# Single－slit diffraction of the arbitrary vector beams 

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#### Abstract

We present the single－slit diffraction of the arbitrary vector fields with different parameters $m$ ，$n$ ，and $\varphi_{0}$ theoretically and experimentally．The single slit covers the polarization singularity in the center and therefore the influence of the polarization singularity on the diffraction fringes is analyzed．The experimental results which agree well with the simulation results show that the total intensity of the diffraction field is related only to the topological charge $m$ ，but the polarization distribution of the diffraction field is related to all the parameters $m, n$ ，and $\varphi_{0}$ ．Therefore，the diffraction patterns allow to determine all the parameters of the arbitrary vector fields．


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The homogeneous or inhomogeneous state of polarization of light is one of the most fundamental properties of the electromagnetic field．This important property is widely used in various fields，such as optical imaging ${ }^{[1]}$ ， particle trapping ${ }^{[2]}$ ，particle manipulation ${ }^{[3]}$ ，and nonlin－ ear optics ${ }^{[4]}$ ．Especially，when the reliable and flexible method of generating arbitrary vector beams was pro－ posed by Wang et al．${ }^{[5]}$ and the method is improved to generate an arbitrary space－variant vector beam with structured polarization and phase distributions ${ }^{[6]}$ ．There are many studies on the effects and the application of the vector beams ${ }^{[7-10]}$ ，for instance，the peculiar interfer－ ence behaviors of the vector fields in Young＇s two－slit configuration was given by Li et al．${ }^{[11]}$ ，and the results have potential applications such as characterizing the topological properties of the arbitrary vector fields．But Young＇s two－slit configuration blocked the polariza－ tion singularity of the vector beams，while the polar－ ization singularity is the important part of the vector beam and have important applications in the precise measurement of the deformation and displacement of submicrometer particles and life science research ${ }^{[12-14]}$ ． Besides，the motion of polarization singularities takes place by varying waist width ratio，amplitude ratio，and propagation distance ${ }^{[15]}$ ，especially in the paraxial ${ }^{[16,17]}$ and diffraction conditions ${ }^{[18,19]}$ ．Diffraction has been employed extensively to reveal the unusual phase distri－ bution of the phase singular beam ${ }^{[20]}$ ．In order to design optical components and imaging systems using vector beams，the single－slit diffraction of the arbitrary vector beams is applied to explain the phase and other charac－ teristics，in particular，phase and other characteristics containing the polarization singularity of the arbitrary vector beams．

In this letter，we explore the single－slit diffraction of the arbitrary vector fields theoretically and experimen－ tally．The method of generating arbitrary vector beams proposed in Ref．［5］is applied in this letter and the
final vector beams are synthesized from the left－and right－hand polarized beams which load the phases with special spatial distribution．Thus，the diffraction pat－ terns of these fields are firstly observed，because there are many novel effects of the interference in the double slit ${ }^{[21]}$ and diffraction in the single slit ${ }^{[22]}$ ．The results show that the polarization singularity at the center of the light has a great influence on the diffraction pattern． Because the single－slit diffraction patterns structure carry the information of the phase and polarization dis－ tributions of the arbitrary vector field，so the structure and polarization distribution of the diffraction patterns will also unveil the characteristics of the arbitrary vec－ tor light field．Besides，our research on the phenomenon of vector near－field diffraction has a guiding significance for the study of the limited transmission ${ }^{[23]}$ ，the spatial and temporal evolutions of the vector optical field ${ }^{[24]}$ ， and the interaction of vector light with matter ${ }^{[25]}$ ．

The field distribution of the single－slit diffraction results from the interference of the coherent light and the fringe distribution is determined by the phase dis－ tribution．It is noticed that the arbitrary vector beams in our experiment are the superpositions of two orthog－ onal circularly polarized beams．Firstly，we propose the single－slit diffraction of the left－and right－hand circu－ larly polarized beams carrying phase which controls the polarization of the vector beams．

In the method，the phase loading on the spatial light modulator is $\delta=m \varphi+2 n \pi \rho / \rho_{0}+\varphi_{0}$ ，-1 order diffraction light modulated for left－hand polarized beam of unit amplitude illuminates the narrow slit，and the field just behind the slit is given by

$$
\begin{equation*}
u^{-1}=\operatorname{rect}\left(\frac{x}{b}\right) \exp \left[\mathrm{i}\left(m \phi+2 n \pi \rho / \rho_{0}+\phi_{0}\right)\right], \tag{1}
\end{equation*}
$$

where $b$ is the slit width，$\rho_{0}$ is the radius of the arbi－ trary vector field and initial phase $\varphi_{0}$ ，while $m$ and $n$ are the indices of the angular and radius，respectively．


Fig. 1. Field just behind the slit of the left-hand polarized beam.

