# Theoretical studies on optimum design of acousto－optic modulator using lithium niobate crystal 

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#### Abstract

We theoretically study the optimum design of anisotropic acousto－optic modulator（AOM）based on lithium niobate crystal．Four different kinds of operating modes in the XOZ and $Y O Z$ acousto－optic（AO）planes are systematically analyzed by the tangent condition theory and the optimized operating mode is determined． Furthermore，the dependence of the AO merit on the operating frequency and the off－axis angle of the AOM are also obtained by numerical simulations．


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Acousto－optic modulator（AOM）is used to generate modulated diffracted laser beam by the refractive Bragg＇s diffraction of the acousto－optic（AO）crystal， which is widely applied in Q －switched laser ${ }^{[1-3]}$ ，mode－ locked fiber laser ${ }^{[4]}$ ，optical tweezers ${ }^{[5]}$ ，microscopy ${ }^{[6]}$ ， acoustic velocity measurement ${ }^{[7]}$ ，and sensors ${ }^{[8]}$ ．The bandwidth and diffractive efficiency are the two most important design parameters of the AOM，which are determined by the operating mode of the AOM．The AOM is commonly designed by the tangent momen－ tum match condition ${ }^{[9,10]}$ ，where the vector of the ultra－ sonic wave is designed tangent to the refractive index ellipsoid of the AO crystals to realize the broadband design of the AOM．However，in the previous study the diffractive efficiency properties of lithium niobate （LN）AOM under the momentum match condition have not been systematically carried out，which restricts the improvement of the properties of AOM based on the LN．In this letter，we systematically study the optimum design of AOM based on LN crystal，where the dif－ fractive efficiency properties under four different kinds of AO interaction modes in the operating plane of the $X O Z$ and $Y O Z$ planes are analyzed and the optimized operating mode is determined．The detailed features of the dependence of the AO merit on the operating frequency of the AOM are also obtained by numerical simulation．
The schematic diagram of LN anisotropic AO interac－ tion under the tangent momentum match condition is shown in Fig． 1.
Note that the circular curves in Fig． 1 are the refrac－ tive index ellipsoids of the ordinary light and the ellipse curves are the refractive index ellipsoids of the extraor－ dinary light，respectively．Here $\vec{k}_{\mathrm{i}}$ ，$\vec{k}_{\mathrm{d}}$ ，and $\vec{K}$ are the wave vectors of the incident light，the diffraction light， and the acoustic wave，$\theta_{\mathrm{a}}$ is named as the off－axis angle， which is between the vector of the ultrasonic wave and
the $X$－axis；$\theta_{\mathrm{i}}$ is the incident angle which is between the vector of incident light and the $Z$－axis，and $\theta_{\mathrm{d}}$ is the diffractive angle which is between the vector of the diffractive light and the $Z$－axis．In order to realize the wideband design of the AOM，the $o \rightarrow e$ operating mode is selected for both $\pm$ order diffractions，thus the vector of the incident light is connected to the circu－ lar index curve（ordinary light）and the vector of the diffractive light is connected to the ellipse index curve （extraordinary light；Fig．1）．Furthermore，the vec－ tor of the ultrasonic wave is configured as tangent to the index ellipsoid of the extraordinary light for both the $\pm$ order diffractions，which contributes to realize the wideband design of the AOM．
Based on Figs．1（a）and（b），the geometrical relation－ ships of LN crystal under the tangent condition can be derived as

$$
\begin{gather*}
\theta_{\mathrm{i}}^{(1)}=\theta_{\mathrm{a}}-\cos ^{-1}\left[\frac{n_{\mathrm{d}}}{n_{\mathrm{o}}} \cdot \cos \left(\theta_{\mathrm{a}}-\theta_{\mathrm{d}}\right)\right],  \tag{1}\\
\theta_{\mathrm{i}}^{(2)}=2 \theta_{\mathrm{a}}-\theta_{\mathrm{i}}^{(1)},  \tag{2}\\
\theta_{\mathrm{d}}=\tan ^{-1}\left(\frac{n_{\mathrm{e}}^{2}}{n_{\mathrm{o}}^{2}} \cdot \tan \theta_{\mathrm{a}}\right),  \tag{3}\\
f_{\mathrm{m}}^{(i)}=\frac{v}{\lambda} \sqrt{n_{\mathrm{o}}^{2}+n_{\mathrm{d}}^{2}-2 n_{\mathrm{o}} n_{\mathrm{d}} \cos \left(\theta_{\mathrm{d}}-\theta_{\mathrm{i}}^{(i)}\right)}, \tag{4}
\end{gather*}
$$

where $f_{\mathrm{m}}$ is the extreme frequency of the AOM，which is also defined as the central frequency of the AOM．The corner mark $i$ means the order of the diffraction，where $i=1$ means the +1 order diffraction and $i=-1$ means the -1 order diffraction．

The acoustic velocity in the $X O Z$ and $Y O Z$ planes of LN crystal can be calculated by the Christoffel equation

$$
\begin{equation*}
\left(\Gamma_{i j}-\rho v^{2} \delta_{i j}\right) \cdot u_{j}=0, \tag{5}
\end{equation*}
$$



Fig. 1. Schematic diagram of the anisotropic AO interaction: (a) +1 order diffraction and (b) -1 order diffraction.
where $\Gamma_{i j}=L_{i J} c_{J I} L_{I j}, \rho$ is the density of the crystal and $v$ is the velocity of the ultrasonic. The ultrasonic velocity in the $X O Z$ and $Y O Z$ planes can be obtained by solving the eigenvalue of Eq. (5), the reciprocal velocity curves are shown in Figs. 2(a) and (b).

