The influence of the characteristics of a collection of particles on the scattered spectral density and its applications

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The far-zone scattered spectral density of a light wave on the scattering from a collection of particles is investigated, and the relationship between the character of the collection and the distribution of the scattered spectral density is discussed. It is shown that both the number of particles and their locations in the collection play roles in the distribution of the far-zone scattered spectral density. This phenomenon may provide a potential method to reconstruct the structure character of a collection of particles from measurements of the far-zone scattered spectral density.

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The far-zone property of light waves on scattering from a medium is a topic of considerable importance due to its potential applications in areas such as remote sensing, detecting, and medical diagnosis. During the past two decades, a lot of papers that discussed the relationship between the character of a scattering medium and the property of the far-zone scattered field were published. For example, the spectrum of polychromatic light waves on scattering both from a continual medium[1,2] and from a collection of particles[3] was discussed, and a condition for the isotropy of the far-zone scattered spectrum was presented by Wolf[4]. The spectral coherence of a light wave on scattering from a quasi-homogenous medium was discussed by Visser *et al.*, and a reciprocity relation between the correlation function of the scattering medium and the property of the scattered field was studied[5]. The far-zone scattered field of light waves on scattering from anisotropic particles and ellipsoid particles was discussed by Du[6] and Mei *et al.*[7], respectively. Recently, the scattering of light waves with different density distributions and different correlation functions was also discussed, and it is shown that the property of the incident light wave is also a critical factor that affects the far-zone scattered field[8–12].

In any discussion of light wave scattering, the inverse problem, i.e., the determination of the characteristics of a scatterer from measurements of the far-zone scattered field, is always a highlight[13,14]. In 2007, Zhao and his collaborators presented a method to reconstruct the correlation function of a homogeneous medium from the measurement of the far-zone scattered spectral density[15]. Soon afterwards, this method was generalized to reconstruct the correlation function of a quasi-homogeneous medium[16]. In practice, the scatterer one encounters is not always a continuous medium, but a collection of particles. In this case, one should determine the number, the type, and the location of each particle in the collection to reconstruct the structure of the collection. In this manuscript, the relationship between the characteristics of a collection of particles and the distribution of the far-zone scattered spectral density will be discussed. It will be shown that the far-zone scattered spectral density is closely related to the number of particles and their locations in the collection. This phenomenon may have potential application in the reconstruction of the structure of a collection of particles.

Assuming that a spatially coherent polychromatic plane light wave with a propagation direction specified by a unit vector $\mathbf{s}_0$ is incident on a scattering medium, the cross-spectral density function of the incident field at a pair of points specified by position vectors $(\mathbf{r}'_1,\ \mathbf{r}'_2)$ within the domain of the scattering medium can be defined as[17]

$$W^{(\mathrm{i})}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = \langle U^{(\mathrm{i})*}(\mathbf{r}'_1, \omega) U^{(\mathrm{i})}(\mathbf{r}'_2, \omega)\rangle, \tag{1}$$

where the asterisk denotes the complex conjugate and the angular bracket denote the ensemble average. $\{U^{(\mathrm{i})}(\mathbf{r}', \omega)\}$ is a statistical ensemble of a random function, which takes the following form:

$$U^{(\mathrm{i})}(\mathbf{r}', \omega) = a(\omega)\exp(ik\mathbf{s}_0 \cdot \mathbf{r}'), \tag{2}$$

where $a(\omega)$ is a random function and $k = \omega/c$, with c being the speed of light in a vacuum. Upon substituting the values from Eq. (2) into Eq. (1), one can rewrite the cross-spectral density function of the incident field as

$$W^{(\mathrm{i})}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = S^{(\mathrm{i})}(\omega)\exp[ik\mathbf{s}_0 \cdot (\mathbf{r}'_2 - \mathbf{r}'_1)], \tag{3}$$

where

$$S^{(i)}(\omega) = \langle a^*(\omega)a(\omega)\rangle \tag{4}$$

represents the spectrum of the incident field.