Accordingly, diffracted +1 order light modulated for right-hand polarized beam illuminates the narrow slit, and the field just behind the slit is

$$
\begin{equation*}
u^{+1}=\operatorname{rect}\left(\frac{x}{b}\right) \exp \left[-\mathrm{i}\left(m \phi+2 n \pi \rho / \rho_{0}+\phi_{0}\right)\right] \tag{2}
\end{equation*}
$$

Figure 1 shows the field just behind the slit of the lefthand polarized beams, where the white circular spot represents the arbitrary vector beam. The single slit is in the middle covered by the polarization singularity. The phase on different positions of the single slit is marked in Fig. 1, where the phase $2 n \pi \rho / \rho_{0}$ is ignored, which only changes the phase distribution in vertical direction.

For the plane wave, the phase in the slit has uniform distribution. So the diffraction fringes are parallel to the slit and have sinc ${ }^{2}$ intensity distribution ${ }^{[10]}$ as

$$
\begin{equation*}
I=I_{0} \frac{\sin ^{2} \beta}{\beta^{2}} ; \beta=\frac{\pi}{\lambda} b \sin \alpha \tag{3}
\end{equation*}
$$

where $\beta$ represents the phase difference between the waves reaching the point of observation from two points in the slit and $\alpha$ is the diffraction angle. For the plane wave, the phase in the slit is the same, so the straight fringes will appear in the diffraction pattern. For the phase difference caused by path difference between the interfering waves at any point, the arbitrary vector beams remain the same as a uniform plane wave, but will change the phase distribution in the plane just behind the slit.

Thus, when an arbitrary vector beam illuminates the slit, the intensity pattern will be a $\operatorname{sinc}^{2}$ distribution on the whole. But owing to the change in the phase distribution, the value of $\beta$ will be modified as

$$
\begin{equation*}
\beta^{\prime}=\frac{\pi}{\lambda} b \sin \alpha+\gamma \tag{4}
\end{equation*}
$$

where $\gamma$ is the additional phase due to the non-uniform phase distribution of the arbitrary vector beam in the slit, and can be given by phase change along left and right edges of the slit.

Here the width of the slit is considered to be infinitely small, so the relative phase distribution in the bottom


Fig. 2. Analysis of the polarization state in the slit.
of the slit is uniform plane, whereas $2 m \pi$ is added in the upper portion because of the abrupt phase. However, the relative phase distribution changes largely in the center where the polarization singularity is located as shown in Fig. 2.
In order to compute the additional phase $\gamma$, we write the complex amplitude of +1 order diffraction beam as

$$
\begin{equation*}
u(x, y)=(x+\mathrm{i} y)^{m}=r^{m} \exp (\mathrm{i} \delta(x, y)) \tag{5}
\end{equation*}
$$

where $\delta(x, y)=m \varphi+\varphi_{0}=\arg \left[(x+\mathrm{i} y)^{m}\right]$. Equation (5) can be solved as

$$
\begin{equation*}
\delta(x, y)=m \phi+\phi_{0}=\frac{m}{\mathrm{i}} \ln \left[\frac{x+\mathrm{i} y}{\sqrt{x^{2}+y^{2}}}\right]+\phi_{0} . \tag{6}
\end{equation*}
$$

As shown in Fig. 1 we obtain the additional phase $\gamma$ along $y$ from Eq. (6) by setting $x=-b / 2$ and $x=b / 2$ as

$$
\begin{equation*}
\gamma=\delta_{l}-\delta_{r}=\frac{m}{2 \mathrm{i}} \ln \left(\frac{-(b / 2)+\mathrm{i} y}{b / 2+\mathrm{i} y}\right) \tag{7}
\end{equation*}
$$

Therefore, we divide the whole into three portions, $\sigma_{1}$ in the upper, $\sigma_{3}$ in the bottom, and $\sigma_{2}$ in the center. In this case, in the top $\sigma_{1}$ and the bottom $\sigma_{3}$ portions, $\gamma$ values are $2 m \pi$ and 0 , respectively. In the center $\sigma_{2}$ portion, the value of $\gamma$ varies continuously from 0 to $2 m \pi$.

