# Testing of a large rectangular mirror based on sub-aperture stitching method

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In order to solve the difficulty of testing large mirror, the sub-aperture stitching interferometry (SSI) is proposed and expatiated. The basic theory and principle of this method are introduced and analyzed. A reasonable stitching algorithm and mathematical model are established based on least-squares fitting, triangulation algorithm, homogeneous coordinate transformation, etc., and the relative program and flow chart are established. Some marked points are used to accomplish the alignments between sub-apertures and calibrate the relationship between the coordinate of the mirror and the pixel. With engineering examples, a large rectangular mirror with an irregular aperture of  $720 \times 165$  (mm) is tested by SSI. The peak-to-valley and root mean square of the stitched surface error are 0.451  $\lambda$  and 0.042  $\lambda$  ( $\lambda$  is 632.8 nm), respectively.

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Fabrication of large mirror includes some processing stages such as milling, grinding, and polishing. For milling and grinding processes, we can measure the surface map of the mirror by contact measurements such as coordinate measuring machine and profilometry, and the distribution of optical surface can be obtained by the data analysis and calculation, so as to provide the basis for the further processing<sup>[1–3]</sup>. For mirror polishing and final stage of the surface error measurement, the contact measurement will no longer apply, because its measuring accuracy normally is only 0.2  $\mu$ m root mean square (RMS), what is more, the contact measurement in this phase will bring some scratches on the surface.

Due to the interferometry with high resolution, high precision, high sensitivity, and good repeatability, the technology has become the most commonly used method for testing surface map during the polishing and final stages<sup>[4,5]</sup>. The Ritchey–Common interferometry can be used for testing large mirror, but the data processing of this method is complex. In addition, this method requires large auxiliary spherical mirror, the aperture of auxiliary spherical mirror is usually 1.2 to 1.3 times of the tested mirror. The high precision and large spherical mirror is very difficult to fabricate. Therefore, large mirror Ritchey–Common testing is difficult to achieve<sup>[6,7]</sup>.

SSI can test large mirror without any assistant optics. It broadens the testing scope of the traditional interferometer, and the figure error of the whole aperture can be obtained by measuring and stitching several sub-apertures<sup>[8–10]</sup>.

For the purpose of testing large optical system without large reference mirror, Kim introduced the basic idea of the sub-aperture stitching technology in 1982<sup>[11]</sup>, the large optical system can be measured by a series of smaller reference mirrors. In 1986, Stuhlinger proposed the discrete phase algorithm<sup>[12]</sup> and the least-squares fitting method to calculate the misalignment errors was developed by Otsubo in 1992<sup>[13,14]</sup>. The automatic stitching interferometer was fabricated by Queues Enforth Development in 2003<sup>[15–17]</sup>, it can test large departure convex asphere by variable optical null technology<sup>[18]</sup>. Recently, the optical center of Arizona University has successfully stitched a large mirror with the aperture of 1.6 m based on the maximum-likelihood algorithm, the reference error was considered and corrected according to the method<sup>[19,20]</sup>.

Here we introduce a reasonable and precise stitching method for measuring large mirror based on triangulation algorithm, least-squares method, and homogeneous coordinate transformation<sup>[21–24]</sup>. The figure error of the large mirror can be obtained by a synthetical optimization stitching method without large reference mirror or other auxiliary optics.

When the large mirror is divided into several subapertures, the slope of each sub-aperture is decreased. SSI can test the phase distribution of each small area (sub-aperture) of the large mirror successively, and then the surface error of the large mirror can be obtained through data analysis and stitching algorithm.

The process of the stitching method is shown in Fig. 1, and the specific steps are as follows:

#### 1) Determine sub-apertures

Firstly, the number and size of the sub-aperture are calculated according to the aperture of the interferometer and the parameters of the tested mirror. In order to solve the stitching coefficients, the sub-apertures should have some overlaps. In order to ensure the precision, the overlapping area is generally greater than a quarter of the sub-aperture<sup>[13]</sup>.



Fig. 1. Flow chart of SSI.

### 2) Locate benchmark targets

To determine the sub-aperture mutual position relation and realize the full aperture coordinates transformation between the pixel coordinates and the tested mirror, some benchmark targets will be posted in the overlapping areas.

## 3) Sub-aperture testing

By adjusting the tested mirror and interferometer, let the interferometer aim at each sub-aperture. Then we can acquire the phase data of each sub-aperture by interferometry.

## 4) Delaunay triangulation interpolation<sup>[24]</sup>

After testing each sub-aperture, the coordinate of individual sub-aperture can be transformed to the physical coordinate of the large mirror by the benchmark targets. Then the stitching coefficients can be calculated by phase data of the overlapping areas, but the overlapping data of the adjacent sub-apertures is not consistent. Assuming that phase data of all subapertures have been transformed to the corresponding global 3D cooordinates, the problem of overlapping correspondence can be solved as follows:

Firstly, we can define a uniform grid on the X-Y plane. The phase data of the *i*th and *j*th sub-aperture can be projected onto the X-Y plane (Fig. 2(a)). The phase data of the *i*th sub-aperture (random sub-aperture) can be calculated by Delaunay triangulation (Fig. 2(b)), and we can characterize the result of triangulation by



Fig. 2. Calculation in the overlapping area. Sketch map of the (a) projection and (b) Delaunay triangulation.

