Two-atom distributed entanglement by detecting the transmission spectrum of a coupled-cavity quantum electrodynamics system

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A scheme is presented to generate atomic entanglement by detecting the transmission spectrum of a coupled-cavity system. In the scheme, two 3-level atoms are trapped in separate cavities coupled by a short optical fiber, and the atomic entanglement could be realized in a heralded way by detecting the transmission spectrum of the coupled-cavity system.

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A cavity quantum electrodynamics (QED) system is one of the ideal candidates for studying quantum information[1,2] and optical physics[3–7]. The strong coupling between the atoms and cavity-field modes can be used to coherently control the light–matter interactions. Many theoretical schemes and experimental demonstrations based on cavity QED have been presented in both the optical and microwave domains. In particular, Serafini *et al.* suggested distributed quantum computation with separate optical cavities coupled by an optical fiber[8]. This coupled-cavity system has the advantage of easily addressing individual sites with external control and the ability of scalability. As a result, it is relatively easy to read out information and to realize a scalable quantum network. Following this seminal work by Serafini *et al.*[8], a variety of schemes were proposed for quantum information processing, including quantum entanglement[9–17], quantum phase gate[18–26], and quantum communication[27–31] through dynamics evolution, an adiabatic process, or the dissipative state technique.

In this Letter, we present an alternative scheme to generate atomic entanglement of a coupled-cavity system by detecting a cavity transmission spectrum. In our scheme, two 3-level atoms are trapped in separate cavities connected by a short optical fiber[8]. The cavity modes interact with the atomic transition with a large detuning, then the interactions lead to state-dependent shifts of the cavity transmission spectrum[32]. When one detects the spectrum of the coupled system, the atoms would collapse to a certain entangled state. Compared with previous protocols for generation of atomic entanglement of the coupled-cavity system[9–17], our scheme could work in a heralded way by detecting the cavity transmission spectrum.

We consider two separate cavities connected by an optical fiber, as shown in Fig. 1(a). The coupling between the cavity modes and the fiber mode is modeled by the Hamiltonian[8]

$$H_1 = J(a_1^\dagger b + b^\dagger a_1 + a_2^\dagger b + b^\dagger a_2), \tag{1}$$

where J is the tunnel constant[8], b is the annihilation operator for the fiber mode, and a_i $(i = 1, 2)$ denotes the annihilation operator for the ith cavity mode. Assume that each cavity traps a single atom and the atomic transition $|g_i\rangle \to |e_i\rangle$ of the ith atom is coupled to cavity mode a_i [Fig. 1(b)]. The Hamiltonian for this process can be written as

$$H_2 = \sum_{i=1}^{2}(\Delta|e_i\rangle\langle e_i| + g a_i|e_i\rangle\langle g_i| + g a_i^\dagger|g_i\rangle\langle e_i|), \tag{2}$$

where Δ is the detuning of the atomic transition from the cavity mode, and g is the coupling rate. In the case of large detuning, i.e., $\Delta \gg g$, the high level $|e_i\rangle$ can be adiabatically eliminated, leading to a state-dependent shift of the cavity frequency[33,34]

$$H_2' = \sum_{i=1}^{2}\frac{g^2}{\Delta}a_i^\dagger a_i|g_i\rangle\langle g_i|. \tag{3}$$

Hence, the effective Hamiltonian of the coupled system is $H = H_1 + H_2'$.

According to the Langevin equation of cavity modes[35], $\dot{\Xi}(t) = -\frac{i}{\hbar}[\Xi(t), H] - \frac{\kappa}{2}\Xi(t) + \sqrt{\kappa}\Xi_{\text{in}}(t)$ where $\Xi = a_1,\ a_2,$

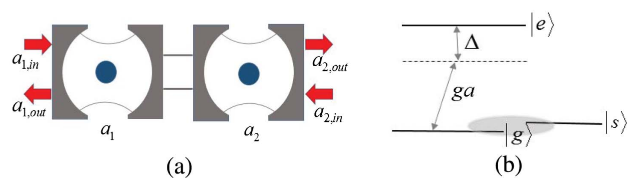

Fig. 1. (a) Generation of atomic entanglement in separated cavities coupled by a short optical fiber. Two-atom entanglement can be realized in a heralded way by detecting the transmission spectrum; (b) concrete atomic level structure and relevant transition.

