# Studies on the wideband design of acousto－optic deflector using lithium niobate crystal 

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#### Abstract

We present theoretical studies on the wideband design of bulk lithium niobate（LN）acousto－optic deflector （AOD）through the walk－off design of the ultrasonic vector，which satisfies the momentum match condition． The ultrasonic properties of LN crystal are studied by solving the Christopher equation and the reciprocal velocity curves in the operating planes $X O Z$ and $Y O Z$ are systematically obtained．The calculation results show that the bandwidth of the AOD is highly dependent on the incident angle of light beam and the velocity of the ultrasonic waves，which show strict linear properties in the operating bandwidth．Furthermore，the dependence of the central frequency of the AOD on the angle of incident light and the ultrasonic velocity are also analyzed．


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Acousto－optic（AO）effect was first proposed by Brillouin in $1922^{[1]}$ and experimentally demonstrated by Lucas et al．${ }^{[2]}$ in France，and Debye and Sears ${ }^{[3]}$ in the United States．The basic principle of the AO deflector（AOD）is that the light beam is diffracted by the periodic distri－ bution of the refractive index of the AO crystal，which is caused by the propagation of the ultrasonic waves in the crystal．The AOD devices can be used for the laser beam steering ${ }^{[4]}$ ，imaging spectrometer ${ }^{[5]}$ ，laser beam shaping ${ }^{[6]}$ ，fringe projectors ${ }^{[7]}$ ，multichannel communi－ cations ${ }^{[8]}$ ，optical tweezers ${ }^{[9]}$ ，and image scanners ${ }^{[10-13]}$ ， The bandwidth of AO diffraction is one of the most important parameters of the AOD and the optimizing design of wideband operating mode is highly required for most applications．
We systematically studied the wideband design of the AOD through the walk－off design of the off－axis angle of the ultrasonic wave，where the bandwidth of the AOD can be determined by solving the Dixon equation of the anisotropic AO interaction，which indicates the geometrical relationships of the vectors of the diffrac－ tive optical wave，the ultrasonic wave，and the incident optical wave under the momentum match condition in the operating plane．The bandwidth features of the AOD in the operating planes of $X O Z$ and $Y O Z$ are sys－ tematically calculated and the optimizing design of the wideband AOD is determined．Furthermore，the central frequency features of the AOD are also calculated and analyzed．
Anisotropic AO interaction can be described by the momentum match condition

$$
\begin{equation*}
\vec{k}_{d}=\vec{k}_{i} \pm \vec{K} \tag{1}
\end{equation*}
$$

where $\vec{k}_{d}, \vec{k}_{i}$ and $\vec{K}$ are the wave vectors of the diffrac－ tion light，the incident light，and the acoustic wave，the
expressions of $\vec{k}_{d}, \vec{k}_{i}$ and $\vec{K}$ can be written as

$$
\begin{equation*}
k_{i}=\frac{2 \pi n_{i}}{\lambda_{0}}, k_{d}=\frac{2 \pi n_{d}}{\lambda_{0}}, K=\frac{2 \pi}{\Lambda}=\frac{2 \pi f}{V}, \tag{2}
\end{equation*}
$$

where $n_{d}$ is the refractive index of the diffractive light， $n_{i}$ is the refractive index of the incident light，$V$ is the velocity of the ultrasonic wave，$f$ is the frequency of the ultrasonic wave，and $\lambda_{0}$ is the wavelength of the light wave．
The schematic diagram of the momentum match of lithium niobate（LN）anisotropic AO interaction in $X O Z$ and $Y O Z$ planes is shown in Fig． 1
It is noted that $\theta_{\alpha}$ is the angle between the vector of the ultrasonic wave and the $X$－axis and $\theta_{d}$ is the angle between the vector of diffractive light and the $Z$－axis． It can be observed from Fig． 1 that the refractive indexes of the incident and diffractive lights are differ－ ent due to the change in polarization associated with the interaction．In our study the incident light is polar－ ized perpendicularly to the $\mathrm{XOZ}(Y O Z)$ plane in order to realize the wideband operating mode both for the +1 order diffraction and the -1 order diffraction．Thus the vector of the incident light is connected to the cir－ cular index curve（ordinary light）and the vector of the diffractive light is connected to the ellipse index curve （extraordinary light）．At the same time，the acoustic wave vector connects the index curves of the incident and diffracted waves，as illustrated in Figs．1（a）and（b）．
From geometrical relationships of Figs．1（a）and（b）， the Dixon equation of LN anisotropic AO interaction can be derived as

