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Radiative force on atoms from the view of photon emission

Zhuo Song (宋 卓), Yonggang Peng (彭勇刚), and Yujun Zheng (郑雨军)*

School of Physics, Shandong University, Jinan 250100, China

**Corresponding author: yzheng@sdu.edu.cn*

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In this Letter, we present a possible methodology to directly "read" the force on an atom via the photons emitted from the atom. In this methodology, the mean radiative force on an atom exerted by external fields can be expressed as a function of the average number of emitted photons $\langle N \rangle$ and its derivatives via the generating function approach developed by us recently.

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The force originating from the momentum of light is the foundation of optically manipulating neutral particles[1–3]. Along with the magneto-optical trap (MOT) and cavity, laser cooling has also become an important tool in controlling the dynamics and exploring the new physics of atoms[4–8]. Early studies of Doppler cooling, which has become the most common method of laser cooling, were proposed in Refs. [9,10]. The first observation of radiation-pressure cooling was reported by Wineland *et al.*[11]. Ashkin, who developed optical tweezers, reported the first observation of a single-beam gradient force pressure particle trap in Ref. [12]. Lett *et al.* employed optical molasses and obtained ultracold atomic vapor[13]. Cohen-Tannoudji proposed the theory of Sisyphus cooling[14]. Recently, Sagi *et al.* demonstrated anomalous diffusion behavior using the Sisyphus cooling method on ^{87}Rb atoms in a one-dimensional optical lattice[15]. The anomalous diffusion of atoms can help us study the complicated forces acting on atoms[16].

In this Letter, we study the radiative forces exerted by two types of laser waves on the ^{87}Rb atom. Based on the generating function methodology of photon counting statistics developed recently, the force on the atom in external fields can be expressed by the average number of emitted photons $\langle N \rangle$ and its time derivatives. This means that we can obtain the force via the photon statistical quantities that are closely related to those in experiments. The results provide us with a new perspective to study the mean radiative force on the atom.

We consider the force exerted by an external laser field on a ^{87}Rb atom composed of a ground state $|g\rangle = |5^2S_{1/2}, F=2\rangle$ and an excited state $|e\rangle = |5^2P_{3/2}, F'=3\rangle$[17]. The transition frequency and transition dipole moment are ω_{eg} and $\boldsymbol{\mu}$, respectively. The external field is described by $\boldsymbol{E}_L(\boldsymbol{r},t) = \hat{\boldsymbol{\epsilon}}\mathcal{E}_0(\boldsymbol{r})\cos[\omega_L t + \Phi(\boldsymbol{r})]$, where $\hat{\boldsymbol{\epsilon}}$ is the polarization unit vector, ω_L is the angular frequency, and $\mathcal{E}_0(\boldsymbol{r})$ and $\Phi(\boldsymbol{r})$ are the amplitude and phase at position $\boldsymbol{r}$, respectively.

The evolution of this system can be described by its reduced density matrix $\sigma(t) \equiv \mathrm{Tr}_{\mathrm{R}}\{\rho(t)\}$, where $\rho(t)$ is the density operator[18]. $\sigma(t)$ satisfies

$$\dot{\sigma}_{ij}(t) = \mathcal{L}_{ij;kl}\sigma_{kl}, \tag{1}$$

in Liouville space, where $\mathcal{L}_{ij;kl}$ is the Liouville superoperator[19].

Equation (1) can be solved via various methods, e.g., in terms of the iterative expansion of $\mathcal{L}_{ij;kl}$. However, we prefer the generating function method developed recently, which can help us obtain the information with respect to the photon statistics of the system. We define the generating function as $\mathcal{G}_{ij}(s,t) \equiv \sum_{n=0}^{\infty}\sigma_{ij}^{(n)}s^n$, where $\sigma_{ij}^{(n)}$ corresponds to the emission of n photons in the time interval $[0,t]$, s is an auxiliary counting variable, and $i,j = e,g$[20,21].