Let us assume that the scattering medium is composed of a collection of particles. In the case that all of the particles and their locations in the collection are deterministic, then the scattering potential $F(\mathbf{r}', \omega)$ of the whole collection is a well-defined function of the position $\mathbf{r}'$. If all of the particles in the collection are the same and located at points denoted by position vectors $\mathbf{r}'_1, \mathbf{r}'_2, \ldots \mathbf{r}'_n$, the scattering potential of the whole collection then can be defined as[18]

$$F(\mathbf{r}', \omega) = \sum_n f(\mathbf{r}' - \mathbf{r}'_n, \omega), \tag{5}$$

where n is the number of particles and $f(\mathbf{r}', \omega)$ is the scattering potential of each particle.

Assuming that the scatter is weak so that the scattering can be analyzed within the accuracy of the first-order Born approximation[19]. Then, the cross-spectral density function of the far-zone scattered field, at two positions specified by a pair of position vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$ can be expressed as[18]

$$W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{S^{(\mathrm{i})}(\omega)}{r^2}\tilde{F}^*[k(\mathbf{s}_1 - \mathbf{s}_0), \omega]\tilde{F}[k(\mathbf{s}_2 - \mathbf{s}_0), \omega], \tag{6}$$

where

$$\tilde{F}(\mathbf{K}, \omega) = \int_D F(\mathbf{r}', \omega)\exp(-i\mathbf{K}\cdot\mathbf{r}')\mathrm{d}^3 r' \tag{7}$$

is the three-dimensional Fourier transform of the scattering potential with

$$\mathbf{K} = k(\mathbf{s} - \mathbf{s}_0). \tag{8}$$

The far-zone scattered spectral density can be obtained from the cross-spectral density function by letting the two position vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$ coincide, i.e.,

$$\begin{aligned} S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) &\equiv W^{(s)}(r\mathbf{s}, r\mathbf{s}, \mathbf{s}_0, \omega) \\ &= \frac{S^{(\mathrm{i})}(\omega)}{r^2}\tilde{F}^*[k(\mathbf{s} - \mathbf{s}_0), \omega]\tilde{F}[k(\mathbf{s} - \mathbf{s}_0), \omega]. \end{aligned} \tag{9}$$

As an example, let us assume that the scattering potential of each particle in the collection has a distribution of the Gaussian function[20], i.e.,

$$f(\mathbf{r}', \omega) = \mathrm{A}\exp\left(-\frac{\mathbf{r}'^2}{2\sigma^2}\right), \tag{10}$$

where A is a constant, and σ, which is dependent on the frequency, is the effective width of the scattering potential of each particle. Upon substituting the values from Eq. (10) into Eq. (5), one can readily find the scattering potential of the whole collection, using the following formula:

$$F(\mathbf{r}', \omega) = \mathrm{A}\sum_n \exp\left[-\frac{(\mathbf{r}' - \mathbf{r}'_n)^2}{2\sigma^2}\right]. \tag{11}$$

Upon substituting the values from Eq. (11) into Eq. (7), and manipulating the six-dimensional Fourier transform, one can obtain

$$\tilde{F}(\mathbf{K}, \omega) = \mathrm{A}(2\pi)^{3/2}\sigma^3 \exp\left[-\frac{1}{2}\sigma^2\mathbf{K}^2\right]\sum_n \exp[-i\mathbf{K}\cdot\mathbf{r}'_n]. \tag{12}$$

Upon substituting the values from Eq. (12) into Eq. (9), one can find the far-zone scattered spectral density of the light wave on the scatter from a collection of particles, which takes the form of

$$\begin{aligned} S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = &\frac{\mathrm{A}^2 S^{(\mathrm{i})}(\omega)(2\pi)^3\sigma^6}{r^2} \\ &\times \exp(-\sigma^2\mathbf{K}^2)\sum_n \exp(i\mathbf{K}\cdot\mathbf{r}'_n) \\ &\times \sum_n \exp(-i\mathbf{K}\cdot\mathbf{r}'_n). \end{aligned} \tag{13}$$

As shown in Eq. (13), both the number of particles (i.e., n) and their locations in the collection (i.e., $\mathbf{r}'_1, \mathbf{r}'_2, \ldots \mathbf{r}'_n$) play roles in the far-zone scattered spectral density. In the following, some numerical results will be presented to illustrate the influence of the number of particles and their locations in the collection on the distribution of the far-zone scattered spectral density.