Therefore, we can consider that the diffraction fringes in $\sigma_{1}$ and $\sigma_{3}$ regions are parallel to the slit. Because of the additional phase $2 m \pi$, there will be displacement of the diffraction fringes in the upper and bottom portions. The bright fringes are where $\sin \beta=1$ can be deduced from Eq. (3). In other words, the bright fringes in the bottom are near $\beta_{i}= \pm i \pi+\pi / 2$ where $i$ is constant. But in the upper portion, bright fringes are near $\beta_{i}=-2 m \pi \pm i \pi+\pi / 2$, so there are $2 m \pi$ differences for them in the upper and lower halves of the slit. But in the middle $\sigma_{2}$ portion, the value of $\gamma$ varies continuously from 0 to $2 m \pi$, so each bright fringe in the upper half portion bends near the center and joins the $m$ th bright fringe to the left in the bottom. Certainly, if opposite topological charge $-m$ is used, the bending will take place in the opposite direction and there will be two opposite phenomena in both cases for $\pm 1$ order diffraction beams.

But for the distribution of the stripe observed on the CCD, the displacement for the fringe will also be influenced a lot by $a$ and $b$, where $a$ is the diffraction distance. It is well-known that there will be dark fringes when $\beta=b \pi \sin \alpha / \lambda= \pm \pi, \pm 2 \pi, \pm 2 \pi, \ldots$ in the bottom, it means $\sin \alpha_{i}= \pm i \lambda / b$. We can consider the diffraction angle $\alpha_{i}= \pm i \lambda / b$ in paraxial approximation. So the dark fringe of the diffraction will present at $x_{i}= \pm i a \lambda / b$, but it is $x_{i}= \pm(m+i) a \lambda / b$ for the upper portion of the fringes. So the displacement is $\tau_{x}= \pm \operatorname{ma\lambda } / b$, and the symbol + expresses that the fringes bends to right, whereas the symbol - is the opposite.

In our experiment, we can deem that the left- and right-hand polarized beams are not coherent, so the total intensity of the single-silt diffraction field is the superposition of that of the left- and right-hand polarized beams. Therefore, the intensity of the single-slit diffraction field can be given as

$$
\begin{equation*}
I=I_{-1}+I_{+1}=\frac{1}{2}\left(I_{0} \frac{\sin ^{2} \beta_{-1}^{\prime}}{{\beta_{-1}^{\prime 2}}^{2}}+I_{0} \frac{\sin ^{2} \beta_{+1}^{\prime}}{\beta_{+1}^{\prime 2}}\right) \tag{8}
\end{equation*}
$$

On the basis of above theories and the Huygens-Fresnel principle, the polarization distribution of single-slit diffraction field in Fig. 2 can be analyzed as follows. In the $\sigma_{1}$ region, the polarization distribution can be considered constant across the narrow slit, so the diffraction patterns of the $x$-component can be given as

$$
\begin{equation*}
I_{x}=I_{-1} \cos ^{2} \theta+I_{+1} \cos ^{2}(-\theta) \tag{9}
\end{equation*}
$$

where $\theta$ is the polarizing angle when $\varphi=0$. In the same way, the $y$-component is

$$
\begin{equation*}
I_{y}=I_{-1} \sin ^{2}(-\theta)+I_{+1} \sin ^{2} \theta \tag{10}
\end{equation*}
$$

It is also the same in the $\sigma_{3}$ region but $\theta$ is the polarizing angle when $\varphi=\pi$. In the $\sigma_{2}$ region, the polarization state changes largely, but in order to analyze this polarization distribution, the approximation can be given as

$$
\begin{align*}
I_{x}^{\left(y=y_{0}\right)} & =c_{1}^{\left(y=y_{0}\right)} I\left(\cos ^{2} \theta^{\left(x=\frac{b}{2}, y=y_{0}\right)}+\cos ^{2} \theta^{\left(x=\frac{-b}{2}, y=y_{0}\right)}\right) \\
& =c_{2}^{\left(y=y_{0}\right)} I \cos ^{2} \theta^{\left(x=\frac{b}{2}, y=y_{0}\right)}  \tag{11}\\
I_{y}^{\left(y=y_{0}\right)} & =c_{1}^{\left(y=y_{0}\right)} I\left(\sin ^{2} \theta^{\left(x=\frac{b}{2}, y=y_{0}\right)}+\sin ^{2} \theta^{\left.\left(x=\frac{-b}{2}, y=y_{0}\right)\right)}\right. \\
& =c_{2}^{\left(y=y_{0}\right)} I \sin ^{2} \theta^{\left(x=\frac{b}{2}, y=y_{0}\right)}, \tag{12}
\end{align*}
$$