We further introduce the generalized Bloch vectors: $\mathcal{U} \equiv \frac{1}{2}(\mathcal{G}_{ge}e^{-i\omega_L t} + \mathcal{G}_{eg}e^{i\omega_L t})$, $\mathcal{V} \equiv \frac{1}{2i}(\mathcal{G}_{ge}e^{-i\omega_L t} - \mathcal{G}_{eg}e^{i\omega_L t})$, $\mathcal{W} \equiv \frac{1}{2}(\mathcal{G}_{ee} - \mathcal{G}_{gg})$ and $\mathcal{Y} \equiv \frac{1}{2}(\mathcal{G}_{ee} + \mathcal{G}_{gg})$. In the interaction picture, by involving the rotating wave approximation (RWA), $\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$, and $\mathcal{Y}$ satisfy the generalized optical Bloch equations[20,21]:

$$\begin{aligned} \dot{\mathcal{U}} &= -\frac{\Gamma}{2}\mathcal{U} + \delta_L\mathcal{V}, \\ \dot{\mathcal{V}} &= -\delta_L\mathcal{U} - \frac{\Gamma}{2}\mathcal{V} - \Omega(\boldsymbol{r})\mathcal{W}, \\ \dot{\mathcal{W}} &= \Omega(\boldsymbol{r})\mathcal{V} - \frac{\Gamma}{2}(1+s)\mathcal{W} - \frac{\Gamma}{2}(1+s)\mathcal{Y}, \\ \dot{\mathcal{Y}} &= -\frac{\Gamma}{2}(1-s)\mathcal{W} - \frac{\Gamma}{2}(1-s)\mathcal{Y}, \end{aligned} \tag{2}$$

where $\Omega(\boldsymbol{r}) = -\boldsymbol{\mu}\cdot\hat{\boldsymbol{\epsilon}}\mathcal{E}_0(\boldsymbol{r})/\hbar$ is the Rabi frequency, $\delta_L = \omega_L - \omega_{eg} + \frac{\partial\Phi(\boldsymbol{r})}{\partial t}$ is the detuning frequency, and Γ is the spontaneous emission rate from state $|e\rangle$ to state $|g\rangle$.

When $\mathcal{Y} = 1/2$ and $s = 1$, Eqs. (2) reduce to the ordinary Bloch equations[14,18,22,23]. Based on Refs. [14,22] and Eqs. (2), the mean force exerted by the laser field on the atom can, after some algebra, be written as

$$\mathcal{F}(\boldsymbol{r},t) = -\hbar\Omega(\boldsymbol{r})[\mathcal{U}(\boldsymbol{r},t)\boldsymbol{\alpha}(\boldsymbol{r}) + \mathcal{V}(\boldsymbol{r},t)\boldsymbol{\beta}(\boldsymbol{r})], \tag{3}$$

where

$$\boldsymbol{\alpha}(\boldsymbol{r}) \equiv \frac{\nabla\Omega(\boldsymbol{r})}{\Omega(\boldsymbol{r})}, \qquad \boldsymbol{\beta}(\boldsymbol{r}) \equiv \nabla\Phi(\boldsymbol{r}). \tag{4}$$

We usually separate the total force $\mathcal{F}(\boldsymbol{r}, t)$ into two parts: the *reactive force* $\mathcal{F}_{\text{react}}(\boldsymbol{r}, t)$ and the *dissipative force* $\mathcal{F}_{\text{dissip}}(\boldsymbol{r}, t)$, defined as

$$\begin{aligned} \mathcal{F}_{\text{react}}(\boldsymbol{r}, t) &\equiv -\hbar\Omega(\boldsymbol{r})\mathcal{U}(\boldsymbol{r}, t)\boldsymbol{\alpha}(\boldsymbol{r}), \\ \mathcal{F}_{\text{dissip}}(\boldsymbol{r}, t) &\equiv -\hbar\Omega(\boldsymbol{r})\mathcal{V}(\boldsymbol{r}, t)\boldsymbol{\beta}(\boldsymbol{r}), \end{aligned} \tag{5}$$

respectively.

We first consider a laser plane wave propagating along the negative direction of the x axis with wave vector $\boldsymbol{k}_L = -k_L\hat{\boldsymbol{i}}$ and angular frequency ω_L, where k_L is the wave number and $\hat{\boldsymbol{i}}$ is the unit vector of the x axis. Without a loss of generality, we assume that the atom also moves along the x axis, and the position of the atom $\boldsymbol{r}$ can be replaced by $\boldsymbol{x} = x\hat{\boldsymbol{i}}$. The velocity of the atom is $\boldsymbol{v}_0 = \frac{dx}{dt}\hat{\boldsymbol{i}}$.

