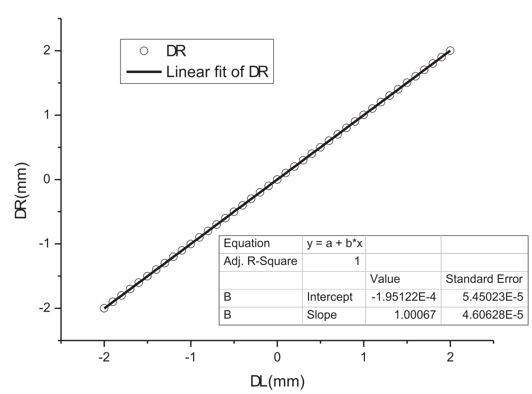


Fig. 3. Relationship curve of $\Delta R$ and $\Delta L$.

**Table 2.** Ray Tracing Results for $L$ and $R$

| No. | $L$ | | $R$ | |
|---|---|---|---|---|
| | Value (mm) | Deviation (mm) | Value (mm) | Deviation (mm) |
| 1 | 1899.53 | −2.0 | 1966.195 | −2.001 |
| 2 | 1899.63 | −1.9 | 1966.295 | −1.901 |
| 3 | 1899.73 | −1.8 | 1966.395 | −1.801 |
| 4 | 1899.83 | −1.7 | 1966.495 | −1.701 |
| 5 | 1899.93 | −1.6 | 1966.595 | −1.601 |
| 6 | 1900.03 | −1.5 | 1966.695 | −1.501 |
| 7 | 1900.13 | −1.4 | 1966.795 | −1.401 |
| 8 | 1900.23 | −1.3 | 1966.895 | −1.301 |
| 9 | 1900.33 | −1.2 | 1966.995 | −1.201 |
| 10 | 1900.43 | −1.1 | 1967.095 | −1.101 |
| 11 | 1900.53 | −1.0 | 1967.195 | −1.001 |
| 12 | 1900.63 | −0.9 | 1967.295 | −0.901 |
| 13 | 1900.73 | −0.8 | 1967.395 | −0.801 |
| 14 | 1900.83 | −0.7 | 1967.495 | −0.701 |
| 15 | 1900.93 | −0.6 | 1967.595 | −0.601 |
| 16 | 1901.03 | −0.5 | 1967.695 | −0.501 |
| 17 | 1901.13 | −0.4 | 1967.795 | −0.401 |
| 18 | 1901.23 | −0.3 | 1967.895 | −0.301 |
| 19 | 1901.33 | −0.2 | 1967.995 | −0.201 |
| 20 | 1901.43 | −0.1 | 1968.095 | −0.101 |
| 21 | 1901.53 | 0 | 1968.196 | 0 |
| 22 | 1901.63 | +0.1 | 1968.296 | +0.1 |
| 23 | 1901.73 | +0.2 | 1968.396 | +0.2 |
| 24 | 1901.83 | +0.3 | 1968.496 | +0.3 |
| 25 | 1901.93 | +0.4 | 1968.596 | +0.4 |
| 26 | 1902.03 | +0.5 | 1968.696 | +0.5 |
| 27 | 1902.13 | +0.6 | 1968.796 | +0.6 |
| 28 | 1902.23 | +0.7 | 1968.896 | +0.7 |
| 29 | 1902.33 | +0.8 | 1968.997 | +0.8 |
| 30 | 1902.43 | +0.9 | 1969.097 | +0.901 |
| 31 | 1902.53 | +1.0 | 1969.197 | +1.001 |
| 32 | 1902.63 | +1.1 | 1969.297 | +1.101 |
| 33 | 1902.73 | +1.2 | 1969.397 | +1.201 |
| 34 | 1902.83 | +1.3 | 1969.497 | +1.301 |
| 35 | 1902.93 | +1.4 | 1969.597 | +1.401 |
| 36 | 1903.03 | +1.5 | 1969.697 | +1.501 |
| 37 | 1903.13 | +1.6 | 1969.797 | +1.601 |
| 38 | 1903.23 | +1.7 | 1969.897 | +1.701 |
| 39 | 1903.33 | +1.8 | 1969.997 | +1.801 |
| 40 | 1903.43 | +1.9 | 1970.097 | +1.901 |
| 41 | 1903.53 | +2.0 | 1970.197 | +2.001 |

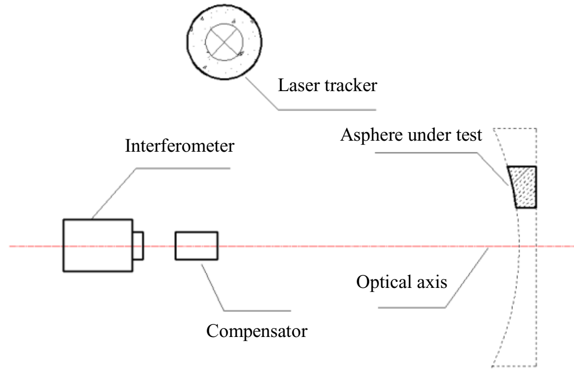


Fig. 4. Sketch of VROC measurement using laser tracker.

In conclusion, we present a method to measure the VROC of asphere using portable and high precision laser tracker. It is suitable for the measurement of the VROC of asphere when interferential null testing with compensator is employed. The tested asphere can be convex or concave. It can be applied to long ROC measurement, if only it is among the measuring range (±55 m) of laser tracker, and the accuracy decreases along with increase in measuring distance. Although the method is indirect, it has good reproducibility and its accuracy can be analyzed clearly. More importantly, theoretically, the accuracy is very high. The accuracy can reach up to 0.056 mm for an aspherical mirror with 2 m of VROC and the relative error is 0.003%.

This work was supported by the Youth Foundation of National Natural and Science Foundation of China under Grant No. 51305422.

## References

1. J. Ruckman, E. M. Fess, and D. V. Gee, Proc. SPIE. **3782**, 2 (1992).
2. B. Du, L. Li, and Y. Huang, Chin. Opt. Lett. **8,** 569 (2010).
3. F. Yan, D. Fan, B. Zhang, L. Yin, R. Li, and X. Zhang, Chin. Opt. Lett. **7,** 534 (2009).
4. X. Wang, L. Wang, L. Yin, B. Zhang, D. Fan, and X. Zhang, Chin. Opt. Lett. **5,** 645 (2007).
5. L. A. Selberg, Opt. Eng. **31,** 1961 (1992).
6. C. Zhao, R. Zehnder, and J. H. Burge, Opt. Eng. **44,** 090506 (2005).
7. H. Wang, Y. Li, L. Zeng, C. Yin, and Z. Feng, Opt. Commun. **232,** 61 (2004).
8. R. Diaz-Uribe and A. Cornejo-Rodriguez, Appl. Opt. **25,** 3731 (1986).
9. D. Baiocchi and J. H. Burge, Proc. SPIE **4093,** 58 (2000).
10. D. Malacara, *Optical Shop Testing* (Wiley, 1992).
11. J. A. Connelly, B. J. Bos, and P. S. Davila, Proc. SPIE. **7068,** 70680F (2008).
12. S. Aguado, D. Samper, J. Santolaria, and J. J. Aguilar, Proc. Eng. **63,** 582 (2013).
13. A. Nubiola and I. A. Bonev, Robot. Comput.-Integr. Manuf. **29,** 236 (2013).