Performance analysis of LDPC code based on Watermark scheme in high-speed optical communication system

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We study the decoding performance of low-density parity-check (LDPC) code based on Watermark scheme under different percentages of water-mark bits over 100 Gb/s differential quadrature phase shift keying (DQPSK) high-speed optical communication system. We also find the optimum percentage of water-mark bits according to the performance comparison. Simulation result shows an improvement of 0.5 dB net code gain by using the optimum Watermark scheme at a post-forward error correction bit error rate (BER) of 10^{-9}, comparing with the traditional log-likelihood ratios belief propagation decoding algorithm. Also, for the same BER, there is a decrease in the number of iterations used in LDPC decoding.

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In order to meet the rapid development of channel capacity of optical communication[1,2], multilevel modulation and multiplexing technology have been widely studied[3–5]. With the increasing complexity of optical communication system, crosstalk between codewords becomes apparent[6]. Therefore, advanced forward error correction (FEC) codes are becoming more and more important for high-speed optical communication system[7,8]. Among various kinds of FEC codes, low-density parity-check (LDPC) code proposed by Gallager in 1962[9] was proved to have outstanding error correction performance[10,11]. In recent years, several schemes of LDPC have been proposed to get better error correction performance[12–14].

However, the algorithms proposed in the above literature and the traditional LDPC algorithm do not consider the interactions among adjacency codewords by the channel knowledge. Obviously, neighbor codes received have certain relativity due to the increasing complexity of the optical transmission channel. We propose a new modified LDPC scheme which can extract the channel knowledge by involving priori characteristic bits in the original LDPC[15]. In this letter, we study the problem of how the density of water-mark bits affects the performance of Watermark scheme. We elaborate on the Watermark scheme and deduce the corresponding formula. We study the decoding performance of four typical proportions of water-mark bits, and then we compare the performance of the optimum scheme with the traditional one.

Log-likelihood ratios belief propagation algorithm (LLR-BPA) is the most commonly used iterative decoding algorithm of LDPC and Fig. 1 shows its iterative process. From Fig. 1, we know that in the process of LLR-BPA, initial information is an important factor which can affect the decoding performance of the algorithm. The LLR vector of initial information is denoted by $L(Q_i)$, the initial value of $L(Q_i)$ is denoted by $L(P_i)$, and $L(P_i) = \ln(P_i(0)/P_i(1))$. We calculated the $L(Q_i)$ vector according to the characteristics of communication

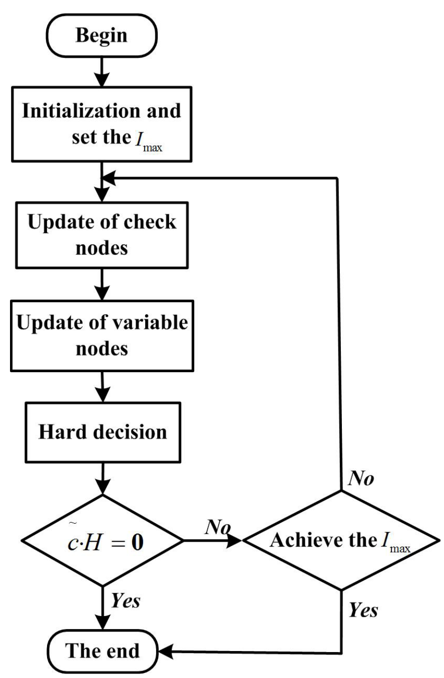

Fig. 1. Process of BP iteration decoding.

channel. The amplified spontaneous emission noise is assumed to be the dominant transmission noise in optical channel, and it can be simulated by the additive white Gaussian noise (AWGN) channel model.

