Group velocity dispersion effect on the collapse of femtosecond laser pulses in air: a revised form for Marburger's law

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The influence of group velocity dispersion (GVD) on the self-focusing of femtosecond laser pulses is investigated by numerically solving the extended nonlinear Schrödinger equation. By introducing the GVD length L_{GVD} into the semi-empirical, self-focusing formula proposed by Marburger, a revised one is proposed, which can not only well explain the influence of GVD on the collapse distance, but also is in good agreement with the numerical results, making the self-focusing formula applicable for more cases.

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The propagation of femtosecond laser pulses in air has been a hot area of research for nearly two decades since its first experimental observation[1]. Numerous theoretical and experimental studies have explored the nonlinear phenomena during this process, such as high-order harmonic generation[2], terahertz radiation[3,4], and supercontinua[5,6], etc. Investigating these effects can be helpful to applications in remote sensing[7,8] and lightning protection[9,10], etc.

As one of the most fundamental and important phenomena in the propagation process, self-focusing has been the subject of intensive investigation in the past 50 years[11–15]. It has been shown that quite a few effects, such as plasma formation, self-steepening, and group velocity dispersion (GVD), etc., affect the propagation of laser pulses. For the GVD effect, its influence on laser propagation has been well studied in previous works, most of which mainly focus on the processes after the nonlinear focus of the laser pulses[16,17]. In practice, after the nonlinear focus, there are quite a few factors that arrest the pulse collapse, such as energy absorption resulting from multiphoton ionization, plasma formation, and even plasma explosion, including a Coulomb explosion, which can be very complicated. In our work, attention is paid to the factors that affect the collapse distance, i.e., the process from the onset of propagation to the first occurrence of pulse collapse.

In our recent work, we investigated the influence of GVD on the self-focusing of femtosecond laser pulses in air at different pressures by freezing the ratio of input pulse power to the critical one of self-focusing[18]. When GVD is considered, along with an increase in pressure and a decrease in pulse duration, the collapse distance L_c will become larger. In this case, the semi-empirical Marburger law[13,14] cannot describe the collapse distance in the case of strong GVD anymore. However, in that work, we failed to propose a formula to describe the collapse distance quantitatively. Usually, the collapse distance can be obtained numerically by the nonlinear Schrödinger equation (NLSE); however, it is not that easy due to a great amount of computation, which is especially the trouble in engineering applications. Can a concise and explicit formula applicable for more cases be found? In this Letter, we mainly focus on the question.

The propagation of a laser pulse in air can be described by the extended NLSE, which governs the evolution of the electric field envelope $E(r,z,t)$ ($I=|E|^2$ is the pulse intensity given in units of $\mathrm{W/m^2}$) of the pulse traveling at the group velocity $v_g=\partial\omega/\partial k|_{\omega_0}$:

$$\frac{\partial E}{\partial z}=\frac{i}{2k_0}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)E-\frac{ik''}{2}\frac{\partial^2 E}{\partial \tau^2}+ik_0 n_2|E|^2E -\frac{\sigma}{2}(1+i\omega_0\tau_c)n_eE-\frac{\beta_K|E|^{2K-2}E}{2}. \quad (1)$$

The equation applies to femtosecond laser pulses moving in their group-velocity frame ($\tau=t-z/v_g$), with central wavenumber $k_0=2\pi/\lambda_0$ and angular frequency $\omega_0=2\pi c/\lambda_0$. Here, the first term on the right-hand side accounts for the transverse diffraction, and the remaining ones refer to the normal GVD with the coefficient $k''=\partial^2k/\partial\omega^2|_{\omega_0}$, the Kerr effect of air with the nonlinear index of refraction n_2, and the plasma absorption (real part) and plasma defocusing (imaginary part) with an inverse bremsstrahlung cross section σ and electron collision time τ_c. The last term describes multi-photon absorption with the coefficient $\beta_K=K\hbar\omega_0 n_{\mathrm{air}}\sigma_K$, where σ_K accounts for the multi-photon ionization coefficient, n_{air} denotes the density of air, $K=\mathrm{mod}(U/\hbar\omega_0+1)$ is the minimum number of photons needed in the multi-photon ionization process, and $U = 11$ eV is the characteristic ionization energy of air.

The evolution of electron density n_e can be given by

$$\frac{\partial n_e}{\partial \tau} = \frac{n_e}{U}\sigma|E|^2 + \frac{\beta_K|E|^{2K}}{K\hbar\omega_0} - an_e^2. \quad (2)$$

The first and second terms on the right-hand side of Eq. (2) account for the avalanche ionization and multi-photon ionization, and the last one describes the electron recombination with coefficient $a = 5.0 \times 10^{-13}$ $\mathrm{m^3/s}$.

