Analogy of light propagation in spatial and temporal domains

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The propagation characteristics of lightwave in spatial and temporal domains are reviewed, and a general analogy of spatial diffraction and temporal dispersion is presented in details. By using the transformation pairs of spatial location and time, wave number and dispersion parameter, some more general expressions, such as spot size/wavefront curvature and pulse-width/chirps, space-angle spectrum product and time-bandwidth product, spatial Fresnel number and temporal Fresnel number, focal length of lens and focusing time, are derived.

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The propagation and transformation of laser beams are the fundamental problems in the field of laser technique, optical communication and optical information processing, etc. The propagation and transform of laser beams include two aspects: spatial and temporal domains[1–3]. The propagating equation of transverse lightwave is identical in form to that of equation describing the temporal evolution of an optical pulse in a dispersive medium. This observation enables us to transfer directly many concepts dealing with diffraction to the problem of temporal propagation. M. Nakazawa *et al.* developed a time-domain ABCD matrix formalism to deal with the temporal evolution of an optical pulse by analogy with the ABCD formalism in the spatial domain[4]. In this paper, we play the emphasis on the analogy of some basic concepts in time-domain and space-domain, and some general transformation pairs are obtained.

In the space- domain, based on Maxwell's equations, considering the two-dimensional case (x, z) for simplicity, under the slow-changing approximation, the master equation of the optical field amplitude can be written as[1]

$$\frac{\partial^2 E(x,z)}{\partial x^2} = -2ik\frac{\partial E(x,z)}{\partial z}. \tag{1}$$

We take E in the form of

$$E(x,z) = \frac{m}{\sqrt{q}}\exp\left(-\frac{ikx^2}{2q}\right), \tag{2}$$

where $i = \sqrt{-1}$, m is a constant, k is the wave number and q is the complex radius of curvature of Gaussian beam.

In time-domain, for pulse width >100 fs, the contribution of the third-order dispersion term is quite small, and ignoring nonlinearity and all loss, nonlinear Schrödinger (NLS) equation that governs propagation of optical pulses can written in the form[2]

$$\frac{\partial^2 A(z,T)}{\partial T^2} = \frac{2i}{\beta_2}\frac{\partial A(z,T)}{\partial z}, \tag{3}$$

where A is the slowly varying amplitude of the pulse envelope and T is measured in a frame of reference moving with the pulse at the group velocity v_g, $T = t - z/v_g$. β_2 is group velocity dispersion (GVD) parameter.

We take A in the form of

$$A(z,t) = \frac{A_0}{\sqrt{p}}\exp\left(\frac{iT^2}{2\beta_2 p}\right), \tag{4}$$

where p is a complex parameter of Gaussian beam and A_0 is the initial amplitude. According to the principle that analogical equations have the same solutions, we can derive the analogical pairs of T (time-domain) and x (space-domain), $-1/\beta_2$ (time-domain) and k (space-domain), and z (time-domain) and z (space-domain).

The propagation of fundamental Gaussian beam can be described by q parameter, the beam spot size $\omega(z)$ and radius of curvature $R(z)$ at the plane z and they are

$$\omega(z) = \omega_0\sqrt{1+\left(\frac{2z}{k\omega_0^2}\right)^2}, \tag{5}$$

$$R(z) = z\left[1+\left(\frac{k\omega_0^2}{2z}\right)^2\right], \tag{6}$$

where ω_0 is the beam spot size at the plane $z = 0$.

The propagation of ultrashort pulses can be described by the ABCD matrix, assuming that the initial chirp of input pulse (at the plane $z = 0$) is 0, and the pulse-width $\tau(z)$ and chirp $C(z)$ at the plane z are[4]

$$\tau(z) = \tau_0\sqrt{1+\left(\frac{\beta_2 z}{\tau_0^2}\right)^2}, \tag{7}$$

$$\frac{1}{C(z)} = \beta_2 z\left[1+\left(\frac{\tau_0^2}{\beta_2 z}\right)^2\right]. \tag{8}$$

The space and angular spectrum product of fundamental mode Gaussian beam is

$$\omega_0 \times \theta = 0.6367\lambda. \tag{9}$$

The product of pulse-width and spectrum-width in ultrashort pulses is a constant, which relates with the waveform of a pulse laser. For the unchirped Gaussian pulse, the constant is 0.441, and

$$\tau \times \Delta v = 0.441, \tag{10}$$

where Δv is the full-width at half-maximum (FWHM) bandwidth of the pulse beam.

The Fresnel number of spherical wave in space-domain (N) and pulse beam in time-domain (N_t) can be written as[5]

$$N = \frac{a^2}{\lambda}\left(\frac{A}{B} + \frac{2i}{k\omega_0^2}\right), \tag{11}$$

$$N_t = -\frac{c\tau^2}{\lambda}\left(\frac{A}{B} - \frac{2i\beta_2}{\tau_0^2}\right), \tag{12}$$

where a is the half-width of the slit, R is the radius of curvature, L is the transmission distance, c is the speed of light in a vacuum, τ is the pulse-width, λ is the wavelength, and A and B are the matrix elements of system transform matrix.

With the paraxial approximation, the penetration function of lens in space-domian is given by[4]

$$t = \exp\left(ik\frac{(x_1^2 + y_1^2)}{2f}\right), \tag{13}$$

where (x_1, y_1) are the coordinates of input plane, f is the focal length of lens.

When an unchirped Gaussian pulse passes through a kerr medium, it will be chirped. The chirp is treated as a time-lens, and neglecting the constant phase and the group delay, the transmission function of the time-lens is described as[6]

$$tt = \exp\left[\frac{i\omega_0 t^2}{2f_t}\right], \tag{14}$$

where f_t is the focusing time and ω_0 is the central wavelength of ultrashort pulse.

Table 1. Analogy of beam propagation in space-domain and time-domain

Time-domain	**Space-domain**
T	x
$-1/\beta_2$	k
z	z
$1/C$	$R(z)/k$
$\tau \times \Delta v = 0.441$	$\omega_0 \times \theta = 0.6367\lambda$
$N_t = -c\tau^2(1/R + A/B)/\lambda$	$N = a^2(1/\mathrm{R} + A/B)/\lambda$
ω_0/f_t	k/f

Comparing Eqs. (13) and (14), we can obtain the analogy of k/f and ω_0/f_t.

In summary, by the rule that similar equations have similar solutions, the solutions of propagational equations in space-domain and time-domain are derived. The analogies of space-angle spectrum product and time-bandwidth product, spatial Fresnel number and temporal Fresnel number, and focal length of lens and focusing time are included in Table 1.

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