one can obtain the equations of motion for the internal fields

$$\dot{a}_1 = -\frac{ig^2}{\Delta} a_1 |g_1\rangle\langle g_1| - iJb - \frac{\kappa}{2} a_1 + \sqrt{k} a_{1,\mathrm{in}}, \quad (4)$$

$$\dot{a}_2 = -\frac{ig^2}{\Delta} a_2 |g_2\rangle\langle g_2| - iJb - \frac{\kappa}{2} a_2 + \sqrt{k} a_{2,\mathrm{in}}, \quad (5)$$

$$\dot{b} = -iJ(a_1 + a_2), \quad (6)$$

where κ is the cavity decay rate, and $a_{1(2),\mathrm{in}}$ is the input field. Define the Fourier components of the fields by $\Lambda = \frac{1}{\sqrt{2\pi}} \int \mathrm{d}\omega \Lambda(\omega) e^{-i(\omega-\omega_a)t}$, where $\Lambda = a_1,\ a_1,\ b,\ a_{1,\mathrm{in}}$, $a_{2,\mathrm{in}}$, and ω_a is the resonant frequency of the bare cavity mode[35]. The equations of motion become

$$i(\omega - \omega_a) a_1(\omega) - \frac{ig^2}{\Delta} a_1(\omega) |g_1\rangle\langle g_1| - iJb(\omega) - \frac{\kappa}{2} a_1(\omega) + \sqrt{\kappa} a_{1,\mathrm{in}}(\omega) = 0, \quad (7)$$

$$i(\omega - \omega_a) a_2(\omega) - \frac{ig^2}{\Delta} a_2(\omega) |g_2\rangle\langle g_2| - iJb(\omega) - \frac{\kappa}{2} a_2(\omega) + \sqrt{\kappa} a_{2,\mathrm{in}}(\omega) = 0, \quad (8)$$

$$i(\omega - \omega_a) b(\omega) = iJ[a_1(\omega) + a_2(\omega)] = 0. \quad (9)$$

With the initial input fields $a_{1,\mathrm{in}} \neq 0$ and $a_{2,\mathrm{in}} = 0$, we find

$$a_2 = \frac{J^2 \sqrt{\kappa} a_{1,\mathrm{in}}(\omega)}{J^2(M_1 + M_2) + i(\omega - \omega_a) M_1 M_2}, \quad (10)$$

where $M_i = i[\omega - \omega_a - |g_i\rangle\langle g_i| g^2/\Delta] - \kappa/2$. By making use of input-output formulation $a_{2,\mathrm{out}} + a_{2,\mathrm{in}} = \sqrt{\kappa} a_2$[35], we can obtain the transmission spectrum of coupled system given by

$$T = \left| \frac{a_{2,\mathrm{out}}}{a_{1,\mathrm{in}}} \right|^2 = \left| \frac{J^2 \kappa}{J^2(M_1 + M_2) + i(\omega - \omega_a) M_1 M_2} \right|^2. \quad (11)$$

Now, we show how to realize entanglement of the couple-cavity system by detecting the transmission spectrum. We assume that each atom has an auxiliary state $|s\rangle$ and both atoms are initially prepared in the superposition states, i.e., $|\Psi_0\rangle = 1/2(|g_1\rangle + |s_1\rangle) \otimes (|g_2\rangle + |s_2\rangle)$. If a weak classical field inputs from the left side of the coupled system [Fig. 1(a)], one can obtain the transmission spectrum (Fig. 2). From Fig. 2 we can see that the transmission spectrum has three spectral lines. The spectral line A corresponds to the state $|\psi\rangle_A = |s_1, s_2\rangle$ [here $|m_1, n_2\rangle$ ($m, n = g, s$) denotes that the atom in the first cavity stays in state $|m\rangle$ and the atom in the second cavity stays in state $|n\rangle$], the spectral line B corresponds to the state $|\psi\rangle_B = 1/\sqrt{2}(|g_1, s_2\rangle + |s_1, g_2\rangle)$, and the spectral line C corresponds to the state $|\psi\rangle_C = |g_1, g_2\rangle$. If one obtains the spectral line B in the transmission spectrum, the entire system collapses to the entangled state