Under the AWGN channel model, the priori conditional probability density formula is

$$p(y|x_i = x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(y-x)^2}{2\sigma^2}\right). \quad (1)$$

According to the mathematical relationship between prior and posterior probabilities, assuming the probabilities of both $l = 0$ and $l = 1$ are 1/2. The posterior probability can be expressed as

$$P_r(x_i = x|y) = \frac{1}{(1+\exp(-2xy/\sigma^2))}. \quad (2)$$

When $l = 1$, $x_i = +1$, or $l = 0$, $x_i = -1$ LLR vector can be stated as

$$L(Q_i) = -2y/\sigma^2. \quad (3)$$

In an optical communication system, the code rate R and signal-to-noise ratio (SNR $= E_b/N_0$) is fixed, then the standard deviation σ is a constant.

$$\sigma = \frac{1}{\sqrt{2\cdot R\cdot \mathrm{SNR}}}. \quad (4)$$

In order to meet the increasing complexity of the optical communication channel, Watermark scheme is designed to have the capability to extract channel information. So, we can expect the performance of this scheme to improve greatly. In this scheme, we insert some fixed value bits (water-mark bits), such as "0" into the original LDPC codewords uniformly, so that we can obtain new codewords. Then, the new codewords are transmitted through the communication channel and we receive them at the end of the receiver. Due to the effect of noise in channel, some error codes appear in the received code. The process of inserting water-mark bits and extracting them is as shown in Fig. 2.

According to the relativity among the neighbor codewords, if the corresponding water-mark bit received is correct, it is reasonable to guess that the performance of the optical channel is well at least in a small neighborhood around the water-mark bit, and the probability

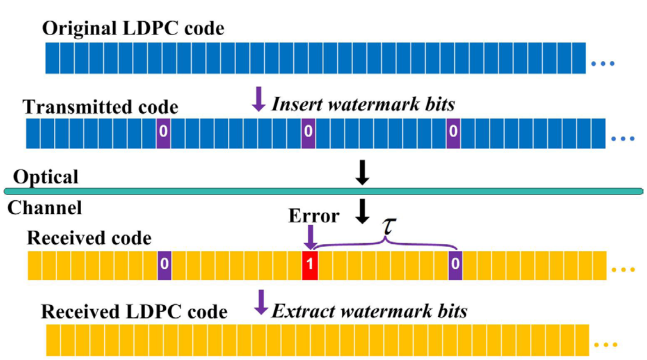

Fig. 2. Process of watermark bits insertion and extraction.

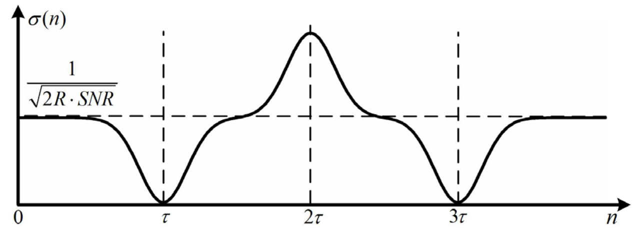

Fig. 3. Curve of $\sigma(n)$.

of correction of the adjacent codewords is bigger. In contrast, if the water-mark bit received is wrong, it is reasonable to guess that the performance of the channel is worse around the Water-mark bit, and the probability of correction of the adjacent codewords is smaller. These assumptions can be described as

$$\sigma(n) = \sigma\cdot\left(\sum_{i=1}^{l/\tau} x(i)e^{-a(n-\tau i)^2} + 1\right),\ n = (1, 2, \ldots, l), \quad (5)$$

where $x(i)$ is defined as

$$x(i) = \begin{cases} -1, & \text{the water-mark bit is correct} \\ +1, & \text{the water-mark bit is wrong} \end{cases}, \quad (6)$$

and n is the index number of bits in the received codewords, τ is the interval between each water-mark bit, l is the length of original codewords, and the integer division of (l/τ) is the number of water-mark bits. Figure 3 shows the curve of the new σ. By the definition of $\sigma(n)$, we know that the influence involved by water-mark bits changes with interval τ. When τ is too big, the change in σ will be too small and its influence on initial information may not be enough. However, when τ is too small, σ will be too sensitive to the large number of water-mark bits. Meanwhile, parameter a is also an important factor in this scheme. It determines the influence region of each water-mark bit to $\sigma(n)$. If a is too big, one water-mark bit can affect a few codewords. Otherwise, if a is too small, overlapping effect will be produced. Hence, with larger interval τ, the modified factor a should be smaller to get the optimum performance and when τ is smaller, the value of a should be set bigger. In the following the effects of parameters τ and a are also considered.

Figure 4 shows the generation of bit sequence, LDPC encoding and decoding, the differential quadrature phase shift keying optical modulation, and direct detection.