In this case, the Rabi frequency $\Omega(x) = \Omega$ is a constant, and the phase is $\Phi(x) = -\boldsymbol{k}_L \cdot x\hat{\boldsymbol{i}} = k_L x$. From Eq. (4), we have

$$\boldsymbol{\alpha}(x) = 0, \qquad \boldsymbol{\beta}(x) = k_L, \tag{6}$$

and the detuning frequency is $\delta_L = \delta_L^{(0)} + \frac{\partial\Phi}{\partial t} = \delta_L^{(0)} + k_L v_0$, where $\delta_L^{(0)} = \omega_L - \omega_{eg}$.

Since $\boldsymbol{\alpha}(x) = 0$, only the dissipative force in Eq. (5) is preserved. We can solve the generalized Bloch vector $\mathcal{V}$ from Eq. (2) as

$$\mathcal{V}(v_0, t) = \frac{1}{\Omega}\left(I + \frac{1}{\Gamma}\frac{\partial I}{\partial t}\right), \tag{7}$$

where $I = 2\frac{d}{dt}(\frac{\partial}{\partial s}\mathcal{Y})\big|_{s=1}$ is the photon emission intensity[21]. The mean force exerted by the laser plane wave on the atom is

$$\mathcal{F}(v_0, t) = -\hbar k_L\left(I + \frac{1}{\Gamma}\frac{\partial I}{\partial t}\right). \tag{8}$$

In the long time limit, $\frac{\partial I}{\partial t} = 0$, the force in Eq. (8) reduces to the time independent form[14],

$$\mathcal{F}(v_0) = -\hbar k_L I. \tag{9}$$

In Fig. 1 we plot the mean force $\mathcal{F}$ as a function of Γt with different δ_L. For $\delta_L \neq 0$, $\mathcal{F}$ exhibits a damping oscillation and finally reaches a constant value. The larger $|\delta_L|$ is, the more violently $\mathcal{F}$ oscillates, and the faster it approaches a constant value. For $\delta_L = 0$, the absolute value of $\mathcal{F}$ monotonically increases to a maximum without oscillation (note that the symbol of $\mathcal{F}$ only indicates its direction). If we change the symbol of δ_L while keeping other parameters unchanged, the dependence of $\mathcal{F}$ on Γt does not change, as shown by the overlapping blue solid line and green dotted line.

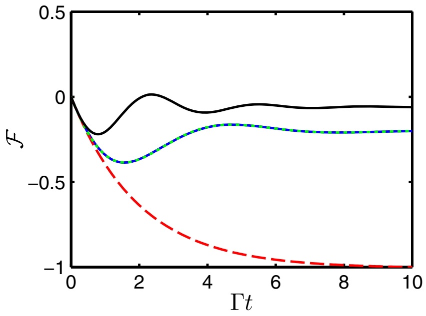

Fig. 1. Mean force $\mathcal{F}$ as a function of Γt in a laser plane wave. Parameters are $\Gamma = 2\pi \times 6$ MHz, $\Omega = -2\pi \times 0.6$ MHz, $k_L = 2\pi \times 1.28 \times 10^6$ m^{-1}, $\delta_L = -\Gamma$ (blue solid line), $\delta_L = 0$ (red dashed line), $\delta_L = \Gamma$ (green dotted line), $\delta_L = 2\Gamma$ (black solid line). Note that the blue solid line and green dotted line overlap each other. $\mathcal{F}$ is scaled by its maximum absolute value.

In Fig. 2 we plot the emission intensity I and mean force $\mathcal{F}$ as functions of v_0 and $\delta_L^{(0)}$ in the long time limit. When $-k_L v_0 = \delta_L^{(0)}$, I and the absolute value of $\mathcal{F}$ reach their peaks. Note that in the long time limit the shapes of I and $\mathcal{F}$ are identical, as indicated by Eq. (9). Since the emission intensity I can be measured in experiments, this result may provide us with a way to “read” the force on the atom directly.

We next consider the force exerted by a laser standing wave on a ^{87}Rb atom fixed on the x axis. The laser standing wave along the x axis and linearly polarized along the z axis can be written as

$$\boldsymbol{E}_L(x, t) = \boldsymbol{\epsilon}_z\mathcal{E}_0(x)\cos(\omega_L t), \tag{10}$$

where $\boldsymbol{\epsilon}_z$ is the unit vector of the z axis, $\mathcal{E}_0(x) = 2\mathcal{E}_0 \cos(k_L x)$, x is the ordinate of the atom, and k_L is the wave number.