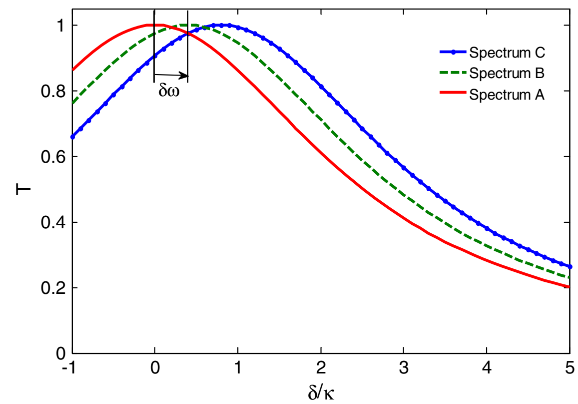

Fig. 2. Transmission spectrum T as a function of the scanning frequency $\delta = \omega - \omega_a$. Spectral line A corresponds to the state $|\psi\rangle_A = |s_1, s_2\rangle$, spectral line B corresponds to the state $|\psi\rangle_B = 1/\sqrt{2}(|g_1, s_2\rangle + |s_1, g_2\rangle)$, spectral line C corresponds to the state $|\psi\rangle_C = |g_1, g_2\rangle$, and $\delta\omega$ is the minimal shift of three spectral lines. Parameters are $\kappa = \gamma$, $g = 4\kappa$[36], $J = 100$ g[8], and $\Delta = 15$ g.

$$|\psi\rangle_B = \frac{1}{\sqrt{2}}(|g_1, s_2\rangle + |s_1, g_2\rangle). \quad (12)$$

Next, we discuss the physical understanding of the aforementioned result. In our scheme, the interactions of the cavity mode and atoms only lead to state-dependent shifts of the resonance frequency of the system. Based on the transmission spectrum T in Eq. (11), the state-dependent shift of the spectrum for two atoms in different cavities are equivalent and indistinguishable. Thus, if we obtains the spectral line B, we know only one of the atoms is in state $|g\rangle$, but we do not know to which cavity this atom locates. As a result, the atoms collapse to the entangled state $|\psi\rangle_B$. If one obtains the spectral line A or C, it denotes that the number of atoms staying in state $|g\rangle$ is 0 or 2, and the atoms collapse to state $|\psi\rangle_A$ or $|\psi\rangle_C$. Thus, the atomic entanglement can be realized in a heralded way by detecting the transmission spectrum.

Now we briefly give some discussion of our scheme. First, in our scheme the atoms collapse to the entangled state by detecting the cavity transmission spectrum. Compared with previous distributed-entanglement protocols[9–17] in the coupled-cavity system through dynamics evolution, an adiabatic process, or the dissipative state technique, our scheme is probabilistic, but in a heralded way (once the spectral line B is detected, one knows that the atoms collapse to the entangled state). Second, our scheme should prepare the special initial atomic state $|\psi_0\rangle$ in advance, compared with previous protocols through the dissipative state technique[11]. Fortunately, coupled-cavity systems have the advantage of easily addressing individual cavities; one could prepare the initial state $|\psi_0\rangle$ by driving two atoms with the external laser fields independently. After the preparation of the special initial state $|\psi_0\rangle$, the atomic entanglement can be realized