Most of the parameters in Eqs. (1) and (2) are related to the pressure: $n_2(p) = n_2(p_0)p$, $\tau_c(p) = \tau_c(p_0)/p$, $\sigma(p) = \sigma(p_0)\frac{p(1+\omega_0^2\tau_0^2(p_0))}{p^2+\omega_0^2\tau_0^2(p_0)}$, $k''(p) = k''(p_0)p$, $n_{\mathrm{air}}(p) = n_{\mathrm{air}}(p_0)p$, and $\beta_K(p) = \beta_K(p_0)p$. Here, p is atmospheric pressure expressed in atm and p_0 denotes the atmospheric pressure at sea level. At a standard atmospheric pressure, the values of the nonlinear refractive index, GVD coefficient, multi-photon absorption coefficient, electron collision time, and inverse bremsstrahlung cross section are $n_2 = 3.2 \times 10^{-23}$ $\mathrm{m^2/W}$, $k'' = 2 \times 10^{-29}$ $\mathrm{s^2/m}$, $\beta_7 = 6.5\times 10^{-104}$ $\mathrm{m^{11}/W^6}$, $\tau_c = 3.5\times10^{-13}$ s, and $\sigma = 5.1\times10^{-24}$ $\mathrm{m^2}$, (the values of parameters β_7, τ_c, and U are from Ref. [19]), respectively.

Figure 1 presents the change of the maximum intensity and beam radius with the propagation distance, as the initial pulse duration and pressure are different (we select a Gaussian pulse whose envelope is written as $E(r,t) = E_0 \exp(-r^2/w_0^2)\exp(-t^2/\tau_0^2)$ throughout this Letter). It can be clearly seen from the figure that along with an increase in pressure and a decrease in initial pulse duration, the collapse distance (propagation length of the self-focusing beam until collapse) L_c becomes larger. Usually, the collapse distance can be well described by a semi-empirical formula proposed by Marburger[13,14]:

$$L_c = L_{\mathrm{DF}}/\{[(P_{\mathrm{in}}/P_{\mathrm{cr}})^{1/2} - 0.852]^2 - 0.0219\}^{1/2}, \quad (3)$$

where $L_{\mathrm{DF}} = k_0 w_0^2/2$ refers to the Rayleigh length, which is also called the diffraction length, and $P_{\mathrm{cr}} = 3.77\lambda_0^2/8\pi n_0 n_2 = 2.815$ GW is the critical power of self-focusing[14]. P_{in} denotes the power of the incident laser pulse, and w_0 accounts for the initial beam radius. From Eq. (3), we see that in the case where $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p)$ and the beam radius w_0 are fixed, the collapse distance L_c is independent of the pressure and initial pulse duration of the femtosecond laser. Obviously, Marburger's law fails to describe the collapse in these cases.

For the above phenomenon, two explanations have been proposed. In our previous work, it is attributed to the effect of GVD[18]. Since the GVD plays the role of defocusing during propagation, and because it is enhanced with the increase in pressure and decrease in initial pulse duration, the collapse distance is larger in the case of a longer pulse duration and higher pressure. Skupin *et al.* attributed it to the increase of the self-focusing threshold P_{th} with decreasing pulse duration[15,20], which has already been verified in experiments[21,22], and used the $\gamma - P_{\mathrm{in}}/P_{\mathrm{th}}$ curve to distinguish the self-focusing dominated region (where self-focusing can occur) and dispersion-dominated region (where self-focusing cannot occur):

$$\gamma \simeq \left\{\sqrt{3.38 + 5.2[(P_{\mathrm{in}}/P_{\mathrm{cr}})^2 - 1]} - 1.84\right\} \times \left[\left(\sqrt{P_{\mathrm{in}}/P_{\mathrm{cr}}} - 0.0852\right)^2 - 0.0219\right]\Big/ 2P_{\mathrm{in}}/P_{\mathrm{cr}}, \quad (4)$$

where $\gamma = L_{\mathrm{DF}}/L_{\mathrm{GVD}}$ and $L_{\mathrm{GVD}} = \tau_0^2/k''$ accounts for the GVD length. From their explanation, we know that as the input power of the laser pulse is fixed, according to Marburger's law, the collapse distance will increase with the decreasing initial pulse duration. Therefore, we can say that the two explanations are the same in a way, for both can adequately explain the influence of the initial pulse duration on the pulse collapse. In addition, after the collapse, GVD will act in combination with the plasma effect to enhance the defocusing effect and thereby arrest the increase of the intensity. As a result, we see in Figs. 1(a) and 1(a′) that the clamping intensity shows a tendency to decrease as the GVD effect is enhanced.