Curve (1) is named as the longitudinal wave and it can result in the isotropic AO interactions; curves (2) and (3) are named as the fast sheared ultrasonic waves and slow sheared ultrasonic waves, which can lead to the anisotropic $A O$ effect. In this letter, the four different sheared ultrasonic waves in the $X O Z$ and $Y O Z$ planes are discussed for the optimum design of the AOM, respectively.

The diffraction efficiency of the AOM can be written as

$$
\begin{equation*}
\eta \approx \frac{\pi^{2} M_{2} L}{2 \lambda^{2} H} P_{a} \tag{6}
\end{equation*}
$$

where the parameter $M_{2}$ is the AO merit of the AOM, $H$ and $L$ are the height and length of the AO crystal, $P_{\mathrm{a}}$ is the power of the ultrasonic. From Eq. (6) it is easy to conclude the linearity relationship between the diffraction efficiency and the AO merit $M_{2}$, therefore the AOM designed with higher $M_{2}$ is required to improve the diffraction efficiency of the modulator. The expression of $M_{2}$ can be written as

$$
\begin{equation*}
M_{2}=\frac{n_{\mathrm{i}}^{3} n_{\mathrm{d}}^{3} p_{\mathrm{eff}}^{2}}{\rho v^{3}} \tag{7}
\end{equation*}
$$

where $P_{\text {eff }}$ is the AO coefficient of the AO crystal.


Fig. 2. Reciprocal velocity curves of LN crystal in (a) $X O Z$ and (b) YOZ planes.


Fig. 3. Curves of $M_{2}$ versus $\theta_{\mathrm{a}}$ under different operating conditions. Operating mode with the (a) fast sheared ultrasonic wave of XOZ plane, (b) slow sheared ultrasonic wave of $X O Z$ plane, and (c) fast sheared ultrasonic wave of YOZ plane.

Based on Eqs. (1)-(7), the relationship between the parameters $M_{2}$ and $\theta_{\text {a }}$ under different operating modes can be calculated and the curves of $M_{2}\left(\theta_{\mathrm{a}}\right) \sim \theta_{\mathrm{a}}$ are shown in Figs. 3(a)-(c).

Different from the results illustrated in Figs. 3(a)-(c), the calculation results show that $M_{2}$ equals zero in the range of $\theta_{\mathrm{a}} \in(0, \pi / 2)$ under the operating mode of the fast sheared ultrasonic wave in the $Y O Z$ plane, which means that there is no AO effect under this operating mode. Thus, the optimum design of the AOM can be determined from the operating modes illustrated in Figs. 3(a)-(c).

Considering that the operating frequency of the AOM is usually designed at the order of $10^{8} \mathrm{~Hz}$, which means that the off-axis angle $\theta_{\mathrm{a}}$ is set far less than $10^{\circ}$. Based on this deduction, it can be found that the operating mode with the fast sheared ultrasonic wave in the $X O Z$ plane has the highest value of $M_{2}$, which can be determined as the optimum operating mode to obtain the higher performance of the AOM.


Fig. 4. Curves of $M_{2}$ versus $f_{\mathrm{m}}$ under different operating conditions: (a) curves of AO merit versus extreme frequency for slow ultrasonic wave in $X O Z$ plane, (b) curves of AO merit versus extreme frequency for fast ultrasonic wave in XOZ plane, and (c) curves of AO merit versus extreme frequency for fast ultrasonic wave in $Y O Z$ plane.

For the convenience to design AOM with the parameter of central frequency, the relationships between $M_{2}$ and $f_{\mathrm{m}}$ have also been derived by numerical simulation, and the curves of $M_{2} \sim f_{\mathrm{m}}$ are shown in Figs. 4(a)-(c).

From Figs. 4(a)-(c), it can also be observed that the operating mode under the fast sheared ultrasonic
wave in the $X O Z$ plane achieves the highest value of $M_{2}$ under the condition of $f_{\mathrm{m}}<500 \mathrm{MHz}$. Furthermore, it can be observed that the operating mode with the -1 order diffraction achieves higher $M_{2}$ than that of the +1 order at a certain extreme frequency (Fig. 4(b)). Therefore, the optimum operating mode of the AOM can be determined as the operating mode of the -1 order diffraction with the fast sheared ultrasonic wave in the $X O Z$ plane.

In conclusion, we carry out the optimum design of the LN AOM by the tangent momentum match condition. The properties of the AO merit are systematically calculated and the dependence of $M_{2}$ on the operating frequency and on the off-axis angle are both figured out by the numerical simulation and the optimum operating mode of the AOM is determined. This research work is useful for the improvement on the performance of the AOM and shows potential prospect in the applications of AO device.

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