where $c\left(y=y_{0}\right)$ is constant which means approximate.
We now experimentally explore the single-slit diffraction of the arbitrary vector field. The arbitrary vector beam is experimentally generated using $\mathrm{He}-\mathrm{Ne}$ laser at 632.8 nm as the light source. The created vector field falls normally on the slit with $b=0.2 \mathrm{~mm}$, but the diffraction distance is not a fixed value which will be changed as the complexity of the diffraction patterns. A polarizer can be inserted between the slits and CCD, so the diffraction patterns of the $x$ - and the $y$-components


Fig. 3. Experimental and simulation results of the single-slit diffraction patterns of vector field with $m=1, n=0, \varphi_{0}=0$, and $m=-1$, $n=1, \varphi_{0}=0$.
will be acquired. According to the analysis of Fig. 2, the stripe displacement of the upper and lower parts in the diffraction field is $\tau_{x}= \pm \operatorname{ma\lambda } / b$, which is $0.63-1.20 \mathrm{~mm}$ in our experimental conditions ( $b=0.1 \mathrm{~mm}, a=10-30 \mathrm{~cm}$ ), and the displacement can be observed in our CCD $(576 \times 768$ pixels, $6 \times 8(\mathrm{~mm}))$.
Firstly the single-slit diffraction patterns with different arbitrary vector beams are observed as shown in Fig. 3. The first row shows the field distribution through the horizontal polarizer and the diffraction patterns of the vector fields with $m=1, n=0, \varphi_{0}=0$, the left- and right- hand polarized beams are exhibited orderly from left to right. The second row shows the simulation results, while the third and fourth rows show that of $m=-1, n=1$, and $\varphi_{0}=\pi / 4$.
Figure 3 shows the displacement of the stripes in the upper and bottom regions and the slope of the stripes in the middle agrees well with our theory. All the fringes have the same period of $\tau_{x}=b \lambda / a$ in the $x$-direction, but the stripes in the upper region have one order displacement to the left for the -1 order diffraction light and that is to the right for the +1 order. The polarization singularity in the middle region leads to the slope of the fringes in $\sigma_{2}$ region and the inclined stripes connect the vertical stripes in the upper and bottom region. The single-slit diffraction field is the superposition of that with the left- and right-hand polarized beams. Besides, the single-slit diffraction patterns of the arbitrary vector beams are only related to the topological charge $m$.

Figure 4 shows the experimental and simulation results for the complex vector beams with $m=2, n=0, \varphi_{0}=0$, and $m=3, n=0, \varphi_{0}=0$. The diffraction patterns become complicated especially in the middle region with the increase in $m$, the total intensity matches well with the superposition of that of the $\pm 1$ order beams. In the center, the diffraction pattern exhibits a chessboard structure, because they are the superpositions of the slope stripes in the center of the -1 and +1 order diffraction lights. In


Fig. 4. Experimental and simulation results of the single-slit diffraction patterns of vector field with $m=2, m=3, n=0$, $\varphi_{0}=0$.
other words, there are periodic bright and dark stripes both in the $x$ - and $y$-directions in $\sigma_{2}$ region, and the number of the bright and dark fringes in the $y$-direction is $2 m-1$. It is bright in the center when $m$ is even as shown in the first row, whereas it is dark when $m$ is odd as shown in the third row.

Figures 3 and 4 show that the single-slit diffraction fields only comply with the topological charge $m$ and the other parameters $n$ and $\varphi_{0}$ do not work. But these parameters play a very important role in the distribution of the polarization, so we explore the polarization distribution of single-slit diffraction fields of the arbitrary vector beams with different $m, n, \varphi_{0}$. Figure 5 (a) shows the distribution of the polarization of the vector beams with $m=1,2$, and $3, n=0, \varphi_{0}=0$ and Fig. 5(b) shows the simulation results.