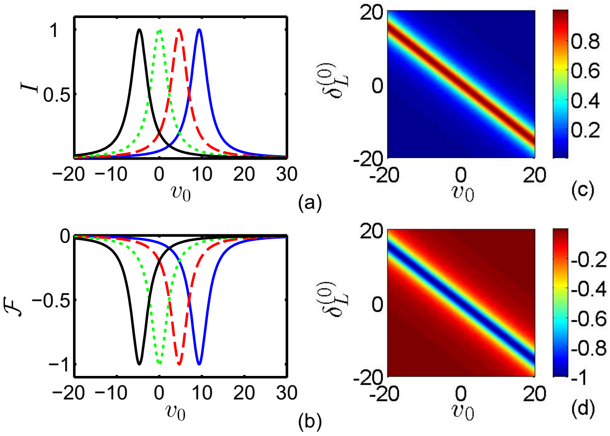

Fig. 2. Emission intensity I [(a),(c)] and mean force $\mathcal{F}$ [(b),(d)] as functions of v_0 and $\delta_L^{(0)}$ in the long time limit. For (a) and (b), $\delta_L^{(0)} = -2\Gamma$ (blue solid line), $\delta_L^{(0)} = -\Gamma$ (red dashed line), $\delta_L^{(0)} = 0$ (green dotted line), and $\delta_L^{(0)} = \Gamma$ (black solid line). The other parameters are the same as in Fig. 1. $\mathcal{F}$ and I are normalized.

In this case, the Rabi frequency is position dependent:

$$\Omega(x) = -\frac{\boldsymbol{\mu}\,\cdot\,\boldsymbol{\epsilon}_z\mathcal{E}_0(x)}{\hbar} = 2\Omega_0\,\cos(k_L x), \tag{11}$$

where $\Omega_0 = -\boldsymbol{\mu}\,\cdot\,\boldsymbol{\epsilon}_z\mathcal{E}_0/\hbar$. The phase $\Phi(x)$ is a constant, yielding $\delta_L = \delta_L^{(0)} + \frac{\partial\Phi(x)}{\partial t} = \delta_L^{(0)}$, where $\delta_L^{(0)} = \omega_L - \omega_{eg}$.

From Eq. (4) we obtain

$$\alpha(x) = -k_L\,\tan(k_L x), \qquad \beta(x) = 0. \tag{12}$$

In this case, only the reactive force in Eq. (5) is preserved. The generalized Bloch vector $\mathcal{U}$ can be obtained from Eq. (2) as

$$\begin{aligned}\mathcal{U}(x,t) = {} & \frac{1}{2\Omega(x)\delta_L}\frac{\partial I}{\partial t} + \frac{4\delta_L^2 + 3\Gamma^2}{4\delta_L\Omega(x)\Gamma}I \\ & + \frac{4\delta_L^2 + \Gamma^2 + 2\Omega(x)^2}{4\delta_L\Omega(x)}\langle N\rangle - \frac{\Omega(x)\Gamma}{4\delta_L}t,\end{aligned} \tag{13}$$

where $\langle N\rangle = 2\frac{\partial}{\partial s}\mathcal{Y}|_{s=1}$ is the average number of the photons emitted by the system in time interval $[0, t]$[20,21]. The force exerted on the atom can be written as

$$\begin{aligned}\mathcal{F}(x,t) &= -\hbar\Omega(x)\alpha(x)\mathcal{U}(x,t) \\ &= \hbar k_L\,\tan(k_L x)\left\{\frac{1}{2\delta_L}\frac{\partial I}{\partial t} + \frac{4\delta_L^2 + 3\Gamma^2}{4\delta_L\Gamma}I\right. \\ &\quad \left. + \frac{4\delta_L^2 + \Gamma^2 + 2\Omega(x)^2}{4\delta_L}\langle N\rangle - \frac{\Omega(x)^2\Gamma}{4\delta_L}t\right\}.\end{aligned} \tag{14}$$

In the long time limit, $\mathcal{F}(x,t)$ reduces to

$$\mathcal{F}(x) = \hbar k_L\,\tan(k_L x)\frac{2\delta_L}{\Gamma}I. \tag{15}$$

In Fig. 3 we plot the mean force $\mathcal{F}(x,t)$, average emitted photon number $\langle N\rangle$, emission intensity I, and the first derivative of I with respect to time, $\dot{I}$, as functions of Γt with different δ_L. For small t, there is fluctuation in $\mathcal{F}$. As t increases, the fluctuation in $\mathcal{F}$ attenuates and finally disappears. When t is large enough, $\mathcal{F}$ reaches a steady value. For large t, $\langle N\rangle$ is approximately proportional to t, with a gradient inversely proportional to $|\delta_L|$. Similarly to $\mathcal{F}$, I and $\dot{I}$ fluctuate when t is small and finally reach constant values when t is large enough.

