Incoherently coupled soliton pairs in parity-time symmetric Bessel potentials

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We present a theory to investigate the existence and the propagation properties of incoherently coupled single-hump and dipole soliton pairs in self-defocusing media with parity-time symmetric lattice. These soliton pairs can exist provided that they are composed of two optical beams with the same polarization and wavelength. It is found that single-hump soliton pairs are always stable when the components copropagate in the lattice, whereas high-power dipole soliton pairs are unstable. If one of the components is absent, the propagation behavior of the other one is also studied.

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Incoherent coupling of two optical beams to form incoherently coupled soliton pairs has attracted lots of research interests in the past two decades[1–8]. This kind of soliton pairs was first predicted and observed in biased photorefractive crystals[1–3]. When two mutually incoherent beams with the same polarization and frequency propagate collinearly in a photorefractive crystal, the combined intensity distribution will create an effective refractive-index modulation. Both the beams experience this refractive-index modulation and they can propagate stably in the nonlinear system to form a coupled soliton pair. Later, these kinds of coupled soliton pairs were found in photovoltaic nonlinearity[4], screening-photovoltaic nonlinearity[5,6], biased centrosymmetric photorefractive media[7], biased guest–host photorefractive polymers[8], and two-photon photorefractive crystals[9], etc.

On the other hand, the light propagation in parity-time (PT)-symmetric potentials has been an intriguing issue since the first introduction of PT optics in 2005[10]. In optics, a complex potential can be readily constructed by carefully incorporating regions having optical gain and loss in the media so that the complex refractive index satisfies the PT symmetry $n(x) = n^*(-x)$, where x is the spatial coordinate and "*" stands for complex conjugation. In other words, the refractive index profile has even symmetry and the gain–loss profile has odd symmetry. Such optical PT potentials have been created experimentally in coupled two-channel system[11] and large-scale lattices[12]. In the context of nonlinear optics, PT-symmetric nonlinear lattices can support soliton solutions and these solitons can be stable over a wide range of parameters[13]. Due to the presence of odd loss or gain, optical solitons exhibit different shapes and propagation behaviors from those in real lattices[14–16]. So far, it has proved that bright PT solitons can exist and propagate stably in various kinds of nonlinear systems with PT symmetry[17–22]. Gray solitons[23], vortex solitons[24], and vector solitons[25] have also been obtained and investigated in PT-symmetric potentials. Although there are a large number of investigations on the PT solitons, the propagation of incoherently coupled soliton pairs in PT-symmetric lattice is still an open question. In this letter, we present a theory to study the existence and stability of incoherently coupled single-hump and dipole soliton pairs in self-defocusing media with PT-symmetric Bessel lattice. If one of the components is absent, the propagation behavior of the other one is also discussed.

To start, we assume that two optical beams with the same polarization and wavelength propagate collinearly along the z-axis, and are allowed to diffract only along the x-axis. These two beams can be obtained by splitting a laser beam by a polarizing beam splitter. They are made mutually incoherent at the input face of a self-defocusing Kerr medium with PT-symmetric lattice by making their optical path difference greatly exceed the coherence length. The optical field can be expressed in terms of slowly varying envelopes, that is, $\vec{E}_1 = \vec{\mathrm{i}}u_1(x,z)\exp(-\mathrm{i}\omega t)$ and $\vec{E}_2 = \vec{\mathrm{i}}u_2(x,z)\exp(-\mathrm{i}\omega t)$, where $\vec{\mathrm{i}}$ denotes the unit vector pointing to the x-direction, ω denotes the frequency, u_1 and u_2 are the normalized complex optical field amplitudes, respectively. In this case, the evolution equations can be written as

$$\mathrm{i}\frac{\partial u_{1,2}}{\partial z} + \frac{\partial^2 u_1}{\partial x^2} + \left[V(x) + \mathrm{i}W(x)\right]u_1 - \left(\left|u_1\right|^2 + \left|u_2\right|^2\right)u_{1,2} = 0, \tag{1}$$

