Property of the azimuthal orientation angles turning by the same axis of the system

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We focus on the need for azimuthal orientation angles of a system. If one azimuthal orientation angle is a function of other angles and these angles turn independently by the same axis, the sum of the partial differential of the function to the other angles is 1. We use this property of the azimuthal orientation angles turning by the same axis of the system to analyze the experimental phenomena of the terahertz polarization, and then quantum theory is used to explain the experimental phenomena.

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The bland sameness of space is at the root of angular momentum conservation. It is worth noting that this conservation is effective in macroscopic and microcosmic views. In this letter, we found another property based on the isotropy of space. Using this property, we could analyze a system without any detailed physical processes. Experimental phenomenon demonstrated here includes controlling the polarization angle of the terahertz (THz) wave generated from two-color laser-induced gas plasma coherently by the relative phase[1,2]. It is well known that the process of the generation of the THz radiation in gas was described phenomenologically with a four-wave mixing process[3,4], but the complete explanation of the process and the underlying mechanism are described by the nonperturbative asymmetric field ionization models[5–9]. To explain the controllable THz polarization, they separately simulate the models tunnel ionized electron wave packet dynamics[1] and one-dimensional transient current[2]. Here, the proposal of the property based on the isotropy of the space is used to explain the experiment without using any of the theories mentioned above. Finally, quantum theory[9] is used to explain the experiment, in which the results of the two methods coincide well.

Consider that one azimuthal orientation angle is a function of the other angles. It can be expressed as

$$\lambda_0 = f\left(\lambda_1, \lambda_2, \cdots, \lambda_n\right), \tag{1}$$

where λ_0 is the azimuthal orientation angle determined by the angles λ_i ($i = 1, 2, 3, \ldots n$), λ_i is independent of the other angle λ_j ($i \neq j \neq 0$), and the angles λ_i ($i = 0, 1, 2, 3, \ldots n$) can vary around the same axis of rotation. Then, we define the right circular rotation around the axis as the positive angle.

If the whole system is rotated around the axis with an angle λ_r and λ_i is independent of the other angle λ_j ($i \neq j \neq 0$), it means that all the angles λ_i ($i = 1, 2, 3, \ldots n$) will sum to the angle λ_r based on the isotropy of space,

$$\lambda_0 + \lambda_r = f\left(\lambda_1 + \lambda_r, \lambda_2 + \lambda_r, \cdots, \lambda_n + \lambda_r\right). \tag{2}$$

Owing to the theory of differential for λ_r, we get

$$\lambda_r = f_1 \cdot \lambda_r + f_2 \cdot \lambda_r + \cdots + f_n \cdot \lambda_r, \tag{3}$$

where f_i ($i = 1, 2, 3, \ldots n$) is the partial differential of the function $f\left(\lambda_1, \lambda_2, \cdots, \lambda_i, \cdots, \lambda_n\right)$ to the angle λ_i.

$$f_1 + f_2 + \cdots + f_n = 1. \tag{4}$$

If λ_i ($i = 0, 1, 2, 3, \ldots n$) is the angle of vector, 2π is the period. Then, the coefficients must obey the relationship

$$\frac{1}{2\pi}\int_0^{2\pi} f_i \mathrm{d}\lambda_i = m, (|m| = 0, 1, 2, \cdots). \tag{5}$$

Equations (4) and (5) describe the property of the azimuthal orientation angles turning by the same axis of the system.

Equations (4) and (5) are used to analyze the experimental phenomenon that includes controlling of the polarization angle of the THz wave generated from two-color laser-induced gas plasma coherently by the relative phase. Two femtosecond optical pulses are involved: one is the fundamental light (ω_1), and the other is the second harmonic light (ω_2). The two pulses and the generated THz wave share the same light axis. The angles of ω_1 and ω_2 lights around the light axis are θ_1 and θ_2, and the angle of the polarization of the THz wave is θ_{THz}. As the angles θ_1, θ_2, and θ_{THz} can vary around the same axis of rotation, θ_1 is independent of θ_2, and Eqs. (4) and (5) can be used to describe the relationship among these three angles.

