The dynamics of spatial bright solitons in nonlocality-controlled nonlinear media

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Nonlocality control is investigated in nonlinear media using material combination with self-focusing and self-defocusing media, and the controlling effects on the propagation and interaction of the spatial solitons are analyzed by numerical simulation. The propagation is stabilized, the interaction is suppressed by the proper material combination, and the effects of the control depend strictly on material maps.

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Spatial solitons have been typically considered in the context of so-called local nonlinear media, where nonlocality of the nonlinearity is a property exhibited by many nonlinear optical materials. In such media, the refractive index change induced by an optical beam in a particular point depends solely on the beam intensity in this point^[1]. Several reviews on the development of the nonlocal spatial solitons have been presented; stable spatial bright (dark) soliton states can exist in self-focused (self-defocused) nonlocal media and Gauss function-like bright soliton states can exist in self-focused strongly nonlocal media^[2,3]. The nonlocality governs the diffusion strength of the refractive index in the nonlocal nonlinear material, and the physical features exhibited by spatial optical soliton propagation can be addressed in the nonlocal media by Gaussian-shaped response and exponential-decay response. The nonlocality satisfies the well-known general power law depending on the incident intensity for local models with competing nonlinearities^[4].

Theoretical results and numerical simulations show that the nonlocality plays an important role in the soliton propagation and interaction, and may become leading limiting factors in the application of nonlinear media using ultra-short pulses (such as solitons); hence, a available means to surmount these difficulties is to control effects of the nonlocality. Being analogous to the dispersion management (control) for the ultra-short pulses^[5–7], advances in media manufacturing techniques have made it possible to incorporate this idea into new optical media waveguides or devices (such as a linear X-junction or coupler) by utilizing this technique of the nonlocality control. In this letter, the effects of the nonlocality control are investigated on the propagation and interaction of the spatial solitons in nonlocal nonlinear media, and some novel results are obtained.

Media are considered with nonlinearity, i.e., a nonlocal function of the incident field, propagation of wave packet along the z-axis within the nonlocal nonlinear media, and the envelope of the wave packet can be described by the nonlinear Schrödinger equation^[1–3]

$$j\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \eta \int R(x - x') \ I(x') dx' \ u = 0, \qquad (1)$$

where, u is normalized wave packet function; z and x are distance coordinate and transverse coordinate, respectively; η is a material constant; $\eta = 1$ corresponds to the self-focusing media; and $\eta = -1$ corresponds to the self-defocusing media; $I(x) = |u|^2$ is the pulse intensity; and R(x) is the normalized symmetric response function. The nonlinear contribution to the refractive index n(x) can be given by

$$n(x) = \int R(x - x') \ I(x') dx',$$

$$n(x) - \rho \frac{\partial^2}{\partial x^2} n(x) = |u|^2.$$
(2)

The refractive index can be expanded through the expansion of the weak nonlocality $^{[8]}$

$$\begin{split} n(x) &\approx \left(1 + \rho \frac{\partial^2}{\partial x^2} + \rho^2 \frac{\partial^4}{\partial x^4} + \right. \\ \rho^3 \frac{\partial^6}{\partial x^6} + \rho^4 \frac{\partial^8}{\partial x^8} \right) \left. \left| u \right|^2, \end{split}$$
(3)

where, ρ is the nonlocality degree that governs the diffusion strength of the refractive index. Substituting Eqs. (2) and (3) into Eq. (1) results in following evolution equation

$$j\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + \eta \left(1 + \rho \frac{\partial^2}{\partial x^2} + \rho^2 \frac{\partial^4}{\partial x^4} + \rho^3 \frac{\partial^6}{\partial x^6} + \rho^4 \frac{\partial^8}{\partial x^8}\right) |u|^2 u = 0.$$

$$\tag{4}$$

In our calculation model, the nonlocality control consists of self-focusing media ($\eta = 1$) and the self-defocusing media ($\eta = -1$). In order to discuss the correlation between material map and the dynamics of the soliton propagation and interaction, four material maps are used, which are as follows:

map (a) : $\eta = 1$ (corresponding to the self-focusing media) along the z axis; map (b) : $\eta = -1$ (corresponding to the self-defocusing media) along the z axis; map (c) : $\eta = 1$ and $\eta = -1$ for each equal medium segment along the z axis by turns, and the average material constant is zero over the total segments; map (d) : $\eta = -1$ and $\eta = 1$ for each inequal medium segment along the z axis, and the average material constant is given





Fig. 1. The nonlocality-controlled sketch of four material maps described by the two material constants along the z-axis. $\eta = 1$ corresponds to the self-focusing media and $\eta = -1$ corresponds to the defocusing media.

Figure 1 shows the nonlocality-controlled sketch of four material maps described by two material constants along the z-axis.