where $V(x)$ and $W(x)$ are the real and imaginary parts of the complex potential, respectively. It should be noted that this model was considered for the first time to investigate vector solitons in periodic PT-symmetric optical lattices[25]. We search for the stationary solutions of Eq. (1) in the form of $u_{1,2}(x) = c_{1,2}f(x)\exp(i\mu z)$, where $f(x)$ is a complex function, μ is the corresponding real propagation constant, and $c_1^2 + c_2^2 = 1$. The values of c_1 and c_2 lead to different intensities ratio, but the existence and stability properties of the optical components are similar. For the facility of demonstration, we fix $c_1^2 = 0.7$ and $c_2^2 = 0.3$ hereafter. By substituting these forms into Eq. (1), $f(x)$ should satisfy

$$-\mu f + \frac{\partial^2 f}{\partial x^2} + \left[V(x) + iW(x)\right]f - \left|f\right|^2 f = 0. \qquad (2)$$

For the facility of demonstration, we assume the PT-symmetric lattice be a Bessel potential, that is, $V(x) = V_0 J_0(x)$, and $W(x) = W_0 J_1(x)$, where V_0 and W_0 are the depth of the real and imaginary parts of PT potentials. Note that there exist symmetry-breaking points corresponding to the values of W_0 and V_0. Numerical calculations show that the PT-symmetry breaking takes place at $W_0/V_0 = 1.12$. Below the PT-symmetric breaking points, the eigenfunctions of linear modes are symmetrical. Otherwise, the symmetries of the eigenfunctions break. When $V_0 = 10$ and $W_0 = 3$, which is below the symmetry-breaking points, the profile of the PT-symmetric optical lattice is as shown in Fig. 1.

To elucidate the stability of solitons, we add small perturbations to the soliton solutions, that is, $u_{1,2} = c_{1,2}\,[f(x) + g(x)\exp(\delta z) + t^*(x)\exp(\delta^* z)]\exp(i\mu z)$, where $\left|g\right|, \left|t\right| << \left|u\right|$. Substituting this form into Eq. (1) and linearizing, we get

$$\delta t = g_{xx} - \mu g + Vg - Wt - 2\left|f\right|^2 g - \left[\mathrm{Re}(f)^2 - \mathrm{Im}(f)^2\right]g - 2\,\mathrm{Re}(f)\,\mathrm{Im}(f)t, \qquad (3a)$$

$$\delta g = -t_{xx} + \mu t - Wg - Vt + 2\left|f\right|^2 t - \left[\mathrm{Re}(f)^2 - \mathrm{Im}(f)^2\right]t - 2\,\mathrm{Re}(f)\,\mathrm{Im}(f)g. \qquad (3b)$$

If $\mathrm{Re}(\delta) > 0$, the small perturbation will undergo an exponential growth, and the solitons are linearly unstable. Otherwise, they are linearly stable.

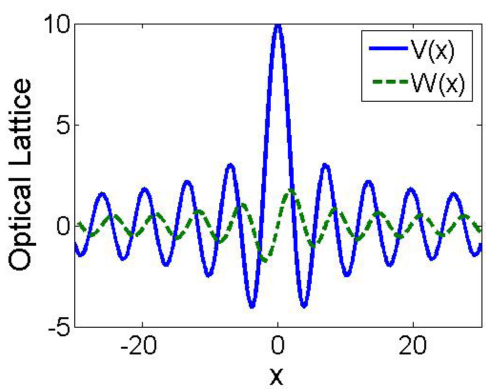

Fig. 1. Profiles of PT-symmetric Bessel lattice, $V_0 = 10$ and $W_0 = 3$.