$$\begin{gathered}\theta_{THz} = f\left(\theta_1, \theta_2\right), \\ f_1 + f_2 = 1, \left(f_1 = \frac{\partial f}{\partial \theta_1}, f_2 = \frac{\partial f}{\partial \theta_2}\right), \\ \frac{1}{2\pi}\int_0^{2\pi} f_i \mathrm{d}\theta_i = m, (|m| = 0, 1, 2, \cdots; i = 1, 2).\end{gathered} \tag{6}$$

The values of f_1 and f_2 cannot be determined by Eqs. (4) and (5). However, in the light of the four-wave mixing process, two ω_1 photons and one ω_2 photon are involved evenly, and then Eq. (6) should be

$$\begin{aligned}\theta_{THz} &= 2\left(a_1\theta_1\right) + a_2\theta_2,\\ 2a_1 + a_2 &= 1,\\ a_1, a_2 &= m, (|m| = 1),\end{aligned} \tag{7}$$

and hence $a_1 = -a_2 = 1$. Correspondingly, the relationship among the angles θ_1, θ_2 and θ_{THz} is

$$\theta_{THz} = 2\theta_1 - \theta_2. \tag{8}$$

Although θ_1 and θ_2 are the rotation angles of ω_1 and ω_2 around the light axis, still the deep physical meaning of the angles θ_1 and θ_1 should be mentioned here. For this purpose, we consider the characterization of an electromagnetic pulse, which can be described as a vector of the electric field:

$$\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix} = \begin{bmatrix}E_{0x}\left(t\right)\cos\left(\omega t - kz\right)\\ E_{0y}\left(t\right)\cos\left(\omega t - kz + \delta\varphi\right)\\ 0\end{bmatrix}, \tag{9}$$

where $E_{0x}(t)$ and $E_{0y}(t)$ are pulse envelopes, ω and k are frequency and wave vector along the z-axis, and $\delta\varphi$ is the phase difference between E_x and E_y components. However, the z-axis is the light axis. The rotation matrix of the vector by θ around the z-axis is

$$R = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{10}$$

If the electromagnetic pulse is linear polarization, we get the phase difference of $\delta\varphi = 0$, the envelopes $E_{0x}(t) = E_0(t)\cos\gamma$ and $E_{0y}(t) = E_0(t)\sin\gamma$, where γ is the polarization angle. Then, after the rotation, the vector of the linear polarization pulse becomes

$$\begin{aligned}\begin{bmatrix}E_x'\\E_y'\\E_z'\end{bmatrix}_{linear} &= R\begin{bmatrix}E_0\left(t\right)\cos\gamma\cos\left(\omega t - kz\right)\\ E_0\left(t\right)\sin\gamma\cos\left(\omega t - kz\right)\\ 0\end{bmatrix}_{linear}\\ &= \begin{bmatrix}E_0\left(t\right)\cos\left(\gamma+\theta\right)\cos\left(\omega t - kz\right)\\ E_0\left(t\right)\sin\left(\gamma+\theta\right)\cos\left(\omega t - kz\right)\\ 0\end{bmatrix}_{linear}.\end{aligned} \tag{11}$$

From Eq. (11) it is concluded that rotation by θ around the light axis means the polarization angle γ plus θ.

If the electromagnetic pulse is right circular polarization, then $\delta\varphi = \pi/2$ and $E_{0x}(t) = E_{0y}(t) = E_0(t)$. Then after the rotation, the vector of the right circular polarization pulse becomes

$$\begin{aligned}\begin{bmatrix}E_x'\\E_y'\\E_z'\end{bmatrix}_{right} &= R\begin{bmatrix}E_0\left(t\right)\cos\left(\omega t - kz\right)\\ E_0\left(t\right)\cos\left(\omega t - kz + \dfrac{\pi}{2}\right)\\ 0\end{bmatrix}_{right}\\ &= \begin{bmatrix}E_0\left(t\right)\cos\left(\omega t - kz - \theta\right)\\ -E_0\left(t\right)\sin\left(\omega t - kz - \theta\right)\\ 0\end{bmatrix}_{right}.\end{aligned} \tag{12}$$

From Eq. (12) it is concluded that rotation by θ around the light axis means the initial phase minus θ_1.