The motion Eq. (4) can be solved numerically to investigate the propagation and interaction characteristics of the spatial bright solitons. Figure 2 shows the normalized one- or three-soliton intensities versus the propagation distance with the material map (a) in Fig. 1. The initially incident one- or three-soliton pulses are $u(x, z = 0) = \sec h(x)$ or $u(x, z = 0) = \sec h(x + \Delta) + \sec h(x + \Delta)$ $h(x) + \sec h(x - \Delta)$, where, Δ is the separation between two neighboring solitons, and $\Delta = 0$ (about 6 times of the initial soliton width) is used. We can observe here that the solitons can propagate stably a very long distance when there is no nonlocality ($\rho = 0$). The nonlocality plays an important role in the evolution of the soliton in the self-focusing media ($\eta = 1$). For example, the effects of the nonlocality on the propagation of the bright soliton are obvious under the very weak nonlocality ($\rho = 0.15$); and the soliton cannot propagate stably a long distance and it cannot stably propagate under the nonlocality ($\rho = 0.5$). It is well-known that when initial separation between solitons is larger than five times the soliton width, the interaction between neighboring solitons can be suppressed effectively in the general soliton system^[9,10]. Also, the effects of the nonlocality on the interaction of the solitons are obvious under the weak nonlocality, the solitons cannot stably propagate under the weak nonlocality ($\rho = 0.5$), and the nonlocality enhances the interaction in the self-focusing media ($\eta = 1$).

Figure 3 shows the normalized one- or three-soliton intensities versus the propagation distance with the material map (b) in Fig. 1. The initially incident oneor three-soliton pulses are the same as in Fig. 2. The solitons cannot propagate even if there is no nonlocality ($\rho = 0$); the effects of the nonlocality on the propagation of the soliton are obvious under the very weak nonlocality ($\rho = 0.15$), and the soliton can propagate nearly a long distance under the weak nonlocality ($\rho = 0.5$). The nonlocality plays an important role in the interaction between the neighboring solitons, and the



Fig. 2. The normalized one- or three-soliton intensities versus the propagation distance with the material map (a) in Fig. 1: (a) $\rho = 0$; (b) $\rho = 0.15$; and (c) $\rho = 0.5$.



Fig. 3. The normalized one- or three-soliton intensities versus the propagation distance with the material map (b) in Fig. 1: (a) $\rho = 0$; (b) $\rho = 0.15$; and (c) $\rho = 0.5$.



Fig. 4. The normalized one- or three-soliton intensities versus the propagation distance with the material map (c) in Fig. 1: (a) $\rho = 0$; (b) $\rho = 0.15$; and (c) $\rho = 0.5$.

interaction is partly suppressed in the self-defocusing media ($\eta = -1$) under the weak nonlocality ($\rho = 0.5$).

Figure 4 shows the normalized one- or three-soliton intensities versus the propagation distance with the material map (c) in Fig. 1. Owing to the different nonlocality with the material map, the nonlocality-controlled solitons need a larger energy relative to the energy of fundamental solitons in a uniform medium with the same value of the general nonlocality, resulting in an energy enhancement. The initial input energy-enhanced soliton pulses are $u(x, z = 0) = 2.5 \times \sec h(x)$ or u(x, z = 0) $= 2.5 [\operatorname{sec} h(x + \Delta) + \operatorname{sec} h(x) + \operatorname{sec} h(x - \Delta)], \text{ and } \Delta = 10$ is used. In this situation, the nonlocality control consists of the self-focusing media ($\eta = 1$) and the self-defocusing media $(\eta = -1)$ with equal medium segments along the z-axis, and the average effective material constant is zero over the total segments. We can observe that the effects of the controlled nonlocality on the soliton propagation and interaction are advantageous, and the solitons can propagate stably a long distance in the combined media. The spatial soliton shape can be recovered and the interaction be suppressed partly by the material combination.

Figure 5 shows the normalized one- or three-soliton intensities versus the propagation distance with the material map (d) in Fig. 1. The initial incident one or three solitons are the same as in Fig. 4. In this situation, the nonlocality control consists of the self-focusing media ($\eta = 1$) and the self-defocusing media ($\eta = -1$) with unequal medium segments along the z-axis, and the average effective material constant is 1/3 over the total segments (the length of the self-focusing medium is two times that of the self-defocusing medium). The solitons can propagate stably in the quasi-focusing media (the corresponding average material constant is positive along the z-axis). The effects of the controlled nonlocality are perfect on soliton propagation and interaction over those with the material maps (a), (b) and (c). For example, the propagation can be stabilized and the interaction be suppressed entirely by the proper material combination as if there is no nonlocality in the map (d) for the arbitrary nonlocality. From Figs. 2–5, we can observe that the controlled nonlocality



Fig. 5. The normalized one- or three-soliton intensities versus the propagation distance with the material map (d) in Fig. 1: (a) $\rho = 0$; (b) $\rho = 0.15$; and (c) $\rho = 0.5$.

improves the physical features of the solitons, and the effects strictly depend on the material maps.

In summary, the nonlocality control in nonlinear media are investigated using the material combination with self-focusing and self-defocusing media, and the controlling effects on the propagation and interaction of the spatial solitons are analyzed. The solitons cannot propagate a long distance under the weak nonlocality in the self-focusing media, but the solitons can propagate nearly a long distance under the weak nonlocality in the self-defocusing media, and the nonlocality affects the interaction. The controlled nonlocality improves physical features of the solitons, and the effects strictly depend on the material maps. Numerical simulation is undertaken to show that the propagation can be stabilized, and the interaction be suppressed by the proper material combination as if there is no nonlocality for the arbitrary nonlocality. The results show that there may be a probability to reduce the disadvantageous effects by the material combination with self-focusing and self-defocusing media.

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