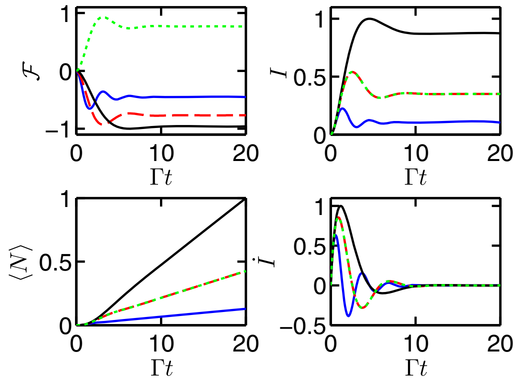

Fig. 3. Mean force $\mathcal{F}$, average emitted photon number $\langle N\rangle$, emission intensity I, and the first-order time derivative of emission intensity, $\dot{I}$, as functions of Γt in a laser standing wave. Parameters are $\delta_L = -2\Gamma$ (blue solid line), $\delta_L = -\Gamma$ (red dashed line), $\delta_L = -\Gamma/2$ (black solid line), and $\delta_L = \Gamma$ (green dotted line). The red dashed lines and green dotted lines overlap in the subfigures of I, $\langle N\rangle$, and $\dot{I}$. The other parameters are the same as in Fig. 1. $\mathcal{F}$, $\langle N\rangle$, I, and $\dot{I}$ are normalized.

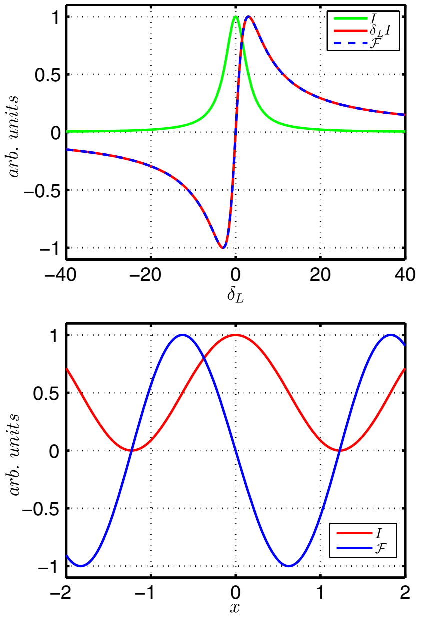

Fig. 4. Mean force $\mathcal{F}$ and emission intensity I as functions of the detuning frequency δ_L (upper panel) and position x (lower panel) in the long time limit. The parameters are $\Gamma = 2\pi \times 6$ MHz, $\Omega_0 = -2\pi \times 0.6$ MHz, $k_L = 2\pi \times 1.28 \times 10^6\ \mathrm{m}^{-1}$, $x = 1$ (for the upper panel), and $\delta_L = -2\pi$ MHz (for the lower panel). $\mathcal{F}$ is scaled by its maximum absolute value and I is normalized.

In Fig. 4 we plot $\mathcal{F}$ and I as functions of δ_L (upper panel) and x (lower panel) in the long time limit. From the upper panel we can see, as functions of δ_L, $\mathcal{F}$ and $\delta_L I$ have the same shape. This indicates that we can "read" the mean force $\mathcal{F}$ via I experimentally. According to the lower panel, both $\mathcal{F}$ and I are periodic, and the period of I is twice as much as that of $\mathcal{F}$. Hence, we can calculate the force after some algebraic operation of I.

In conclusion, we present a way to "read" the mean radiative force $\mathcal{F}$ exerted on a ^{87}Rb atom in a plane wave field and in a standing wave field. By employing the generating function approach, the mean force $\mathcal{F}$ can be expressed by the average emitted photon number $\langle N\rangle$ and (or) its time derivatives. Since $\langle N\rangle$ and its time derivatives can be measured in experiments, this may serve as a way to "read" the mean force exerted by the laser fields on the atom directly.

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