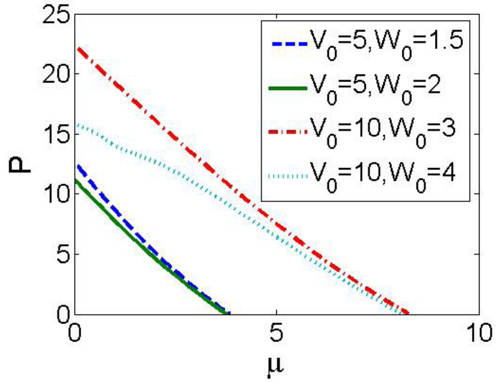

Fig. 2. Total normalized power of single-hump soliton pairs.

Figure 2 shows the existence curves of single-hump soliton pairs for several values of V_0 and W_0. Obviously, the power of the bright soliton pairs (defined as $P = \int_{-\infty}^{\infty}\left(\left|u_1\right|^2 + \left|u_2\right|^2\right)dx$) decreases as the propagation constants increases. We can also see that single-hump soliton pairs can exist when the propagation constant is below a critical value μ_0, which is different from that in PT-symmetric periodic potentials[20]. The value of μ_0 depends on the values of V_0 and W_0. When $V_0 = 10$ and $W_0 = 3$, μ_0 is equal to 8.27; when $V_0 = 10$ and $W_0 = 4$, μ_0 is equal to 8.10. If V_0 is changed to be 5 but the ratios of W_0/V_0 are kept unchanged to be 0.3 and 0.4, then the values of μ_0 are 3.835 and 3.758, respectively. Numerical calculations show that the increase in V_0 or decrease in W_0 leads to the increase in μ_0. The results of stability analysis of Eq. (3) show that $\mathrm{Re}(\delta)$ is always zero and thus all these single-hump soliton pairs can propagate stably when the two components coexist in the optical lattice.

For example, the optical fields of a single-hump soliton pair at $\mu = 2$ and 6 when $V_0 = 10$ and $W_0 = 3$ are shown in Figs. 3(a) and (b), respectively. We can see that both of the optical components have similar optical field profiles. The real parts of optical fields have even symmetry, whereas the imaginary parts have odd symmetry. The relative ratio of imaginary parts to real parts of optical fields decreases when the value of propagation constant increases. When $\mu = 6$, the total power of the single-hump soliton pair is 4.9473. Figures 4(a) and (b) show the stable propagation of single-hump soliton pair at $\mu = 6$ when the components propagate

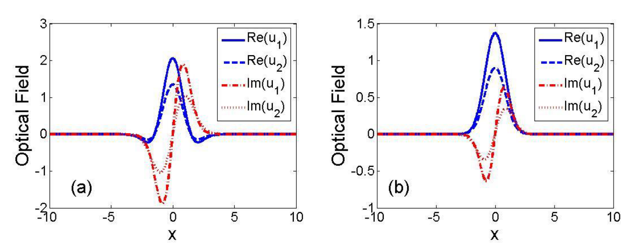

Fig. 3. Optical field profiles of single-hump soliton pairs at (a) $\mu = 2$ and (b) $\mu = 6$ when $V_0 = 10$ and $W_0 = 3$.

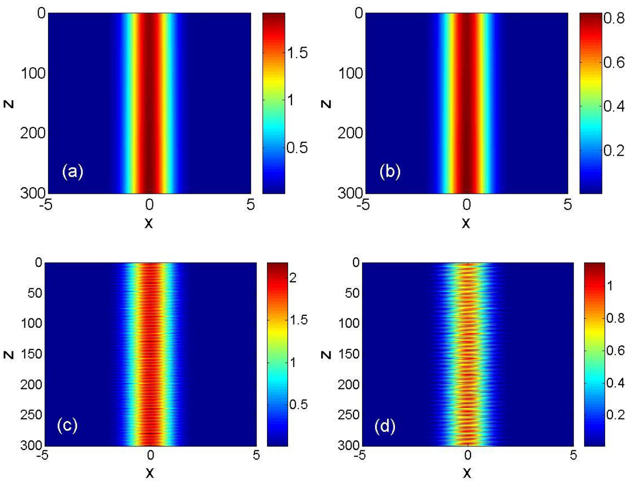

Fig. 4. Stable propagation of the optical components (a) u_1 and (b) u_2 of a single-hump soliton pair at $\mu = 6$. (c) Evolution of u_1 when u_2 is absent and (d) evolution of u_2 when u_1 is absent.

