# Measurement of mild asphere by digital plane 

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#### Abstract

In order to test mild aspheric surface directly without other null optics，the digital plane method is proposed． When departure of the tested aspheric surface is mild，a sphere mirror can be used as the reference surface． The phase distribution can be measured swiftly by the digital interferometer．The surface error can be obtained by subtracting the theory wavefront error（the value of the digital plane）from the phase data and eliminating the translation errors through least－squares fitting．The basic principle and theory of this method are analyzed and researched，and the testing model and flow chart are established．Meanwhile the experiment is carried out with a mild aspheric mirror by this method，the PV and RMS of the surface error are $0.173 \lambda$ and $0.018 \lambda$（ $\lambda$ is 632.8 nm ），respectively．We also design and make a null corrector to the asphere for contrast， the differences in PV and RMS error between them are $0.026 \lambda$ and $0.001 \lambda$ ，respectively．


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Aspheric surfaces can correct aberrations，improve image quality，and reduce the size and weight of the op－ tical system ${ }^{[1-3]}$ ，hence they are extremely important in optical systems and have been applied in various kinds of fields，such as deep space exploration，high－resolution earth observation from space，astronomical observation， and so on ${ }^{[4-6]}$ ．As the use of aspheres in optical systems becomes more and more prevalent，the need for precise and efficient metrology grows．One of the most promis－ ing measurements is interferometry．Owing to its high resolution，high sensitivity，and reproducibility，this technology has become the standard tool for testing optical surfaces and wavefronts ${ }^{[7,8]}$ ．
However，compared with traditional plane and spheri－ cal surfaces，the manufacturing and testing difficulty of the aspherical mirrors are the main technical bottlenecks limiting its wide application．Especially for the aspheri－ cal mirror testing，there is no universal aspherical test－ ing equipment and methods till now．How to accomplish the testing of an aspherical surface quickly and accurately has become difficult．So，we will fall back on auxiliary optics such as null lens and computer－generated hologram （CGH）${ }^{[9-11]}$ ．The auxiliary elements must be specially designed and customized，it needs much more time and cost，what is more it brings other errors including both manufacturing errors and some unavoidable misalignment errors．The cost of making and verifying the null elements conspires to keep aspheres from practical optical designs．
In this letter，we have proposed a simple method for testing mild aspheric surfaces by digital plane．The basic theory and prototype of this method is analyzed and researched；it can test weak aspheric surfaces at high resolution，low cost，and high efficiency without any null optics．
Digital plane method is a quick，real－time method for testing small－departure aspherical mirrors，the setup
of this method is shown in Fig．1．For small－departure aspherical mirrors，a standard spherical surface can be used as a reference surface．The phase distribution of the full aperture can be measured by a digital interfer－ ometer．The accurate aspherical surface can be obtained by removing the deviation between the asphere and the sphere wavefront from the testing result．The basic principle and flow chart are shown in Fig．2，and the specific steps are as follows．
Firstly，the radius of the best－fit sphere is calculat－ ed according to the three－point method based on the aspherical parameters：

$$
\begin{equation*}
r=\frac{D^{2}}{8 h}+\frac{h}{2}, \tag{1}
\end{equation*}
$$

where $D$ is the diameter of the asphere and $h$ is the sag of the edge of the tested asphere；then we calculated the value of the digital plane $\left(W_{\mathrm{d}}\right)$ by the following equation：

$$
\begin{equation*}
W_{\mathrm{d}}=[z(x, y)-s(x, y, r)] \cos \theta, \tag{2}
\end{equation*}
$$

where $z(x, y)$ is the sag distribution along the optical axis direction of the aspherical surface and $s(x, y, r)$ is the height distribution of the best－fit sphere along the optical axis direction．$\theta$ is the normal angle of each point on the aspheric surface．


Fig．1．Sketch of setup for testing aspheric surface by digital plane．

Secondly, adjust the locations of the interferometer and the aspheric surface to make the focus of the reference sphere superpose with the center of the best-fit sphere. The phase data ( $W$ ) can be tested by interferometer with the interference fringes. Finally, the surface error $\left(W_{e}\right)$ of the aspheric surface can be calculated by subtracting the digital plane and alignment errors from the phase data, the alignment errors can be calculated by a least-squares fitting. The calculation process is as follows:

$$
\begin{equation*}
W_{\mathrm{e}}=W-W_{\mathrm{d}}-\left[\left(a_{1}+a_{2} X+a_{3} Y+a_{4}\left(X^{2}+Y^{2}\right)\right)\right] \tag{3}
\end{equation*}
$$

where $a_{1}, a_{2}, a_{3}$, and $a_{4}$ are the coefficients of the alignment errors of the asphere of the displacement, tilt in the $x$ and $y$ directions and power, respectively. By leastsquares fitting, the coefficients of the alignment errors can be obtained as follows:

