Propagation speed calculation of a plasmonic THz wave trapping system

Baoshan Guo (郭宝山)^{*}, Wei Shi (史 伟), and Jianquan Yao (姚建铨)

College of Precision Instrument and Optoelectronics Engineering, Tianjin University,

Tianjin 300072, China

*Corresponding author: gbs@tju.edu.cn

Received August 05, 2013; accepted October 10, 2013; posted online March 20, 2014

A graded metallic grating structure acts as a wave trapping system. Different frequencies of THz waves are trapped at different positions along this structured metal surface grating. The real wave propagation speed of such a system is reduced gradually from the light speed in vacuum to zero, which is demonstrated by calculation and simulation. Different frequencies of THz waves are propagated at a designed propagation speed by a partial graded grating according to the practical demand.

OCIS Codes: 230.3120, 130.2790, 240.0240. doi: 10.3788/COL201412.S12301.

Precise control of the properties of light propagation has always been of great importance in the field of optics, and also in terahertz field. Surface plasmon polaritons (SPPs) help in localizing and guiding electromagnetic (EM) waves in sub-wavelength metallic structures, and also help in constructing miniaturized optoelectronic circuits^[1,2]. The plasmonic structure could confine the EM energy within sub-wavelength dimensions over a wide spectral range^[3-6], and this has been a crucial tool to precisely control the EM waves. Different surface structures or material interfaces can cause significant changes in the properties of the transmitted or propagated EM fields^[7,8]. Based on this feature, it is already demonstrated that a graded depth grating structure could confine, focus, slow down, and even stop THz waves along the metal surface^[9,10]. However, the exact propagation speed of the THz waves and how the propagation speed changes during the propagated and trapped process are not defined yet. These problems have been discussed in this paper, and demonstrated using a twodimensional (2D) finite-difference time-domain (FDTD) simulation technique.

It is known that periodically structured metal surface with grooves can achieve sub-wavelength confinement in metal gratings at THz or microwave frequencies because the dispersion relation of SPPs can be engineered according to the requirement by designing the parameters of surface grating. With such a structure, plasmonics enable THz waves used in many promising fields, such as near-field imaging^[11] or biosensing^[12] with unprecedented sensitivity. Basically, such a one-dimensional (1D) grating engraved in the metal surface has three-dimensional parameters: depth (d), width (w), and period (p).

The metallic structures are designed for working in THz and GHz domains. In this frequency region, the metals could be treated as perfect electrical conductors (PEC). Then, the dispersion relation for TM-polarized (*Ex, Ez, Hy*) EM waves propagating in the *x* direction along the surface grating structure can be expressed as^[13]

$$k = \frac{\boldsymbol{\omega}}{c} \sqrt{1 + \frac{w^2}{p^2} \tan^2\left(\frac{\boldsymbol{\omega}}{c}d\right)},\tag{1}$$

where, c is the light velocity in vacuum, k is wave vector, $\boldsymbol{\omega}$ is angular frequency, w, p, d are width, period, depth of the grating structure, respectively. This is a first-order approximation, and just an approximate expression to simplify and illustrate the principle of grating design. In fact, the cutoff frequency estimated from Eq. (1) will be slightly larger than FDTD simulation. In order to attain a more precise prediction, an eigen value equation in the corrugated conducting plane of a periodic system needs to be introduced, and higher-order scattering components should be taken into account^[14].

The dispersion relations for the 1D groove grating with different depths can be obtained by solving Eq. (1), which is shown in Fig. 1(a). The blue line $(d = 60 \text{ }\mu\text{m})$ and red line $(d = 120 \text{ }\mu\text{m})$ in Fig. 1(a) show different dispersion properties while changing the groove depth (d). Figure 1(b) shows the schematic of the graded depth (grating 1) and partial graded depth (grating 2) of metal grating structure. For grating 1, the period of the grating is 50 μ m, the width is 25 μ m; the depth increases from 25 μ m with an increase of 1 µm. For grating 2, the period and width of the grating are same as grating 1; the depth also increases from 25 μ m with an increase of 1 μ m. The difference in the grating depth does not linearly increase to the end. The second part of the grating has a constant grating depth. For example, when the grating depth is increased to 85 µm, it cannot be increased anymore, but fixed to this depth at the grating end. Such a kind of partial graded grating can transmit the EM waves in a designed ultraslow speed.

For $d = 60 \,\mu\text{m}$, the cutoff frequency of the dispersion curve is close to 1.1 THz, which means the corresponding structure is capable of supporting the surface EM modes in the grating structure at such a frequency. For $d = 120 \,\mu\text{m}$, the cutoff frequency of the grating structure is reduced to about 0.6 THz. Therefore, the EM wave at 0.6 THz could be coupled, confined, and propagated along the grating structure. With an increase in the depth, the cutoff frequency of the dispersion curve decreases. The period (p) and width (w) are set at 50 and 25 µm, respectively.