$$
\begin{equation*}
\sin \left[ \pm\left(\theta_{a}-\theta_{i}\right)=\frac{\lambda}{2 n_{\mathrm{o}} v}\left[f-\frac{v^{2} n_{\mathrm{o}}^{2}}{f \lambda^{2}}\left(\frac{n_{\mathrm{e}}^{2}-n_{\mathrm{o}}^{2}}{n_{\mathrm{e}}^{2}} \sin ^{2} \theta_{d}\right)\right],\right. \tag{3}
\end{equation*}
$$


(a)

(b)

Fig. 1. Schematic diagram of the anisotropic AO interaction: (a) +1 order diffraction and (b) -1 order diffraction.

$$
\begin{equation*}
\sin \left[ \pm\left(\theta_{d}-\theta_{a}\right)\right]=\frac{\lambda}{2 n_{\mathrm{o}} v}\left[f+\frac{n_{\mathrm{o}}^{2} v^{2}}{f \lambda^{2}}\left(\frac{n_{\mathrm{e}}^{2}-n_{\mathrm{o}}^{2}}{n_{\mathrm{e}}^{2}} \sin ^{2} \theta_{d}\right)\right] \tag{4}
\end{equation*}
$$

where the " $\pm$ " means the +1 order diffraction and -1 order diffraction, respectively.

The acoustic property of LN crystal is determined by the Christoffel equation:

$$
\begin{equation*}
\left(\Gamma_{i j}-\rho v^{2} \delta_{i j}\right) \cdot u_{j}=0 \tag{5}
\end{equation*}
$$

where $\Gamma_{i j}=L_{i J} c_{i J} L_{I j}, \rho$ is the density of the crystal and $v$ is the velocity of the ultrasonic.

$$
\begin{align*}
\delta_{i j} & =\left\{\begin{array}{ll}
0 & i=j \\
1 & i \neq j
\end{array},\right.  \tag{6}\\
L_{i J} & =\left(\begin{array}{lllllll}
\mathrm{l}_{x} & 0 & 0 & 0 & \mathrm{l}_{z} & \mathrm{l}_{y} \\
0 & \mathrm{l}_{y} & 0 & \mathrm{l}_{z} & 0 & \mathrm{l}_{x} \\
0 & 0 & \mathrm{l}_{z} & \mathrm{l}_{y} & \mathrm{l}_{x} & 0
\end{array}\right), \tag{7}
\end{align*}
$$

where $l_{x}, l_{y}$, and $l_{\mathrm{i}}$ are the direction cosines of the vector of ultrasonic wave.

The velocity of ultrasonic wave in $X O Z$ and $Y O Z$ planes can be obtained by extracting the eigenvector from the Christoffel equation, the reciprocal curves of the LN crystal is shown in Fig. 2.

It can be found that there are three reciprocal velocity curves in each operating plane. The curve mentioned with (1) is called the longitudinal wave, which leads to the isotropic AO interaction. The curves mentioned with (2) and (3) are named as sheared ultrasonic wave that can lead to the anisotropic AO effect. The values of the sheared ultrasonic waves along the $X$ - and $Y$-axes are figured out as $v_{x 1}=3468 \mathrm{~m} / \mathrm{s}, v_{x 2}=4020 \mathrm{~m} / \mathrm{s}$, $v_{y 1}=3579 \mathrm{~m} / \mathrm{s}$, and $v_{y 2}=4131 \mathrm{~m} / \mathrm{s}$.