As shown in Fig. 5, the diffraction patterns of the vector fields exhibit the spatial structure in the $x$ - and $y$-(slit) directions described by $I_{x}$ and $I_{y}$. The $x$-component of the diffraction patterns is discussed firstly. For the first row, $I_{x}=I_{-1} \times \cos ^{2}(\pi / 2)+I_{+1} \times \cos ^{2}(-\pi / 2)=0$ in $\sigma_{1}$, and $I_{x}=I_{-1} \times \cos ^{2}(3 \pi / 2)+I_{+1} \times \cos ^{2}(-3 \pi / 2)=0$ in $\sigma_{3}$, so there is no fringe in these regions. But in $\sigma_{2}, I_{x}$ can be given as $I_{x}^{\left(y=y_{0}\right)}=c^{\left(y=y_{0}\right)} \times I \times \cos ^{2} \theta^{\left(x=\mathrm{b} / 2, y=y_{0}\right)}$. Therefore, $\theta$ is from $\pi$ to $\pi / 2$ when $y$ is from 0 to $\infty$, then $I_{x}$ is from $I$ to 0 . This trend is contrary to the periodicity of the central column fringe in the $y$-direction, but complies with the next columns, so the fringes of only odd number columns are retained. For the $y$-component, $I_{y}=I_{-1} \times \sin ^{2}(\pi / 2)+I_{+1} \times \sin ^{2}(-\pi / 2)=I$ in $\sigma_{1}$ and $I_{y}=I_{-1} \times \sin ^{2}$ $(3 \pi / 2)+I_{+1} \times \sin ^{2}(-3 \pi / 2)=I$ in $\sigma_{3}$, so the fringes of $I_{y}$ are the same with that of $I$. In $\sigma_{2}, I_{y}^{\left(y=y_{0}\right)}=c^{\left(y=y_{0}\right)^{y}}$ $\times I \times \sin ^{2} \theta^{\left(x=b / 2, y=y_{0}\right)}$, so $I_{y}$ is from 0 to $I$ when $y$ is from 0 to $\infty$. This trend is contrary to the periodicity of the even number columns fringe, but complies with the next columns, so the fringes of only even number columns are retained which are the dark fringes in this region. That means no fringe in this region.


Fig. 5. (a) Experimental results of the single-slit diffraction patterns of vector field with $m=1,2,3, n=0, \varphi_{0}=0$, and (b) simulation of the single-slit diffraction patterns of vector field with $m=1,2,3, n=0, \varphi_{0}=0$. The right columns show the distribution of the $x$ - and $y$-components.

For the vector beam with $m=2$ as the second row in Fig. 5, $I_{x}=I_{-1} \times \cos ^{2}(0)+I_{+1} \times \cos ^{2}(0)=I$ in $\sigma_{1}$ and $I_{x}=I_{-1} \times \cos ^{2}(2 \pi)+I_{+1} \times \cos ^{2}(-2 \pi)=I$ in $\sigma_{3}$, so $I_{x}$ is the same with $\stackrel{-1}{I}$. In $\sigma_{2}, I_{x}^{\left(y=y_{0}\right)}=\mathrm{c}^{\left(y=y_{0}\right)} \times I \times \cos ^{2} \theta^{\left(x=b / 2, y=y_{0}\right)}, \theta$ is from $2 \pi$ to $\pi$ when $y$ is from 0 to $\infty$, then $I_{x}$ is from $I$ to 0 to $I$. This trend is contrary to the periodicity of the even number columns fringes and complies with the next column. As a result, the fringes of only even number columns are retained in this region. For the $y$-component, $I_{y}=\mathrm{I}_{-1} \times \sin ^{2}(0)+I_{+1} \times \sin ^{2}(0)=0$ in $\sigma_{1}$, and $I_{y}=I_{-1} \times \sin ^{2}(2 \pi)+I_{+1} \times \sin ^{2}(-2 \pi)=0$ in $\sigma_{3}$, therefore there is no fringe. In $\sigma_{2}, I_{y}^{\left(y=y_{0}\right)}=c^{\left(y=y_{0}\right)} \times I \times \sin ^{2} \theta^{\left(x=b / 2, y=y_{0}\right)}$, in the same way, $I_{y}$ is from 0 to $I$ to 0 when $y$ is from 0 to $\infty$. Therefore the trend is contrary to that of the $x$-component, so the fringes of only odd number columns are retained in this region.