Numerically solving the NLSE can obtain the accurate collapse distance; however, it is time consuming, and an intuitive self-focusing formula is needed in experiments and engineering applications. Whether a revised self-focusing formula suitable for more cases can be found still remains an open question.

In view of the defocusing role that GVD plays, and here utilizing the classical optical imaging principle and introducing the GVD length into the semi-empirical Marburger's law, i.e., Eq. (3), we present a revised self-focusing formula that may help solve the above problems:

$$\frac{1}{L_c'} = \frac{1}{L_c} - \frac{1}{L_{\mathrm{GVD}}}. \quad (5)$$

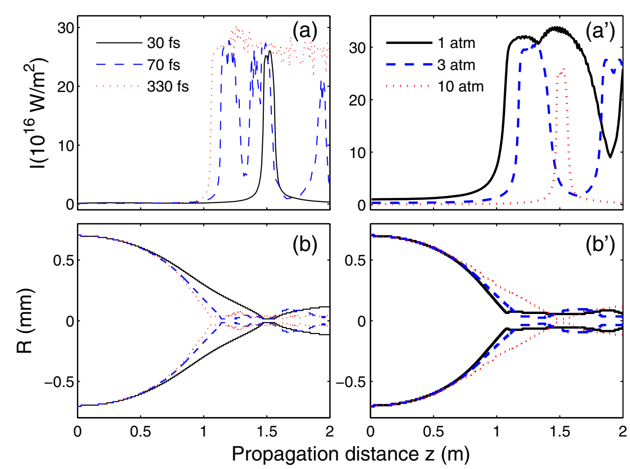

Fig. 1. Change of the (a) maximum intensity and (b) beam radius of 10 atm as the initial pulse duration is 30, 70, and 330 fs. Change of the (a′) maximum intensity and (b′) beam radius for 30 fs pulses as the pressure is 1, 3, and 10 atm. $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p) = 8$ and $w_0 = 1.2$ mm.

We can see from Eq. (5) that for pulses with longer durations (larger τ_0) or the low-pressure case (smaller k''), L_{GVD} is very large, resulting in $\frac{1}{L_c'} \approx \frac{1}{L_c}$, which indicates that Eq. (3) is still applicable. In contrast, for shorter pulse or k'' with a higher value, L_{GVD} is very small. Thus, $L_{\mathrm{GVD}} > L_c$, L_c' will be larger than L_c, indicating that Eq. (3) is no longer applicable, while when $L_{\mathrm{GVD}} < L_c$, L_c' is less than zero, this indicates that self-focusing cannot occur. It means that Eq. (5) can qualitatively describe the influence of GVD on the collapse distance of a self-focusing pulse. In Fig. 2, we present the change of the collapse distance calculated by numerically solving the NLSE in the absence and presence of GVD from Eq. (5). It can be seen from the figure that the collapse distance calculated from Eq. (5) is in agreement with the results of numerically solving the NLSE in the presence of GVD. In addition, by comparing L_{GVD} with L_c, we can divide the collapse and filamentation processes into strong and weak GVD cases: as L_{GVD} is comparable with L_c, it is a strong GVD case, and L_{GVD} is much longer than L_c, it is weak GVD one.

It should be noted that even in the absence of GVD, there is still a little difference between the collapse distance calculated from Eq. (3) and that obtained by numerically solving the NLSE. For instance, the collapse distance calculated from Eq. (3) is $L_c = 1.077$ m, and that obtained by numerically solving the NLSE in the absence of GVD is $L_c \approx 1.055$ m (blue triangles in Fig. 2). To solve this problem, a modified Marburger's law has been proposed by Couairon *et al.* to better match the collapse distance of the simulation result in the absence of GVD[23]:

$$L_c = b_M L_{\mathrm{DF}} / \{[(P_{\mathrm{in}}/P_{\mathrm{cr}})^{1/2} - a_M]^2 - (1 - a_M)^2\}^{1/2}, \quad (6)$$

where a_M and b_M are the free parameters determined by the numerical results. In the original Marburger's law, i.e., Eq. (3), their values are $a_M = 0.852$ and $b_M = 0.376$, while in the modified one in Ref. [21], their values are $a_M = 0.7823$ and $b_M = 0.405$. However, this modified formula still fails to take the influence of GVD into consideration.