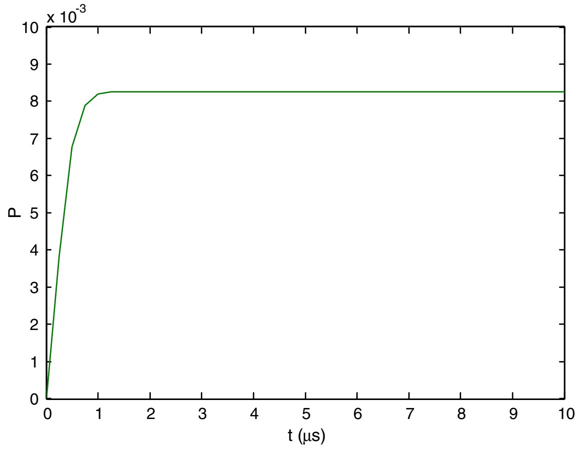

Fig. 3. Probability P of the atomic qubit loss as a function of the time t. Parameters are $(g,\kappa,\gamma)/(2\pi) = (16, 4.2, 2.6)$ MHz[36], $\Omega/2\pi = 4$ MHz, $J = 100$ g[8], and $\Delta = 15$ g.

in heralded way, by detecting the transmission spectrum. Third, to obtain the transmission spectrum T in Eq. (11), we have adiabatically eliminated the high level $|e\rangle$ and ignored the influence of atomic spontaneous emission. However, due to atomic spontaneous emission γ from state $|e\rangle$ and the atomic transition between atomic states $|g\rangle$ and $|e\rangle$, the final state would be outside of the qubit Hilbert space $\{|g\rangle, |s\rangle\}$. Figure 3 shows the probability P of the atomic qubit loss as a function of the time t, using the subsequent master equation for the density operator ρ of the intracavity system[35]

$$\partial_t\rho = -i[H_1 + H_2 + H_3, \rho] + \sum_{i=1}^{2}[\kappa D(a_i)\rho + \gamma D(\sigma_i)\rho], \tag{13}$$

where $D(\Theta)\rho = 2\Theta\rho\Theta^\dagger - \Theta^\dagger\Theta\rho - \rho\Theta^\dagger\Theta$ $[\Theta = a_1, a_1, \sigma_1, \sigma_2$ $(\sigma_i = |g_i\rangle\langle e_i|)]$, and $H_3 = \Omega(a_i + a_i^\dagger)$ is the Hamiltonian for the cavity mode a_1 driven by the probe light[32] with amplitude Ω. From Fig. 3, we see that when the large detuning condition, i.e., $\Delta \gg g$, is satisfied, the probability P would be smaller than 1%.

Finally, we address the experimental feasibility of the present scheme. The atomic configuration can be chosen from the hyperfine states of the cesium atom[37]. For instance, the atomic states $|g\rangle$ and $|s\rangle$ could correspond to the hyperfine levels $|F = 4, m = -1\rangle$ and $|F = 3, m = -1\rangle$ of $6S_{1/2}$, respectively, whereas the state $|e\rangle$ corresponds to $|F = 4, m = 0\rangle$ of $6P_{3/2}$. Then we employ the cavity mode to drive the atomic transition $|g\rangle \leftrightarrow |e\rangle$ with strong coupling strength and large detuning. In the experiment[36], typically the cavity decay rate is $\kappa/(2\pi) = 4.2$ MHz and the coupling rate is $g/(2\pi) = 16$ MHz. Setting the atom-cavity detuning $\Delta/(2\pi) = 320$ MHz, satisfies the large detuning condition $\Delta \gg g$. Remarkably, Zhang *et al.*[32] demonstrated the capability to measure the micro-shift of the spectral lines with mesoscopic ensembles containing 100 or more atoms; thus our scheme here would be possible to realize in the future.

In conclusion, we propose a scheme for generation of atomic entanglement in a coupled-cavity system by detecting the transmission spectrum. In our scheme, two atoms are trapped in separated cavities connected by a short optical fiber. The interactions between the atoms and cavity modes lead to state-dependent shifts of the transmission spectrum of the coupled-cavity system. As the shifts are only decided by the number of atoms staying in state $|g\rangle$, detection of the cavity transmission spectrum of the coupled system could lead the atomic collapse to a certain entangled state in a heralded way.

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