New proposal of modulation of amplitude-squeezed state of light by intensity variation of a low-frequency coherent message signal

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Squeezed state of light explores a new era in noiseless communication and data processing recently breaking the quantum limit of noise. We propose a new mechanism of modulating an amplitude-squeezed signal with the instantaneous intensity variation of a coherent signal. The modulating signal is a coherent light where the amplitude-squeezed light takes the role of a carrier signal.

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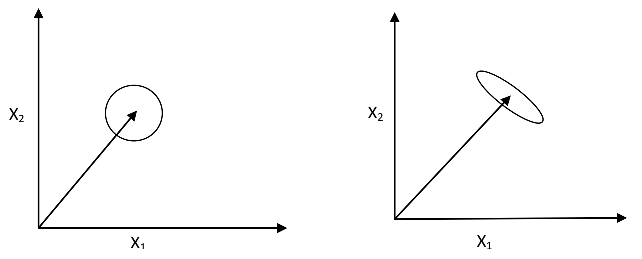
Squeezed state of light has already provided a breakthrough in the field of quantum communication and data processing because it has caused a revolution in noise reduction. In optical communication and computation, we have seen that several types of all-optical logic methods, devices, and switches are proposed for ultra-high-speed operation[1–4]. But it is not possible to avoid the quantum noise produced in those devices because of the inherent quantum limitation of light. And here lies the success of squeezed state of light because it produces noise below the quantum noise limit. Squeezed state of light was first experimentally generated a few decades ago and then several times it has been produced successfully by various techniques till today[5–10]. Various non-linear phenomena such as second-harmonic generation, four-wave mixing, two-photon absorption, and parametric up and down conversion process are mainly responsible for the generation of a squeezed state of light. Recently, various researchers have been able to generate more noise reduced squeezed signal which may result in a more promising world of nearly noise-free communication[11–13]. For its noise reducing property, squeezed light is applied to optical communication and measurement. Few years back squeezed light has shown a new horizon in quantum communication and information processing. By using squeezed light a secret quantum key can be sent safely[14]. Squeezed light can also be used in quantum teleportation[15–18]. Recently, squeezed light has made it possible to enhance the sensitivity of gravitational wave detector successfully which may cause a revolution in the field of astronomy and cosmology[19]. It has been possible to enhance the degree of squeezing of a light beam using beam splitter (BS)[20,21]. Different types of logic devices encoding squeezed light have been also proposed[22–24]. Many properties of squeezed state of light are being explored[25–27]. Again modulation is an essential tool in any communication and there are various types of digital and analog communications. In this letter, we propose a completely different type of modulation where a time-varying coherent signal plays the role of the modulating signal and the carrier signal is an amplitude-squeezed light. After modulation the modulated signal is a squeezed state of light where both its amplitude and its uncertainty vary in accordance with the intensity of a coherent message signal.

Coherent state is a minimum uncertainty state and the uncertainties of two field quadratures are equal. But when the uncertainties of two quadratures are made unequal, that is, the uncertainty of a specific quadrature is squeezed at the expense of an increase in the other, then the state is called squeezed coherent state or simply squeezed state. There are different types of squeezed states such as amplitude-squeezed state, phase-squeezed state, and quadrature-squeezed state. But here we will discuss about amplitude-squeezed light. In quantum optics, a special emphasis is given for treating light as a stream of photons to use the photon statistics with a suitable approach. A coherent light obeys Poissonian statistics, that is, $\Delta n = \sqrt{\langle n \rangle}$, where Δn is the photon fluctuation and $\langle n \rangle$ is the average number of photons. But in amplitude-squeezed light the flow of photons is more controlled than that of a coherent state and it obeys sub-Poissonian photon statistics, that is, $\Delta n < \sqrt{\langle n \rangle}$. Now we will find out the relation between Δn and $\sqrt{\langle n \rangle}$ in the case of amplitude-squeezed light.

Figure 1(a) shows the phasor diagram of a coherent state. The "error circle" indicates the uncertainty both in amplitude and in phase. In Fig. 1(b) the phasor diagram of an amplitude-squeezed light is shown. Here the minor axis of the "error ellipse" has been aligned along the phasor. The mean photon number of a squeezed coherent state can be expressed as[28]

$$\langle n \rangle = |\alpha|^2 + \sinh^2 r, \tag{1}$$

where $|\alpha|$ is the amplitude of the electric field of the squeezed coherent state and r is the squeezed parameter.



(a) (b)

Fig. 1. Phasor diagrams of (a) coherent state and (b) amplitude-squeezed light (X_1 and X_2 are the field quadratures of light).