First of all, let us consider the influence of the locations of the particles in the collection on the distribution of the far-zone scattered spectral density. For the sake of simplicity, let us consider a two-particle collection with different intervals (see Fig. 1). In Fig. 2, the influence of the scattered spectral density of the light wave on scattering from a two-particle collection corresponding to Fig. 1 is presented. Due to the fact that the incident wave is along the z-th direction and the particles in the collection are located in the y-th direction (see Fig. 1), we present the distribution of the scattered spectral density in the y–z plane. As shown in Fig. 2(a), when the interval between the two particles is small, by increasing the scattering angle (i.e., the angle made by $\mathbf{s}$ and $\mathbf{s}_0$), the spectral density decreases, and no secondary maximum appears. However, with the increase of the interval between the two particles, a series of secondary maxima appears in

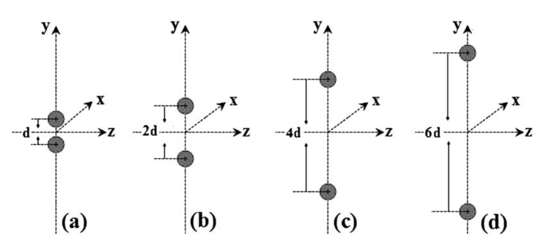

Fig. 1. Two-particle collection with different intervals.

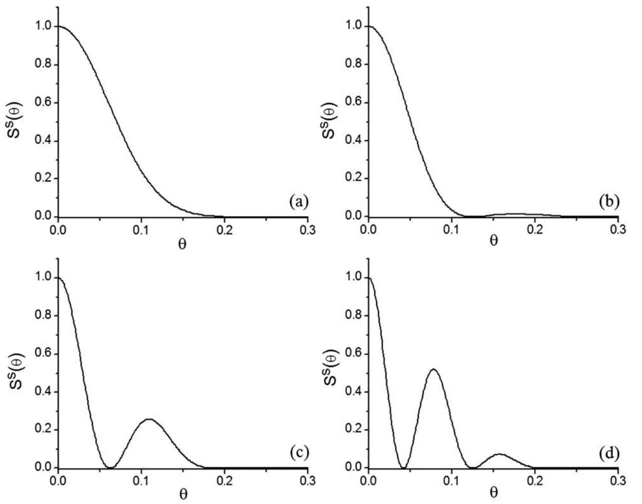

Fig. 2. The normalized scattered spectral density of a light wave on the scattering from a two-particle collection with different intervals corresponding to Fig. 1. The parameters for the calculations are as follows: $\lambda = 0.6283$ μm, $k = 2\pi/\lambda$, $k\sigma = 10$, and $d = 2\lambda$.

the far-zone spectral density. Moreover, it is shown from Fig. 2(b–d) that the scattering angle at which the first minimum value appears decreased with the increase of the interval of the two particles. For an intuitive illustration of above phenomenon, we compiled the distance of the two particles and the scattering angle at which the first minimum and the secondary maximum appeared in the scattered spectral density in Table 1.

Now let us consider the influence of the number of particles on the far-zone scattered spectral density. Assume that all of the particles in the collection are equidistant (see Fig. 3). In Fig. 4, the far-zone scattered spectral density of the light wave on the scattering from a collection with a different number of particles corresponding to Fig. 3 is presented. As shown in Fig. 4(a), for a one-particle collection, the distribution of the spectral density is a Gaussian function, and the spectral density decreased with the increase of the scattering angle. However, for a multi-particle collection, with an increase in the scattering angle, a series of secondary maxima can be found in the scattered spectral density. Moreover, it is shown that the number of the minimum values between two main maximum values in the distribution of the scattered