together in the PT-symmetric Bessel potential. If one of the two optical components is turned off, which means it is absent during the whole propagation distance, the other will suffer great oscillation, and it propagates as a breather (Figs. 4(c) and (d)). For other values of μ, the single-hump soliton pairs have the similar behavior as in the case of $\mu = 6$.

Next, we study the existence and stability of dipole soliton pairs. Figure 5 shows the total normalized power P and the corresponding instability growth rate of dipole soliton pairs for several values of V_0 and W_0. It is shown that this kind of dipole soliton pairs can also exist when the propagation constant is below a certain critical value μ_0, and the total power decreases with the increase in propagation constant. Numerical calculations of Eq. (3) show that the stable range of dipole soliton pairs is $\mu_c < \mu < \mu_0$ where μ_c is critical propagation constant for dipole soliton stability. When $V_0 = 10$ and $W_0 = 1$, the stable range is $5.25 < \mu < 5.486$; when $V_0 = 10$ and $W_0 = 1$, $2.43 < \mu < 5.25$. If V_0 is chosen to be 5, these dipole soliton pairs are stable at $1.578 < \mu < 1.898$ when $W_0 = 0.5$ and $0 < \mu < 1.8$ when $W_0 = 2$. For a given V_0, dipole soliton pairs with higher W_0 have wider stable range.

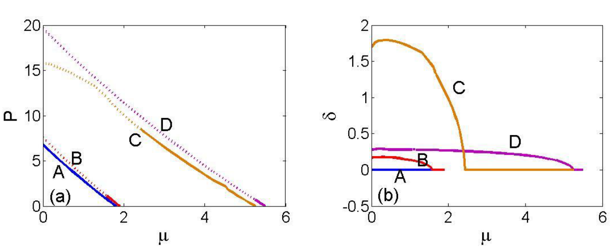

Fig. 5. (a) Total normalized power and (b) instability growth rate of dipole soliton pairs. In (a) solid curve represents stable region while dotted curve represents unstable region. The values of V_0 and W_0 are chosen to be: (A) $V_0 = 5$, $W_0 = 2$; (B) $V_0 = 5$, $W_0 = 0.5$; (C) $V_0 = 10$, $W_0 = 4$; (D) $V_0 = 10$, $W_0 = 4$.

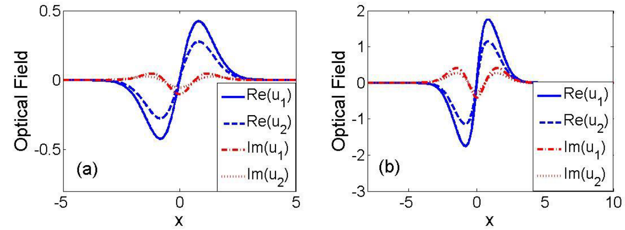

Fig. 6. Optical field profiles of dipole soliton pairs at (a) $\mu = 5.3$ and (b) $\mu = 2$ when $V_0 = 10$ and $W_0 = 1$.