We can deduce that there is no fringe for the $x$-component in $\sigma_{1}$ and $\sigma_{3}$ when $m$ is odd. But in $\sigma_{2}$, the fringes of only odd number columns will be retained and the number of the bright and dark fringes in the $y$-direction
is $2 m-1$. For the $y$-component, $I_{y}$ is the same with $I$ in $\sigma_{1}$ and $\sigma_{3}$, but in $\sigma_{2}$, the fringes of only even number columns will be retained and the number of the bright and dark fringes in the $y$-direction is also $2 m-1$. However, when $m$ is even, $I_{x}$ will swap with $I_{y}$. The last row of Fig. 5 proves our analysis very well.

Figure 6 shows the diffraction patterns and the polarization distribution of the vector beams are $m=1$, $n=0$, and $\varphi_{0}=0, \pi / 4,2 \pi / 4,3 \pi / 4$. The diffraction patterns of vector light fields exhibit different spatial structures as different $\varphi_{0}$. The first row in Fig. 6 is same as Fig. 5 and has been analyzed. For the second row with $\varphi_{0}=\pi / 4, I_{x}=I_{-1} \times \cos ^{2}(\pi / 4)+I_{+1} \times \cos ^{2}(-\pi / 4)=I / 2$


Fig. 6. (a) Experimental results of the single-slit diffraction patterns with vector beam with $m=1, n=0, \varphi_{0}=0, \pi / 4,2 \pi / 4$, $3 \pi / 4$ and (b) simulation results of the single-slit diffraction patterns with vector beam with $m=1, n=0, \varphi_{0}=0, \pi / 4,2 \pi / 4$, and $3 \pi / 4$.
in $\sigma_{1}$ and $I_{x}=I_{-1} \times \cos ^{2}(5 \pi / 4)+I_{+1} \times \cos ^{2}(-5 \pi / 4)=I / 2$ in $\sigma_{3}$. As a result, $I_{x}$ is the same as $I$ with certain weak intensity. In the $\sigma_{2}$ region

$$
\begin{equation*}
I_{x}^{\left(y=y_{0}\right)}=c^{\left(y=y_{0}\right)} I \cos ^{2}\left(\theta+\frac{\pi}{4}\right)+\cos ^{2}\left(-\theta+\frac{\pi}{4}\right)=c_{1}^{\left(y=y_{0}\right)} I \tag{13}
\end{equation*}
$$

where $\theta$ is the polarizing angle when $\varphi_{0}=0$. So fringes of $I_{x}$ remain that of $I$.

For the $y$-component, $I_{y}=I_{-1} \times \sin ^{2}(\pi / 4)+I_{+1} \times \sin ^{2}(-\pi / 4)$ $=I / 2$ in $\sigma_{1}$ and $I_{y}=I_{-1} \times \sin ^{2}(5 \pi / 4)+I_{+1} \times \sin ^{2}(-5 \pi / 4)=I / 2$ in $\sigma_{3}$. In the $\sigma_{2}$ region

$$
\begin{equation*}
I_{y}^{\left(y=y_{0}\right)}=c^{\left(y=y_{0}\right)} I\left(\sin ^{2}\left(\theta+\frac{\pi}{4}\right)+\sin ^{2}\left(-\theta+\frac{\pi}{4}\right)\right)=c_{2}^{\left(y=y_{0}\right)} I, \tag{14}
\end{equation*}
$$

which is the same as $I_{x}$. The vector beam with $m=1$, $n=0, \varphi_{0}=3 \pi / 4$ is the same as shown in the fourth row. In fact, according to the calculated process, we can deduce that the results of vector beam with $m=1$, $n=0, \varphi_{0}=m \pi / 2+\pi / 4$ will all be the same.

The third row of Fig. 6 shows the results of vector beam with $m=1, n=0, \varphi_{0}=2 \pi / 4 . I_{x}=I_{-1} \times \cos ^{2}(\pi)+$ $I_{+1} \times \cos ^{2}(\pi)=I$ in $\sigma_{1}$ and $I_{x}=I_{-1} \times \cos ^{2}(2 \pi)+I_{+1} \times \cos ^{2}(-2 \pi)$ $=I$ in $\sigma_{3}$, so the fringes in these regions are the same as that of $I$. But in $\sigma_{2}$ region,

$$
\begin{align*}
I_{x}^{\left(y=y_{0}\right)} & =c^{\left(y=y_{0}\right)} I\left(\cos ^{2}\left(\theta+\frac{\pi}{2}\right)+\cos ^{2}\left(-\theta+\frac{\pi}{2}\right)\right) \\
& =c_{1}^{\left(y=y_{0}\right)} I \cos ^{2} \theta, \tag{15}
\end{align*}
$$