Now for large average photon number, that is, for $|\alpha|^2 \gg e^{2r}$,

$$\langle n\rangle \approx |\alpha|^2. \quad (2)$$

If the length of the minor axis of the uncertainty ellipse is β then the photon fluctuation Δn can be found out by considering that the phasor is uncertain between $(\alpha + \beta/2)$ and $(\alpha - \beta/2)$[29]. So amplitude-squeezed light can be written as

$$\Delta n = 2\beta\sqrt{\langle n\rangle}. \quad (3)$$

With the help of photon statistics it can be shown that if a coherent light is superimposed with an amplitude-squeezed light using 50/50 (3 dB) BS, an amplitude-squeezed state of light can be obtained[30,31].

Case 1: When the amplitudes of two waves are same

Since the amplitudes are same then the numbers of photons are same (say $\langle n\rangle$)

Now for the coherent signal the photon fluctuation

$$\Delta n_c = \sqrt{\langle n\rangle}. \quad (4)$$

Again for the amplitude-squeezed light the photon number uncertainty is

$$\Delta n_{as} = 2\beta\sqrt{\langle n\rangle}. \quad (5)$$

Therefore the total photon fluctuation of the new wave

$$\Delta n_s = \sqrt{\left\{\left(\Delta n_c\right)^2 + \left(\Delta n_{as}\right)^2\right\}}$$

$$= \frac{\sqrt{\left(1+4\beta^2\right)}}{\sqrt{2}}\sqrt{\langle n_s\rangle}, \text{ where } \langle n_s\rangle = 2\langle n\rangle. \quad (6)$$

Since $\beta < 1/2$ then $\dfrac{\sqrt{\left(1+4\beta^2\right)}}{\sqrt{2}} < 1$.

Therefore $\Delta n_s < \sqrt{\langle n_s\rangle}$.

So the new wave will be amplitude squeezed.

Case 2: When the amplitudes of two waves are different

Here we consider different amplitudes, that is, the photon numbers are different. If there are n_1 and n_2 number of photons in the coherent- and amplitude-squeezed light, then by the same calculation as above we get

$$\Delta n_s = \frac{\sqrt{\left(\langle n_1\rangle + 4\beta^2\langle n_2\rangle\right)}}{\sqrt{\left(\langle n_1\rangle + \langle n_2\rangle\right)}}\sqrt{n_s}. \quad (7)$$

Since $\beta < 1/2$ then $\dfrac{\sqrt{\left(\langle n_1\rangle + 4\beta^2\langle n_2\rangle\right)}}{\sqrt{\left(\langle n_1\rangle + \langle n_2\rangle\right)}} < 1$.

Therefore $\Delta n_s < \sqrt{n_s}$.

So here also the nature of the superposition of light is also amplitude squeezed.

With the help of photon uncertainty one can also determine the nature of the resulting wave after interaction between different types of squeezed states of light (e.g. squeezed vacuum, amplitude squeezed, and phase squeezed)[32].

Amplitude-squeezed light can be generated by various types of non-linear processes[33–36] such as second-harmonic generation, two-photon absorption, and self-phase modulation. The generation of amplitude-squeezed light using frequency-doubling process in a second-order non-linear crystal is shown in Fig. 2[29,37].

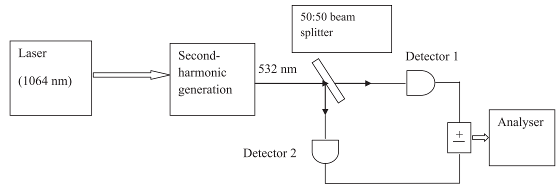


Fig. 2. Schematic arrangement for generating amplitude-squeezed light.

Figure 2 shows the arrangement for generating amplitude-squeezed light. Here the output of the Nd:YAG laser is the input of a second-order non-linear crystal. Since the pump beam of a laser obeys Poissonian photon statistics, the photon stream is random and a higher conversion probability will arise when two photons come close together. Therefore, one can get both the transmitted fundamental and second-harmonic beams in which the photon flow will be more regular than the incoming pump beam. So the photon fluctuations of both beams are smaller than the incoming pump beam and after filtering out the transmitted fundamental beam one can get only the second-harmonic beam at 532 nm. Since the incoming photon stream is Poissonian then the photons of second-harmonic beam obey sub-Poissonian photon statistics. So one can get amplitude-squeezed light which is fed into the input of a +/− balanced detector.

Figure 3 shows a possible arrangement for generating intensity modulated optical signal. At the input port A, an electronic clock pulse and at the input port B a time-varying electrical signal are provided. Then the two signals are fed into a logical AND gate and thus a pulse amplitude modulated (PAM) signal is generated where the signal strength of the out coming wave varies

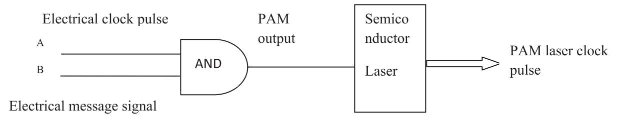


Fig. 3. Schematic arrangement for producing time-varying coherent signal.

in accordance with the strength of the instantaneous variation of the message signal. The AND output then is fed to the input of a semiconductor laser. Therefore, the intensity of each of the output pulse of the laser will vary with time. Therefore, one can express the average photon number of the output beam of the laser pulse as

$$\langle n_c \rangle = \langle n_0 \rangle + \langle n_1 \rangle \sin \omega t. \tag{8}$$

Now we want to modulate an amplitude-squeezed signal by a varying coherent signal as obtained from the system shown in Fig. 3. So we consider such a coherent state whose photon number varies with time. The average photon number of the coherent message signal is

$$\langle n_c \rangle = \langle n_0 \rangle + \langle n_1 \rangle \sin \omega t, \tag{9}$$

where $\langle n_0 \rangle \gg \langle n_1 \rangle$.