To validate the stability analysis, we give two examples of dipole soliton pairs at $\mu = 5.3$ and $\mu = 2$ when $V_0 = 10$ and $W_0 = 1$. Their optical field profiles are plotted in Figs. 6(a) and (b), respectively. It can be seen that the coupled components u_1 and u_2 have similar optical field profiles. The real parts of the optical fields have odd symmetry and the imaginary parts have even symmetry, which is different from the single-hump soliton pairs. The relative ratio of imaginary parts to real parts decreases with the increase in propagation constant μ. Figures 7(a) and (b) show the evolution of the dipole soliton pair at $\mu = 5.3$ when $V_0 = 10$ and $W_0 = 1$. Both these components can keep their intensity profiles and propagate stably in the optical lattice. If one of them is absent, the other one can still propagate stably during the propagation with the intensity profiles slightly changed (Figs. 7(c) and (d)). If we change the value of μ to be 2, however, the soliton pair (whose field profiles are shown in Fig. 6(a)) will suffer from strong modulation instability and they are unstable even if they propagate together in the optical lattice (Figs. 8(a) and (b)). After a short propagation distance, both the optical beams begin to suffer great oscillation because of the exponentially grown random noises during their propagation. If one of the components is absent, the other one still fails to keep its intensity profile (Figs. 8(c) and (d)).

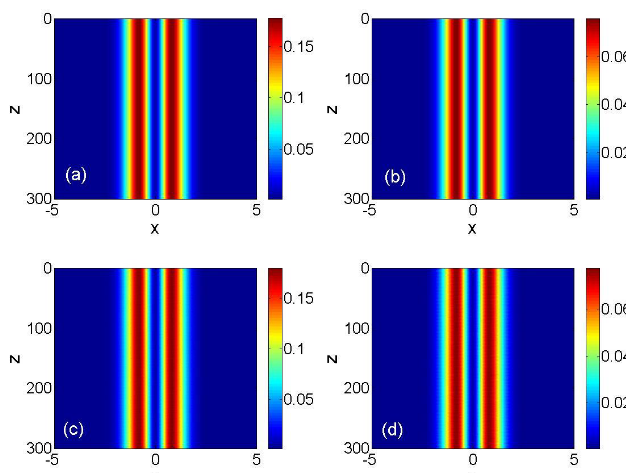

Fig. 7. Stable propagation of the optical components (a) u_1 and (b) u_2 of a dipole soliton pair at $\mu = 5.3$. (c) Evolution of u_1 when u_2 is absent and (d) evolution of u_2 when u_1 is absent.

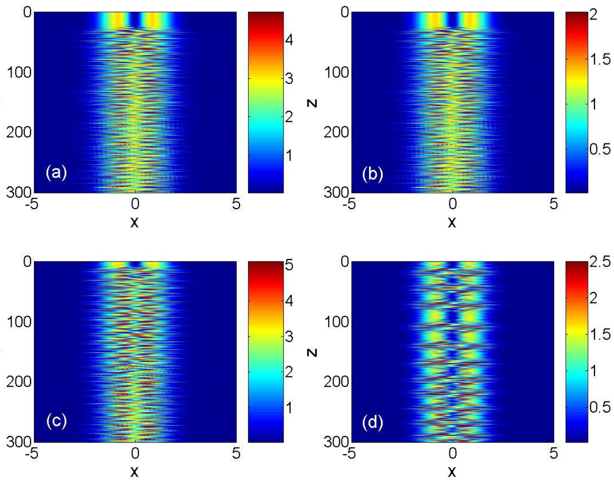

Fig. 8. Unstable propagation of (a) u_1 and (b) u_2 of a dipole soliton pair at $\mu = 2$. (c) Unstable evolution of u_1 when u_2 is absent and (d) unstable evolution of u_2 when u_1 is absent.

In conclusion, we investigate the existence and stability of incoherently coupled single-hump and dipole soliton pairs in the self-defocusing Kerr media with PT-symmetric Bessel potentials. The soliton pairs exist provided that they are composed of two optical beams having the same polarization and wavelength. It is found that both kinds of soliton pairs can exist when the propagation constant is below a certain critical value. The total power of their components decreases as the propagation constant increases. All single-hump soliton pairs can propagate stably in the optical lattice, whereas dipole soliton pairs can be stable only in low-power region. If one of the components is absent, the propagation behavior of the other one is also studied.

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