So, as the left circular polarization, $\delta\varphi = -\pi/2$, which means the initial phase plus θ as shown in

$$\begin{aligned}\begin{bmatrix}E_x'\\E_y'\\E_z'\end{bmatrix}_{left} &= R\begin{bmatrix}E_0\left(t\right)\cos\left(\omega t - kz\right)\\ E_0\left(t\right)\cos\left(\omega t - kz - \dfrac{\pi}{2}\right)\\ 0\end{bmatrix}_{left}\\ &= \begin{bmatrix}E_0\left(t\right)\cos\left(\omega t - kz + \theta\right)\\ E_0\left(t\right)\sin\left(\omega t - kz + \theta\right)\\ 0\end{bmatrix}_{left}.\end{aligned} \tag{13}$$

However, as analyzed above, if both the two-color pulses are linear, right circular, and left circular polarization, Eq. (8) becomes

$$\begin{aligned}&(\mathrm{a}): \theta_{\mathrm{THz}} = 2\gamma_1 - \gamma_2, \left(\mathrm{linear-polarization}\right),\\ &(\mathrm{b}): \theta_{\mathrm{THz}} = -\left(2\varphi_1 - \varphi_2\right), \left(\mathrm{right-circular-polarization}\right),\\ &(\mathrm{c}): \theta_{\mathrm{THz}} = 2\varphi_1 - \varphi_2, \left(\mathrm{left-circular-polarization}\right),\end{aligned} \tag{14}$$

where γ_1 and γ_2 are the polarization angles, and φ_1 and φ_2 are the initial phases.

In order to change the relative phase[1,2], one way is to vary the optical delay difference $\Delta\tau = \Delta l\left(n_{\omega 2} - n_{\omega 1}\right)$. The relationship between the relative phase and the optical delay difference is

$$\begin{aligned}-\left(2\varphi_1 - \varphi_2\right) &= -2\pi\Delta l\left(2n_{\omega1}/\lambda_{\omega1} - n_{\omega2}/\lambda_{\omega2}\right)\\ &= 2\pi\left(n_{\omega2} - n_{\omega1}\right)\Delta l/\lambda_{\omega2} = 2\pi\Delta\tau/\lambda_{\omega2},\end{aligned} \tag{15}$$

where Δl is the step translation stage, $n_{\omega1}$ and $n_{\omega2}$ are the refractive indices of the medium at ω_1 and ω_2, and $\lambda_{\omega1}$, and $\lambda_{\omega2}$ are the wavelengths of the ω_1 and ω_2, respectively.

Equations (14) and (15) have supported the experiment result[1,2], that when the two-color lasers both have right or left circular polarization, the polarization angle of the THz wave is indirectly or inversely proportion to the optical delay difference. However, when they both exhibit linear polarization, the polarization angle of the THz wave has nothing to do with the optical delay difference, but is indirectly proportion to the polarization angle difference of the two-color lasers, which is different as explained in Ref. [1]. Their initial work with the two-color lasers that are orthogonally polarized did not result in Eq. (14a), but they conclude that the THz polarization, in this case, essentially follows the polarization of the ω_2 beam. As the polarization angle $\pm\pi$ denotes the same polarization azimuthal orientation for the linear polarization, both our theory above and the theory by in Ref. [1] agree with the experimental result. To distinguish between the two explanations, we demonstrate a method in which the two-color lasers are polarized at an angle of $\pi/4$. However, by this method, the corrigendum of both the theories can be studied.

It has been discussed[9] with quantum theory when the two-color light pulses exhibit right-circular polarization, whereas the analysis agreed well when they both exhibit left-circular polarization. However, the case when the light pulses are both linear-circular polarization is a little different. Consequently, we obtain Eqs. (14b) and (14c) from the method mentioned in Ref. [9]. Then, our aim is to get Eq. (14a) in microcosmic way.

We use the model in Ref. [9], in which we develope a quantum model to analyze the polarization property of the THz wave generated from two-color laser-induced gas plasma. Here we just used the conclusion.

The polarization of the photon can be expressed as the superposition of the two spherical unit vectors: the right-circular and the left-circular polarizations:

$$\hat{e} = \hat{e}_{+1}e^{-i\gamma}\cos\left(\alpha - \frac{\pi}{4}\right) + \hat{e}_{-1}e^{i\gamma}\sin\left(\alpha - \frac{\pi}{4}\right), \qquad (16)$$

where $\hat{e}_{+1}$ and $\hat{e}_{-1}$ are the spherical unit vectors and correspond with the positive and negative helicity states, α reflects the type of polarization and r is the azimuthal orientation of the polarization. We could get different polarization types by change the parameter a, as show in Fig. 1. We refer to $\hat{e}_{+1}$ as the right-circular polarization and $\hat{e}_{-1}$ as the left-circular polarization.