Table 1. Relationship between the Distance of Two Particles and the Scattering Angle at which the First Minimum and the Second Maximum appeared in the Scattered Spectral Density

	Distance between Two Particles					
	2λ	4λ	6λ	8λ	10λ	12λ
$\theta_{\min 1}$	0.186	0.125	0.083	0.063	0.05	0.042
$\theta_{\min 2}$	–	0.175	0.134	0.109	0.091	0.078

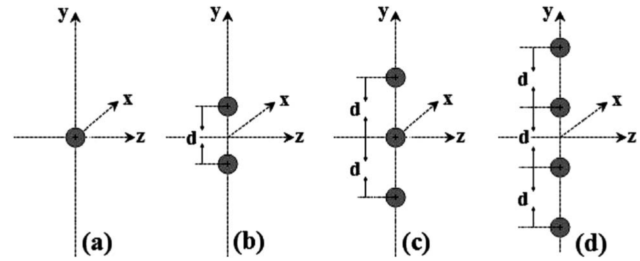

Fig. 3. Particle collection with different numbers of particles.

spectral density is closely related to the number of particles in the collection.

Finally, let us discuss the potential application of the above phenomenon, i.e., the reconstruction of the character of a collection, including the number of particles and their locations in the collection from the measurements of the far-zone scattered spectral density. As shown in Fig. 2, the interval of a two-particle collection is closely related to the direction in which the first minimum appears in the far-zone scattered spectral density. In other words, one can determine the interval of a two-particle collection from the measurement of the direction in which the first minimum appears in the far-zone scattered spectral density. As shown in Fig. 4, the distribution of the far-zone scattered spectral density is influenced by the number of particles in the collection. This can be further concluded as follows: the number of particles in a collection can be determined from the measurement of the number of the minimum values that appear between the two main maximum values in the far-zone scattered spectral density.

It should be noted that the above discussion is based on a collection of identical spherical particles. This method can be generalized to a collection of spherical particles with different sizes, i.e., particles with different effective

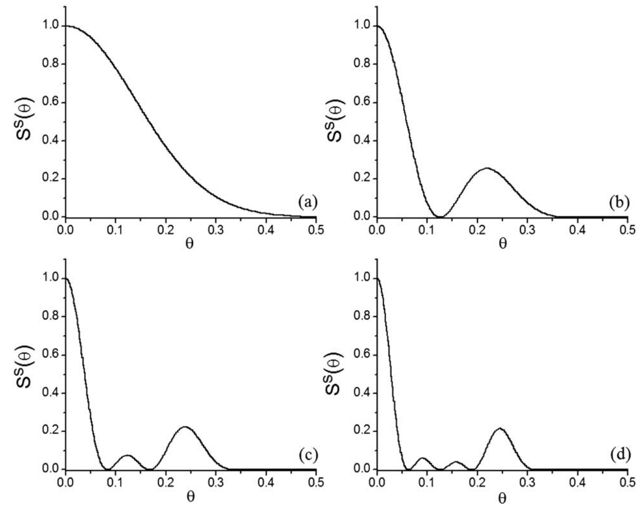

Fig. 4. The normalized scattered spectral density of a light wave on the scattering from collection with different numbers of particles corresponding to Fig. 3. The parameters for the calculations are as follows: $\lambda = 0.6283$ μm, $k = 2\pi/\lambda$, $k\sigma = 5$, and $d = 4\lambda$.

widths of their scattering potentials. For a more complex case, i.e., a collection of particles with non-spherical particles, the scattered field should be discussed with the help of a tensor integral, and the density of the scattered field is then anisotropic[21]. In this case, one needs more information about the scattered field to find the structure information of the collection.

In conclusion, we discuss the far-zone spectral density of a light wave on the scattering from a collection of identical isotropic particles with different distributions. It is shown that the far-zone scattered spectral density is closely related to the location and the number of the particles in the collection. This phenomenon may provide a simple and effective method for the determination of the structure of a collection of identical isotropic particles.

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