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -z_1 \\ -z_2 \\ -z_3 \end{pmatrix},$$
(1)

where  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  are the coordinates of the three points which compose a fundamental triangle, and a, b, and c are the coefficients of

$$z = -ax - by - c. \tag{2}$$

It can be expressed by the plane equation of the triangle, and the phase data of each sub-aperture will be interpolated by Eq. (2). Because the overlapping data must be consistent<sup>[25]</sup>, the stitching factors of each subaperture will be analyzed and calculated in the next step.

#### 5) Synthetical optimization stitching

The phase data of individual sub-aperture can be tested by interferometer and will be interpolated by the abovementioned Delaunay triangulation method, and the coordinates of the benchmark targets can be easily obtained. We can transform the coordinate of each sub-aperture to the globe coordinate by homogeneous coordinate transformation. The random adjacent subapertures can be stitched together by eliminating the relative misalignment errors, but when stitching many sub-apertures one by one, it often induces error transfer and accumulation.

In this letter, we propose a global optimization stitching method and all the stitching coefficients are calculated simultaneously. Assume there are N sub-apertures to cover the full aperture. In order to test and locate the sub-aperture precisely, the central sub-aperture was chosen as fiducial sub-aperture. When testing large aspheric surfaces, the relative piston, tilt, power, astigmatism, coma, and primary spherical errors were induced by misalignment of the adjacent sub-apertures and can be described as

$$w_{i}' = w_{i} + p_{i} + a_{i}x_{i} + b_{i}y_{i} + c_{i}(x_{i}^{2} + y_{i}^{2}) + d_{i}x_{i}y_{i} + e_{i}(x_{i}^{2} - y_{i}^{2}) + f_{i}x_{i}(x_{i}^{2} + y_{i}^{2}) + g_{i}y_{i}(x_{i}^{2} + y_{i}^{2}) + h_{i}(x_{i}^{2} + y_{i}^{2})^{2},$$
(3)

where  $w_i$  is the measured phase data of the *i*th subaperture which is tested by interferometry,  $w'_i$  is the real-phase data of the *i*th sub-aperture,  $p_i$ ,  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$ ,



Fig. 3. Setup of SSI.

 $e_i, f_i, g_i$ , and  $h_i$  are the coefficients of the relative piston, X tilt, Y tilt, power, astigmatism  $(d_i, e_i)$ , coma  $(f_i, g_i)$  and primary spherical errors, respectively.

When testing large sphere, the misalignment errors of adjacent sub-apertures only contain relative piston, tilt, and power, Eq. (3) can be modified as

$$w'_{i} = w_{i} + p_{i} + a_{i}x_{i} + b_{i}y_{i} + c_{i}(x_{i}^{2} + y_{i}^{2}).$$
(4)

If stitching large flat mirror, the misalignment of adjacent sub-apertures will have combinations of different amounts of piston and tilt only, Eq. (3) can be simply rewritten as

$$w_i' = w_i + p_i + a_i x_i + b_i y_i \tag{5}$$

The best stitching coefficients can be calculates by the least-squares fitting. Thus, the sum of the squared differences of all the overlapping data should be minimized as



Fig. 5. Corresponding phase maps of two sub-apertures.

interferometer with the aperture of 600 mm is fixed on a vibration isolator directly. The tested mirror is mounted on an air flotation tilting table with two axes stage, which can adjust the tip and tilt of the mirror precisely, and the air flotation tilting table can be slid on the vibration isolator optionally. The sub-aperture is about 80% of the whole mirror, and there are only two sub-apertures to cover the irregular rectangle mirror (Fig. 4).

By aligning the large mirror, the interferometer can aim at the two sub-apertures successively. The testing beam and reference beam can form an interferogram, and the phase maps of the two sub-apertures are shown in Fig. 5.

The coordinates of the two sub-apertures can be united into the same reference by homogeneous coordinate transformation, and the misalignment errors will be subtracted from individual sub-aperture by least-squares fitting. Then the stitched figure error of

$$\min = \sum_{i=1}^{N} \sum_{\substack{j=1\\j\neq i}}^{N} \{ [(w_i + p_i + a_i x_i + b_i y_i + c_i (x_i^2 + y_i^2) + d_i x_i y_i + e_i (x_i^2 - y_i^2) + f_i x_i (x_i^2 + y_i^2) + g_i y_i (x_i^2 + y_i^2) + h_i (x_i^2 + y_i^2)^2] - \frac{1}{2} [w_j + p_j + a_j x_j + b_j y_j + c_j (x_j^2 + y_j^2) + d_j x_j y_j + e_j (x_j^2 - y_j^2) + f_j x_j (x_j^2 + y_j^2) + g_j y_j (x_j^2 + y_j^2) + h_j (x_j^2 + y_j^2)^2] \}^2.$$

(6)

Therefore, the figure error of each sub-aperture can be corrected and the full stitched surface map can be obtained.

A well-polished large mirror with a rectangular aperture of  $720 \times 165 \text{ (mm)}$  is measured by the proposed SSI. The experimental setup is shown in Fig. 3. The Zygo



In conclusion, in order to test large mirror without any auxiliary optics, we propose a sub-aperture stitching method based on Delaunay triangulation interpolation, homogeneous coordinate transformation, and least-squares fitting. With engineering examples, a large mirror with an irregular aperture of  $720 \times 165$  (mm) is



Fig. 4. Sub-apertures distribution of large mirror.



Fig. 6. Figure error of the large mirror testing by SSI.

tested by SSI, the distribution of the stitched surface map is smooth and continuous. By using targets as the benchmark, the coordinate transformation and alignment of sub-apertures can be calculated directly and precisely, and the adjusting mechanism request of the method is not high, thus is a simple and efficient testing method for large surface.

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