$$
\begin{align*}
& {\left[\begin{array}{l}
a_{2} \\
a_{3} \\
a_{4} \\
a_{1}
\end{array}\right]=\left[\begin{array}{ll}
\sum X X & \sum X Y \\
\sum Y X & \sum Y^{2} \\
\sum\left(X^{2}+Y^{2}\right) X & \sum\left(X^{2}+Y^{2}\right) Y \\
\sum X & \\
\sum Y
\end{array}\right.} \\
& \left.\begin{array}{ll}
\sum X\left(X^{2}+Y^{2}\right) & \sum X \\
\sum Y\left(X^{2}+Y^{2}\right) & \sum Y \\
\sum\left(X^{2}+Y^{2}\right)^{2} & \sum\left(X^{2}+Y^{2}\right) \\
\sum\left(X^{2}+Y^{2}\right) & N
\end{array}\right]^{-1}\left[\begin{array}{c}
\sum X\left(W-W_{\mathrm{d}}\right) \\
\sum Y\left(W-W_{\mathrm{d}}\right) \\
\sum\left(X^{2}+Y^{2}\right)\left(W-W_{\mathrm{d}}\right) \\
\sum\left(W-W_{\mathrm{d}}\right)
\end{array}\right] . \tag{4}
\end{align*}
$$

Thus, we can obtain the accurate figure error of the aspheric surface from Eqs. (1)-(4).

With engineering examples, we measured a mild aspheric mirror by the digital method, the asphere is a ellipsoid whose aperture was 80 mm ; vertex radius of curvature $R$ was 250 mm ; conic constant $K$ was -0.9025 ; and aspheric departure was $4.63 \mu \mathrm{~m}$. The setup of the experiment is shown in Fig. 3.

Firstly, the radius of the best-fit sphere was calculated as 251.44 mm , and the phase distribution of the digital model was computed, as shown in Fig. 4. The full aperture testing result was obtained with a $\Phi 100$ mm Zygo interferometer and an $\mathrm{F}^{\#} 1.5$ standard spherical lens, as shown in Fig. 5. After removing the digital plane and alignment errors, the figure error distribution of the mirror is as shown in Fig. 6, in which the PV and RMS are $0.173 \lambda$ and $0.018 \lambda$, respectively.

In order to contrast and validate, we designed an Offner null corrector to test the aspherical surface, the setup of the null testing is shown in Fig. 7. The null corrector can introduce enough aberration (of the opposite sign) into the test beam so that it eliminates the aberration aroused from the tested aspheric surface at its center of curvature ${ }^{[12,13]}$. It consists of two positive piano-convex lenses: the field and relay lenses. Both lenses are located near the center of curvature of the mirror under test, so they are significantly smaller than


Fig. 2. Flow chart of testing asphere by digital plane.
the tested mirror itself. The function of the field lens is to rearrange in a linear way the rays that arrive to the relay lens. The function of the relay lens is to introduce


Fig. 3. Setup of testing asphere by the digital plane method.


Fig. 4. Phase map of the digital plane.


Fig. 5. Testing results by the digital plane method.


Fig. 6. Figure error testing by digital plane.
the bulk of the optical correction and to obtain a null test point.

The phase map and interferogram from the null test are shown in Fig. 8, where the PV and RMS are $0.199 \lambda$ and $0.019 \lambda$, respectively. Note that the differences in PV and RMS errors between the two methods are $0.026 \lambda$ and $0.001 \lambda$, respectively and the residual error between them is shown in Fig. 9, where the PV and RMS are $0.021 \lambda$ and $0.002 \lambda$, respectively.

In conclusion, we propose the digital plane method to test mild aspheric surface. This method expanded the capabilities of the interferometer, it can test smalldeparture (departure is within $10 \mu \mathrm{~m}$ ) concave, convex aspheric surface directly and precisely without additional optical elements. There are many advantages of this method, such as simple data processing and calculation, convenient experimental operation, short time testing, and low cost. With engineering examples, a $\Phi 80 \mathrm{~mm}$ ellipsoid is tested by this method, and a null corrector to measure the aspheric mirror by null compensation is designed; the surface error of the two methods is


Fig. 7. Setup of testing asphere by compensation.


Fig. 8. Surface map and interferogram from null compensation.


Fig. 9. Residual error of the two methods.
consistent and the residual errors of the PV and RMS are $0.021 \lambda$ and $0.002 \lambda$, respectively. When the departure is more than $10 \mu \mathrm{~m}$, we can combine the digital plane method with other two non-null testing methods. Combining this method with the partial compensation method can test asphere especially for convex asphere swiftly, it simplifies compensation structure, reduces the compensator design difficulty and production costs, and when testing asphere unify this method and subaperture stitching, it widens the horizontal and vertical dynamic range of the interferometer and can test large-aperture concave, convex, and off-axis aspherical surfaces without additional optical components, in low cost and short time.

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