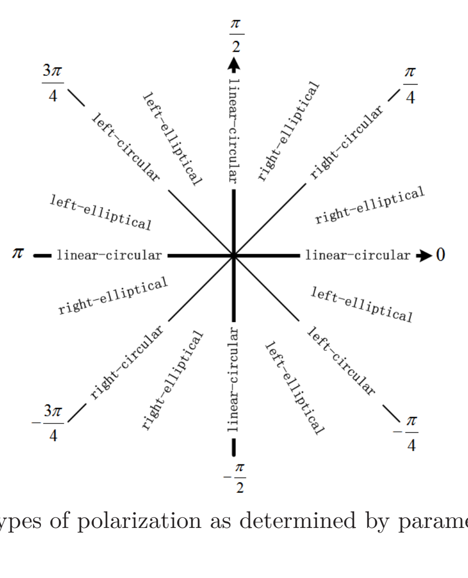

Fig. 1. Types of polarization as determined by parameter α.

The two linear polarized pulses are described as

$$e^{i(\omega_1 t+\varphi_1)}\hat{e}_{\omega 1} = e^{i(\omega_1 t+\varphi_1)}\left(\hat{e}_{+1}e^{-i\gamma_1} + \hat{e}_{-1}e^{i\gamma_1}\right), \qquad (17)$$

$$e^{i(\omega_2 t+\varphi_2)}\hat{e}_{\omega 2} = e^{i(\omega_2 t+\varphi_2)}\left(\hat{e}_{+1}e^{-i\gamma_2} + \hat{e}_{-1}e^{i\gamma_2}\right). \qquad (18)$$

Next we use the method[9] to analyze the right-circular polarization parts of the two pulses, then we can obtain the linear polarization of the THz photon $\hat{e}_{THz+}$ as

$$\hat{e}_{THz+} = M_I e^{i\left[\Delta\varphi-(2\gamma_1-\gamma_2)\right]}\hat{e}_+ + M_{II}e^{-i\left[\Delta\varphi-(2\gamma_1-\gamma_2)\right]}\hat{e}_-, \qquad (19)$$

where $\Delta\varphi = 2\varphi_1 - \varphi_2$ and the polarization angle θ_{THz+} is

$$\theta_{THz+} = -\Delta\varphi + \left(2\gamma_1 - \gamma_2\right). \qquad (20)$$

Similarly, for the left-circular polarization parts, we obtain the linear polarization of the THz photon $\hat{e}_{THz-}$ as

$$\hat{e}_{THz-} = M_I e^{-i\left[\Delta\varphi+(2\gamma_1-\gamma_2)\right]}\hat{e}_+ + M_{II}e^{i\left[\Delta\varphi+(2\gamma_1-\gamma_2)\right]}\hat{e}_-, \qquad (21)$$

and the polarization angle θ_{THz-} is

$$\theta_{THz-} = \Delta\varphi + \left(2\gamma_1 - \gamma_2\right). \qquad (22)$$

The last polarization of the THz photon is the superposition of $\hat{e}_{THz+}$ and $\hat{e}_{THz-}$

$$\hat{e}_{THz} = \hat{e}_{THz+} + \hat{e}_{THz-}, \qquad (23)$$

so the last polarization angle of the THz photon is

$$\theta_{THz} = \tfrac{1}{2}\left(\theta_{THz+} + \theta_{THz-}\right) = 2\gamma_1 - \gamma_2, \qquad (24)$$

which is the same as Eq. (14a). Consequently, the quantum theory has proved the results Eq. (14) from the property of the azimuthal orientation angles turning by the same axis of the system.

In conclusion, we study a property of the azimuthal orientation angles turning by the same axis of the system, which is used to analyze an experimental phenomenon and obtain some interesting results, while these experimental results are proved by quantum theory. This property of the azimuthal orientation angles turning by the same axis of the system is effective not only in macroscopic view but also in microcosmic view.

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