As illustrated by the dispersion relations in Fig. 1(a), the normal surface grating structures can slow down, confine, and propagate the EM waves. In this case, the surface grating structures with 1D periodic sub-wavelength grooves can act as a waveguide for propagating EM modes slowly at various frequencies with specific designed surface gratings^[15,16]. However, it is almost impossible to use a single depth grating as ultra-slow waveguide. As the supported ultra-slow frequency must be close to the corresponding cutoff frequency, the wave vector mismatch (Δk) between the dispersion curve and light line (green dotted line) is too large [Fig. 1(a)]. If the depth of the grating is graded linearly along the whole grating, the dispersion curves of the graded depths differ from those of a constant grating depth. When a graded depth changes from 60 to 120 µm, the dispersion curves will be located between the blue line and red line, as shown in Fig. 1(a). Therefore, the cutoff frequency of the graded structure decreases from 1.1 to 0.6 THz, which means a broadband EM wave within a frequency band of 0.6 - 1.1 THz can be slowed down in such a single-graded depth grating structure, even completely stopped at different locations along the surface grating. The propagation group velocity (v_{e}) can be deduced from Eq. (1), and expressed as Eq. (2):

$$v_g = \frac{d\boldsymbol{\omega}}{dk} = \frac{c}{G + \frac{w^2 d}{G c p^2} \boldsymbol{\omega} \tan^2\left(\frac{\boldsymbol{\omega}}{c}d\right) \sec^2\left(\frac{\boldsymbol{\omega}}{c}d\right)}, \quad (2)$$



Fig. 1. (a) Dispersion curves calculated for different grating depths: $d = 60 \text{ }\mu\text{m}$ (blue line), $d = 120 \text{ }\mu\text{m}$ (red line), light in vacuum (green dotted line). (b) Schematic of a graded depth metal grating 1 and partial graded depth metal grating 2, with width (w), period (p), and depth (d).

where, $G = \sqrt{1 + \frac{w^2}{p^2} \tan^2\left(\frac{\omega}{c}d\right)}$. It can be observed from

Fig. 2 that when the depth is close to 0, the group velocity equals to the light velocity in vacuum. As the grating depth increases, the group velocity slows down (Fig. 2). It drops to 0 at certain depth point, and even attains negative value after that point, which is different for different frequencies. Figure 2 also shows that higher frequencies slow down faster, and stop first at a smaller depth, i.e., frequency of 1.1 THz stops at a depth of 70 μ m, and frequency of 0.6 THz stops at a depth of 120 µm. The graded depth grating structure [grating 1 as shown in Fig. 1(b)] can definitely work as a wave trapper. Except for that, it can also act as an ultra-slow waveguide for different frequencies by just fixing the grating depth when the depth increases to a certain value [grating 2 as shown in Fig. 1(b)]. For example, when the grating depth linearly changes to 85 µm and fixes at this point, the frequency of 0.9 THz (green line in Fig. 2) stops, but the frequency of 0.8THz (black line in Fig. 2) still propagates in a very small group velocity which is about 10^7 m/s (1/30 of light speed in vacuum), which could be reduced further close to zero by using a grating depth of 95 µm. The FDTD simulation demonstrates this further. In order to make the speed calculation more convenient, we take frequency of 0.5 THz as an example because the shorter frequency propagates a longer distance and stops at a far end.

Figure 3 shows FDTD simulation results of grating 1 [Fig. 3(a)–(c)]. The 1D EM intensity ($|E|^2$) distribution of grating 1 shown in Fig. 3(a)–(b) shows the propagation distance of 0.5 THz after a different propagation time. The wave packet of 0.5 THz is propagated to x = -2000 µm after a time t [green line in Fig. 3(a)], and to x = -900 µm after a time $t + t_0$ [blue line in Fig. 3(a)]. The total propagation length in time t_0 is about 1100 µm. The absolute value of t_0 is about



Fig. 2. Calculated group velocities (v_g) of different grating depths (d) at different frequencies: red line, 0.6 THz; blue line, 0.7 THz; black line, 0.8 THz; green line, 0.9 THz; cyan line, 1.0 THz; magenta line, 1.1 THz.



Fig. 3. Results of FDTD simulations. (a) 1D EM field distribution of grating 1 at different times and (b) 2D EM field distribution of grating 1 at the time of $t + 63t_0$.(c) 2D EM field distribution at the time of $t + 63t_0$.

 1.2×10^{-11} s, which is an increased calculation time adding on to the calculation time t. Then, the propagation speed of 0.5 THz equals $1100/(1.2 \times 10^{-11}) =$ $9.2 \times 10^{13} \ \mu m/s$, which is about 0.3 times of light speed in vacuum. This propagation speed gradually reduces as the EM waves propagate further. After a time of $t + 6t_0$ [blue line in Fig. 3(b)] and $t + 31t_0$ [red line in Fig. 3(b)], the wave packet goes to x= 500 μ m, and x = 700 μ m, respectively. The total propagation length in time $25t_0$ is only about 200 μm . Then, the propagation speed equals 200/(1.2) $\times 10^{-11} \times 25$ = 6.7 $\times 10^{11}$ µm/s, which is about 0.002 times of light speed in vacuum. After a time of $t + 31t_0$, the propagation speed already approaches to 0. The wave packet cannot go any further even after a longer time like $t + 63t_0$ [black line in Fig. 3(b)]. And, the real propagation speed can be designed by tuning the grating depth according to the requirement. The micrometer-scale dimensions of the structure for THz waves can be easily realized by current fabrication technologies.