where $\theta$ is from $\pi$ to $3 \pi / 2$ when the $y$ is from 0 to $\infty$, which is the same with $I_{y}$ in the first row. So $I_{x}$ with $m=1$, $n=0, \varphi_{0}=2 \pi / 4$ is the same with the $I_{y}$ with $m=1$, $n=0, \varphi_{0}=0$. Furthermore, $I_{y}$ with $m=1, n=0, \varphi_{0}$ $=2 \pi / 4$ is the same with $I_{x}$ with $m=1, n=0, \varphi_{0}=0$, which is shown in the first and third rows of Fig. 6. Besides, we can deduce that the polarization distribution is one cycle of vector beams with $\varphi_{0}$ from 0 to $\pi$ (Fig. 6).

Lastly, the polarization distribution of vector beams with different $n$ is discussed. Figure 7 (a) shows the diffraction patterns and the polarization distribution of vector beams with $m=1, \varphi_{0}=0$ and different $n$ where $n=0.5,1,1.5$ and Fig. 7(b) shows the simulation results.

As shown in Fig. 7, the diffraction patterns exhibit the spatial periodicity structure in the $y$-(slit) direction when $\delta$ changes as radius. Because the phase $2 n \pi \rho / \rho_{0}$ can be supposed to be uniform from left to right, the change in the phase distribution is only in the vertical direction. In the upper half portion, $I_{x}=I_{-1} \times \cos ^{2}\left(\pi \rho / \rho_{0}\right)$ $+I_{+1} \times \cos ^{2}\left(-\pi \rho / \rho_{0}\right)=I \times \cos ^{2}\left(\pi \rho / \rho_{0}\right)$. In the bottom portion, $I_{x}=I_{-1} \times \cos ^{2}\left(-\pi \rho / \rho_{0}\right)+I_{+1} \times \cos ^{2}\left(\pi \rho / \rho_{0}\right)=I \times \cos ^{2}\left(\pi \rho / \rho_{0}\right)$. $\theta$ is from 0 to $\pi$ when $y$ is from 0 to $\rho_{0}$, then $I_{x}$ is from $I$ to 0 to $I$ which is $n$ periodic change in the $y$-direction.


Fig. 7. (a) Experimental results of the single-slit diffraction patterns with vector beam with $m=1, \varphi_{0}=0, n=0.5,1,1.5$ and (b) simulation results of the single-slit diffraction patterns with vector beam with $m=1, \varphi_{0}=0, n=0.5,1,1.5$.

The center is bright and in the bottom it is the same. For the $y$-component, in the upper half portion, $I_{y}=I_{-1} \times \sin ^{2}\left(\pi \rho / \rho_{0}\right)+I_{+1} \times \sin ^{2}\left(-\pi \rho / \rho_{0}\right)=I \times \sin ^{2}\left(\pi \rho / \rho_{0}\right)$, so $I$ is from 0 to $I$ to 0 , so there is also $n$ cycles periodic change in the $y$-direction but the center is dark, and it is the same in the bottom. Furthermore, it is the same with $n=1.5,2$ as shown in the second and third rows of Fig. 7. We can draw the conclusion that the phase $2 n \pi \rho / \rho_{0}$ does not contribute to the total intensity of the single-slit diffraction but will change both the $x$ - and $y$-components to be periodical which is $2 n$ cycles. The fringe is bright in the center for $I_{x}$ while it is dark for $I_{y}$ when $m$ is odd, and that is opposite when $m$ is even.
In conclusion, we present single-slit diffraction of the arbitrary vector fields with different parameters $m, n$, and $\varphi_{0}$ theoretically and experimentally. The single
slit covers the polarization singularity in the center. Firstly, we figure out the relationship between the diffraction patterns and the parameters $m, n$, and $\varphi_{0}$. In particular, the influence of the polarization singularity on the diffraction fringes is analyzed. The total intensity of the diffraction field is related only to the topological charge $m$, but the polarization distribution of the diffraction field is related to all the parameters $m, n$ and $\varphi_{0}$. Therefore, the diffraction patterns allow to determine all the parameters of the arbitrary vector fields. The experimental results agree well with the simulation results. In addition, our research on the phenomenon of vector near-field diffraction has a guiding significance for the study of the spatial and temporal evolutions of the vector optical field and the interaction of vector light with matter.

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