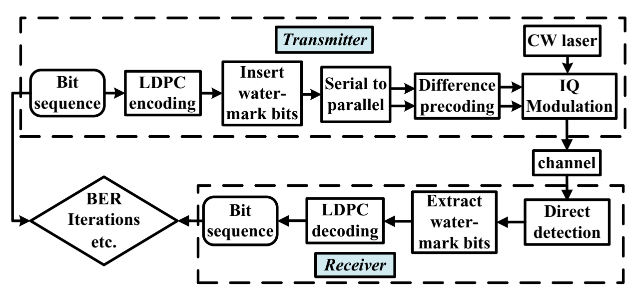

Fig. 4. Simulation platform of optical communication system.

Table 1. List of Parameters for Different Percentages of Water-mark Bits

Percentage of Water-mark Bits	**0.78**	**1.56**	**3.12**	**6.25**
Interval τ (bit)	128	64	32	16
Number of Water-mark bits	8	16	32	64
Optimum Value of a	0.006	0.01	0.04	0.08

We notice that the water-mark bits are inserted after LDPC encoding and extracted before LDPC decoding. LDPC (3, 6, 1024) code was selected as the FEC code in this back-to-back 100 Gb/s optical communication system.

In order to demonstrate the optimum percentage of water-mark bits of the Watermark scheme, we choose several typical proportions as representatives and simulation of their decoding performance are done to find the optimum percentage of the water-mark bits. The simulation system is shown in the upper part, the length of LDPC code is 1024, the code rate R is 1/2, and the maximum number of iteration, $I_{\max}$ is set to 20. Considering that the interval τ between water-mark bits cannot be too large or too small (as shown earlier), we take four proportions 0.78%, 1.56%, 3.12%, and 6.25% as representatives. Table 1 shows the corresponding parameter a for each percentage. For the parameter a selected in Table 1, different proportions can obtain better performance, respectively.

Bit error rate (BER) performance of Watermark scheme for four proportions of water-mark bits and the traditional one is shown in Fig. 5. One can easily notice that at the same optical SNR (OSNR), Watermark scheme for the four percentages shows decrease in BER performance compared with the traditional LLR-BPA. The scheme with 0.78% water-mark bits gains only a little improvement, the improvement gain by scheme with 6.25% water-mark bits is nearly the same as the scheme with 1.56% water-mark bits, but it is bigger than the situation of 0.78%. The Watermark scheme with 3.12% water-mark bits gains the optimum BER performance in

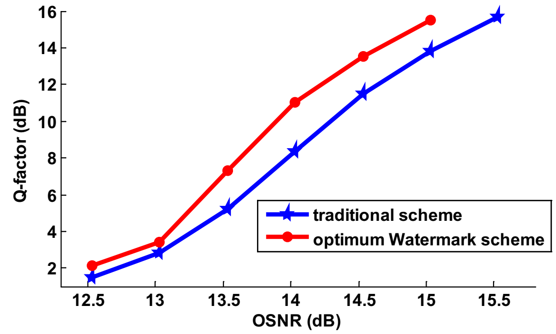

Fig. 6. Q-factor of traditional algorithm versus optimum Watermark scheme.

the four groups of different percentages. So we can conclude that 3.12% is the optimum percentage of water-mark bits, and we can call the Watermark scheme with the optimum percentage of water-mark bits as the optimum Watermark scheme. We can know that when BER performance drops to 10^{-9}, compared with the traditional LLR-BPA, the optimum Watermark scheme shows a net code gain (NCG) improvement of 0.5 dB.

Figure 6 shows Q-factor performance of the optimum Watermark scheme and the traditional scheme and Fig. 7 shows the average iteration number of the two schemes when $I_{\max}$ is set to 20. We can notice that the optimum Watermark scheme shows an improvement of 2.4 dB Q-factor at the OSNR of 14 dB, which is the biggest improvement of Q-factor performance in Fig. 6. For the same BER performance, Fig. 7 shows a decrease in the average iteration number of the optimum Watermark scheme. The number of iteration decreases greatly at the OSNR of 14 dB also. The trend analyses of Figs. 6 and 7 show that the Watermark scheme can obtain better performance compared with the traditional LLR-BP decoding algorithm for lower OSNR. It is the same with the theory of Watermark scheme. We believe that the new scheme can get bigger improvement when the condition of communication channel is worse.