Let the average number of photons of the amplitude-squeezed light be taken as $\langle n_2 \rangle$.

The photon fluctuation of the coherent signal is

$$\Delta n_c = \sqrt{\langle n_0 \rangle + \langle n_1 \rangle \sin \omega t}. \tag{10}$$

Again the photon uncertainty of the amplitude-squeezed light is

$$\Delta n_{as} = 2\beta \sqrt{\langle n_2 \rangle}. \tag{11}$$

If we superimpose the coherent signal after reflecting from mirror M with the amplitude-squeezed signal as depicted in Fig. 4, then the resultant fluctuation of the new wave is

$$\begin{aligned} \Delta n_s &= \sqrt{\left\{ (\Delta n_c)^2 + (\Delta n_{as})^2 \right\}} \\ &= \sqrt{\langle n_0 \rangle + \langle n_1 \rangle \sin \omega t + 4\beta^2 \langle n_2 \rangle} \\ &= 2\beta_s(t) \sqrt{\langle n_s \rangle}, \end{aligned} \tag{12}$$

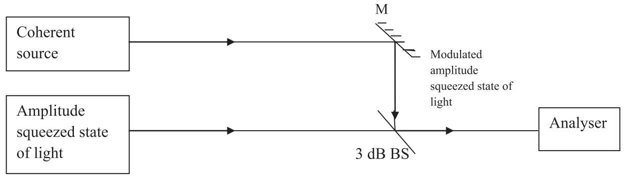


Fig. 4. Schematic diagram of interaction between coherent and amplitude-squeezed signals.

where $\beta_s(t) = \dfrac{1}{2} \sqrt{\dfrac{\langle n_0 \rangle + \langle n_1 \rangle \sin \omega t + 4\beta^2 \langle n_2 \rangle}{\langle n_0 \rangle + \langle n_1 \rangle \sin \omega t + \langle n_2 \rangle}}$ and $\langle n_s \rangle = \langle n_0 \rangle + \langle n_1 \rangle \sin \omega t + \langle n_2 \rangle$

So it is seen that the length of the minor axis of the resultant wave is a function of time. The maximum and minimum values of the fluctuation are

$$\beta_s(\text{max.}) = \frac{1}{2} \frac{\sqrt{\left(\langle n_0 \rangle + \langle n_1 \rangle + 4\beta^2 \langle n_2 \rangle\right)}}{\sqrt{\left(\langle n_0 \rangle + \langle n_1 \rangle + \langle n_2 \rangle\right)}}, \tag{13}$$

$$\beta_s(\text{min.}) = \frac{1}{2} \frac{\sqrt{\left(\langle n_0 \rangle + 4\beta^2 \langle n_2 \rangle\right)}}{\sqrt{\left(\langle n_0 \rangle + \langle n_2 \rangle\right)}}. \tag{14}$$

Therefore, the length of the minor axis of the ellipse varies between β_s(max.) and β_s(min.). Thus, modulated amplitude-squeezed light is obtained. Actually, here the amplitude as well as the photon fluctuation (or the squeezed parameter) is being modulated.

Therefore after some theoretical investigation, we have seen that an amplitude-squeezed light is modulated (more precisely the amount of squeezing) according to the instantaneous value of a time-varying coherent signal. From Eq. (14) it is seen that β_s takes its maximum value $\{\beta_s(\text{max.})\}$ when the signal goes to its highest value $\left(\text{i.e.,} \langle n \rangle = \langle n_0 \rangle + \langle n_1 \rangle + \langle n_2 \rangle\right)$. Similarly β_s is minimum when signal goes to lowest value $\left(\text{i.e.,} \langle n \rangle = \langle n_0 \rangle + \langle n_2 \rangle\right)$. This property can be used in modulation of electronic/optical signal by using amplitude-squeezed state of light as a carrier one. This modulated beam remains also in amplitude-squeezed nature. So when this type of modulated signal is transmitted through the optical fiber the noise of the communication will be reduced below the shot noise limit (SNL) (SNL is the quantum limit in coherent communication). To the best of our knowledge, this proposal shows the reduction of noise below the quantum limit in optical pulse amplitude modulation which can show a tremendous benefit in reduction of signal-to-noise ratio and bit-error rate of optical communication.

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