The 2D EM field distribution of grating 1 at time of $t + 63t_0$ is shown in Fig. 3(c). The speed of 0.5 THz becomes 0 and trapped at that position. The simulation time is already far beyond the EM waves propagating time in free space, that is to make sure that the simulated slowing down effect is reliable. The dimension of the simulated region is 5000×800 (µm) with a uniform cell of $\Delta x = 2 \ \mu m$ and $\Delta z = 2 \ \mu m$. The simulated region is surrounded by a perfectly matched layer absorber. These results are based on a 2D FDTD model [Fig. 1(a)] simulation, and clearly confirm that the speed of propagating THz waves is reduced to 0 gradually and stopped at a certain point along the grating 1. Based on this fact, we can design and control the propagation speed of a specific frequency using grating 2. We can surely tune the working frequency range of the grating structure by using another different grating depth. The dispersed pulse of such a structure has a very steady-state distribution, so that the spectrum information could be obtained conveniently. The thickness of the metal is set at 300 μ m. The incident wave is a pulsed source with a centered frequency at 1 THz. The full width at half maximum (FWHM) of the pulse in frequency domain is about 0.4–1.5 THz, so that the structure response of any frequency in this range can be simulated.

It is already demonstrated that grating 1 can be used to slow down and trap the EM wave packet on the surface, and the propagation speed can be calculated



Fig. 4. (a) 1D and (b) 2D EM intensity $(|E|^2)$ distribution along the structured metal surface (grating 2) at a frequency of 0.5 THz.

exactly. If we just need an ultra-slow waveguide, not a wave trapper, then grating 2 will be very useful. Figure 4 shows the EM wave distribution of 0.5 THz along grating 2. Waves of 0.5 THz propagated along the grating 2 after a long simulation time and remained unchanged, confirming that the grating 2 can work as a slow THz waveguide efficiently. It can be observed that the THz wave is not trapped anymore, but propagated to the end of the waveguide in a slow speed. We can calculate the exact propagation speed using the same wave packed following the method shown in Fig. 3, and we also can design the structure of grating 2, like the depth, to control the propagation speed according to the practical demand.

In conclusion, when the grating depth is graded, the dispersion curves of the surface structure dramatically change and become spatially inhomogeneous. Such grating structures are capable of slowing down EM waves within an ultra-wide spectral band along the surface. We are demonstrating a metal surface structure with graded and fixed depth grating supporting ultra-slow THz SPP modes, which is reduced to be nearly 0 according to the real application demand that is almost impossible for a single depth grating. The working frequency is easily widened or shifted by tuning the depth of the grating structure. Such a feature helps exactly control the EM waves on a chip or even realize novel applications, such as signal processing, data storage, chemical diagnostic integrated on a chip.

This work was supported by the Creative Foundation of Tian Jin University under Grant No. 60305019 and NSFC under Grant No. 61275102.

References

- W. L. Barnes, A. Dereux, and T. W. Ebbesen, Nature 424, 824 (2003).
- 2. E. Ozbay, Science 311, 189 (2006).
- J. B. Pendry, L. Martin-Moreno, and F. J. Garcia-Vidal, Science 305, 847 (2004).
- A. P. Hibbins, B. R. Evans, and J. R. Sambles, Science **308**, 670 (2005).
- 5. S. Maier and S. Andrews, Appl. Phys. Lett. 88, 251120 (2006).
- 6. M. B. Johnston, Nat. Photon. 1, 14 (2007).
- 7. B. Guo, G. Song, and L. Chen, Appl. Phys. Lett. 91, 021103 (2007).
- B. Guo, G. Song, and L. Chen, IEEE Trans. Nanotechnol. 8, 408 (2009).
- S. A. Maier, S. R. Andrews, L. Martin-Moreno, and F. J. Garcia-Vidal, Phys. Rev. Lett. 97, 176805 (2006).
- 10. Q. Gan, Z. Fu, Y. J. Ding, and F. J. Bartoli, Phys. Rev. Lett. 100, 256803 (2008).
- H.-T. Chen, R. Kersting, and G. C. Cho, Appl. Phys. Lett. 83, 1 (2003).
- M. Nagel, P. Haring Bolivar, M. Brucherseifer, H. Kurz, A. Bosserhoff, and R. Buttner, Appl. Phys. Lett. 80, 154 (2002).
- F. J. Garcia-Vidal, L. Martin-Moreno, and J. B. Pendry, J. Opt. A: Pure Appl. Opt. 7, S94 (2005).
- K. Zhang and D. Li, *Electromagnetic Theory for Microwaves and Optoelectronics* (Springer-Verlag, Berlin, Heidelberg, 1998).
- 15. Z. Ruan and M. Qiu, Appl. Phys. Lett. 90, 201906 (2007).
- 16. B. Guo, G. Song, and L. Chen, Opt. Commun. 281, 1123 (2008).