In conclusion, we study the modified scheme based on LDPC which is called Watermark scheme over 100 Gb/s high optical communication system. We find that the optimum percentage of water-mark bits is 3.12%. Comparing

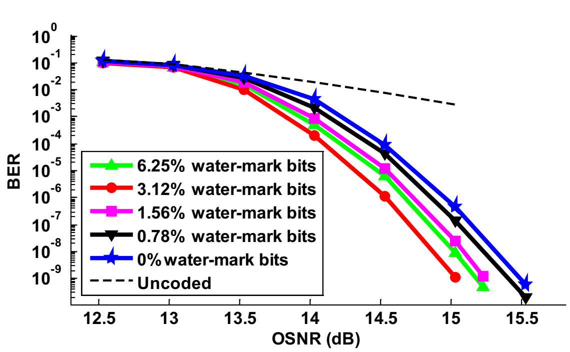

Fig. 5. BER performance of different proportions for Watermark scheme and traditional scheme.

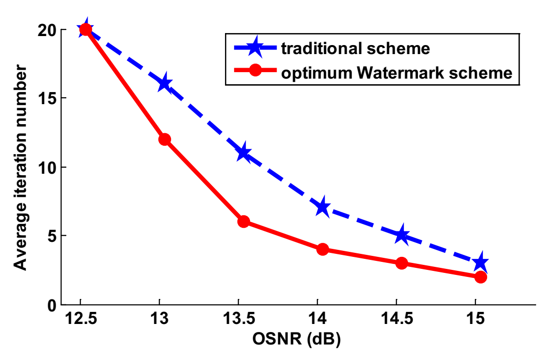

Fig. 7. Average iteration number of traditional algorithm versus optimum Watermark scheme.

with the traditional one, the optimum Watermark scheme shows an improvement of 0.5 dB NCG, the best Q-factor improvement is 2.4 dB. It also shows a decrease in average iteration number. The Watermark scheme can get a better decoding performance when the condition of communication channel is worse.

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References

1. K. Sean, F. Tobias, B. Polina, and K. Robert I, Opt. Express **20,** 4198 (2012).
2. E. Desurvire, J. Lightw. Technol. **24,** 4697 (2006).
3. J. Renaudier, O. Bertran-Pardo, A. Ghazisaeidi, P. Tran, H. Mardoyan, P. Brindel, A. Voicila, G. Charlet, and S. Bigo, in *Proceedings of Optical Fiber Communication Conference* OTu3B.1 (2013).
4. D. Qian, J. Yu, J. Hu, P. Ji, and T. Wang, in *Proceedings of European Conference and Exhibition on Optical Communications* Mo.3.E.4 (2008).
5. F. Li, J. Yu, Y. Fang, Z. Dong, X. Li, and L. Chen, Opt. Express **22,** 8742 (2014).
6. Z. Dong, X. Li, J. Yu, and N. Chi, J. Lightw. Technol. **30,** 3687 (2012).
7. M. Arabaci and I. B. Djordjevic, in *Proceedings of Optical Fiber Communication Conference* OWF2 (2011).
8. F. Chang, K. Onohara, and T. Mizuochi, IEEE Commun. Mag. **48,** S48 (2010).
9. R. G. Gallager, IEEE Inform. Theory **8,** 21 (1962).
10. T. J. Richardson and R. Urbanke, *Modern Coding Theory* (Cambridge University Press, 2008).
11. K. Sugihara, Y. Miyata, T. Sugihara, K. Kubo, H. Yoshida, W. Matsumoto, and T. Mizuochi, in *Proceedings of Optical Fiber Communication Conference* OM2B.4 (2013).
12. E. O. Torshizi, H. Sharifi, and M. Seyrafi, in *Proceedings of International Conference on Electrical and Electronics Engineering* (2013).
13. D. Chang, F. Yu, Z. Xiao, N. Stojanovic, F. N. Hauske, Y. Cai, C. Xie, L. Li, X. Xu, and Q. Xiong, in *Proceedings of Optical Fiber Communication Conference*, OW1H.4 (2012).
14. T. M. N. Ngatched, F. Takawira, and M. Bossert, IEEE Trans. Commun. **57,** 302 (2009).
15. W. Bi, W. Zhang, W. He, and C. Li, in *Proceedings of International Conference on Advanced Infocomm Technology* (2013).