In the above discussion, $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p)$ is fixed as 8 and the initial beam radius is selected as $w_0 = 1.2$ mm. It is natural to raise the question of whether the revised formula works accurately in a wide range of situations. In Figs. 3(a) and 3(b), we present the change of the collapse distance with pressure for 50 fs pulses and with the initial pulse duration at 10 atm when $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p) = 4$ and $w_0 = 1.2$ mm. It can be seen from the figure that the revised formula still works, though the difference between the collapse distance calculated from Eq. (5) and that obtained by numerically solving the NLSE becomes larger. As we increase the initial beam radius to $w_0 = 2$ mm, and $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p)$ is fixed as 8, the collapse distances calculated from the revised formula fit well with those calculated from the numerical simulation, as shown in Figs. 3(c) and 3(d). For this reason, the revised formula works accurately in a wide range of situations, though there exists little difference between the collapse distance calculated by numerically solving the NLSE in the presence of GVD and using the revised formula. Many factors lead to the discrepancy, such as the pulse self-compression during pulse collapse and the intensity- and pulse duration-dependent nonlinear refractive index (n_2), which affect the values of critical power for self-focusing P_{cr}.

In addition, it can be seen from Eq. (5) that as L_{GVD} is close to L_c, the L_c' will become very large, making the laser pulse collapse at a longer distance. As the intensity

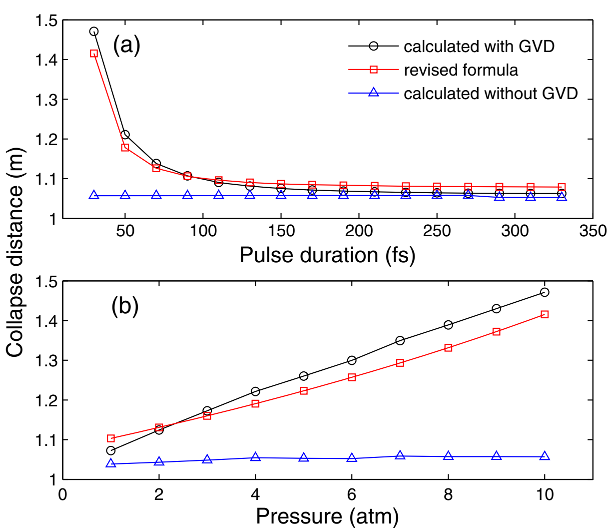

Fig. 2. (a) Variation of the collapse distance with initial pulse duration at 10 atm. (b) Variation of the collapse distance with pressure for 30 fs laser pulse. The black circles and blue triangles show the collapse distance calculated by numerically solving the NLSE in the presence and absence of GVD, respectively; the red squares present the collapse distance obtained from revised formula. $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p) = 8$ and $w_0 = 1.2$ mm.

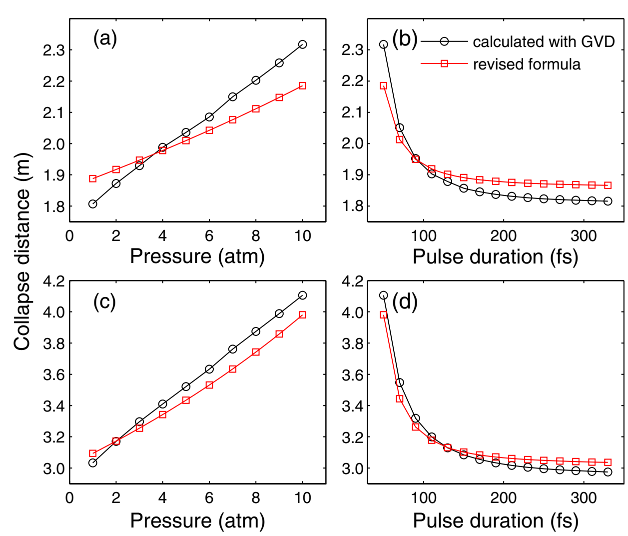

Fig. 3. Change of the collapse distance (a) and (c) with pressure for 50 fs pulses and (b) and (d) with the initial pulse duration at 10 atm. (a) and (b): $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p) = 4$ and $w_0 = 1.2$ mm; (c) and (d): $P_{\mathrm{in}}(p)/P_{\mathrm{cr}}(p) = 8$ and $w_0 = 2.0$ mm. The black circles and red squares show the collapse distance calculated by numerically solving the NLSE in the presence of GVD and the result obtained from the revised formula.

envelope and duration of laser pulse do not change much before the pulse collapse due to the weak nonlinear effect during this process (energy decay is quite small), we can make the intense laser pulse self-focusing at a relatively long distance.

In conclusion, in the weak GVD case, we can calculate the collapse distance directly from the semi-empirical Marburger's law, while in the strong GVD case, Marburger's law is no longer applicable. By introducing the GVD length into Marburger's law, a revised formula for Marburger's law is given. This concise and explicit formula can not only well explain the influence of GVD on the collapse distance, but is also in good agreement with the numerical results in a wide range of situations, making the self-focusing formula applicable for more cases, which may be helpful in engineering applications.

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