In order to realize the wideband design of LN anisotropic AOD by the walk-off design of the ultrasonic vector, the non-monotonicity condition of the $\theta_{\alpha}(f) \sim f$ curves in the operating bandwidth should be satisfied. Based on the calculation with Eqs. (3) and (4) it can be found that this non-monotonicity condition can be satisfied when $\theta_{i} \neq 0$. The $\theta_{\alpha}(f) \sim f$ curves under different incident angles $\theta_{i}$ are given in Figs. 3 (a) and (d).
In the actual design the off-axis angle of $\theta_{\alpha}$ is set as $\theta_{\alpha}=\theta_{a m} \pm \delta \theta_{\alpha}$ to realize the walk-off of the ultrasonic vector, where the $\theta_{a m}$ is named the extremum angle and


Fig. 2. The reciprocal velocity curves of LN crystal in (a) XOZ and (b) YOZ planes:


Fig. 3. $\theta_{\alpha}(f) \sim f$ curves under different incident angles and operating modes: (a) $\theta_{i}=2.5^{\circ},+1$ order diffraction, $(\mathrm{b}) \theta_{i}=2.5^{\circ},-1$ order diffraction, (c) $\theta_{i}=5^{\circ},+1$ order diffraction, and (d) $\theta_{i}=5^{\circ},-1$ order diffraction.
can be determined by the theoretical calculation with the $\theta_{\alpha}(f) \sim f$ curve, the bandwidth of the AOD is calculated by the equation $\Delta f=f_{\mathrm{H}}-f_{\mathrm{L}}$, where $f_{\mathrm{H}}$ and $f_{\mathrm{L}}$ are the two frequencies that meet the $\theta_{\alpha}(f) \sim f$ curve for $\theta_{\alpha}=\theta_{a m} \pm 2 \delta \theta_{\alpha}$, as illustrated in Fig. 3(a). Herein, the $d q_{a}$ is named the walk-off angle and usually set as 0.1 , the " $\pm$ " in the equation means the +1 order diffraction and -1 order diffraction, respectively. The central frequency is calculated by the equation of $f_{\mathrm{c}}=f_{\mathrm{H}}+f_{\mathrm{L}} / 2$.

From Figs. 3(a) and (d), the broadening of the AOD bandwidth with the increasing ultrasonic velocity and the incident angle can be observed qualitatively. Figure 4(a)


Fig. 4. The linearity relationships between: (a) the bandwidth and (b) the incident angle.
shows the calculation results of the bandwidth at certain ultrasonic velocities under the incident angles of $2.5^{\circ}$ and $5^{\circ}$, and Fig. 4(b) shows the results at certain incident angles at the ultrasonic velocities of 3467 and $4131 \mathrm{~m} / \mathrm{s}$.

From Figs. 4(a) and (b) it can be clearly found that the AOD bandwidth is broadened linearly with the increase in the velocity and the incident angle $\theta_{i}$. Furthermore, the gradient of the simulated lines also increase with the increase in the incident angle and the ultrasonic waves. Thus, we can conclude that under the same incident angle of $\theta_{\mathrm{i}}$, the optimum wideband design is to select the operating modes with the most fast ultrasonic wave to obtain the broad bandwidth.

The effects of the incident angle on the central frequency of the $A O D$ are also studied under different sheared ultrasonic waves, and the calculation results for different velocities are shown in Figs. 5(a)-(d).

It can be observed from Figs. 5(a)-(d) that the simulated lines show strict linear relationship between the central frequency and the incident angle. Furthermore, it can also be found that the gradients of the linear relationship increases due to the increasing ultrasonic


Fig. 5. The linearity relationships between the central frequency and the incident angle at different ultrasonic velocities: (a) $v=3468 \mathrm{~m} / \mathrm{s}$, (b) $v=3579 \mathrm{~m} / \mathrm{s}$, (c) $v=4020 \mathrm{~m} / \mathrm{s}$, and (d) $v=4131 \mathrm{~m} / \mathrm{s}$.
velocity, which shows similar properties as the feature the bandwidth analyzed in Fig. 4.

In conclusion, studies on the wideband design of LN AO are systematically carried out and the wideband design of the AOD is realized by the walk-off set of the ultrasonic vector, the bandwidths of the LN AOD are found to show strict linearity properties with the incident angle and with the ultrasonic velocities. Furthermore, the linearity properties of the central frequency with the incident angle and the ultrasonic velocity are also
observed. Here we not only propose a new method for the wideband design of anisotropic AOD but also provide meaningful references for the design of novel devices based on the anisotropic AO diffraction.

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