## A planar hyperlens-based device for super-resolution magnification imaging in a far field

Jun Cui (崔 钩), Guoxing Zheng (郑国兴)\*, Pingan He (何平安), Jiangnan Zhao (赵江南), Jianping Yun (鄧建平), Jinling Yang (杨晋陵), and Song Li (李 松)

School of Electronic Information, Wuhan University, Wuhan 430072, China \*Corresponding author: gxzheng@whu.edu.cn

Received September 22, 2013; accepted October 11, 2013; posted online February 28, 2014

A planar-hyperlens-based imaging device is presented in this paper. Based on the structure of hyperbolic dispersion metamaterial and with the ability of collecting the evanescent waves from the object, the planar hyperlens can deliver and magnify the super-resolution details of a planar object to the extent that a traditional microscopic objective can resolve them. The super-resolution magnification imaging principle of the device was analyzed, and the relations of the imaging resolution and magnification with the structure parameters of the device were deduced. With careful design, the effectiveness of the device was confirmed in a series of numerical simulations.

OCIS codes: 160.1190, 260.3910, 350.3950. doi: 10.3788/COL201412.S11601.

Evanescent waves, which represent an object's detailed information below the diffraction limit, cannot propagate into the far field because of its rapid attenuation in an isotropic medium. As a result, the resolution of conventional optical lens can never reach to half of the operating wavelength<sup>[1]</sup>. In order to improve the image resolution, scientists have proposed a series of various superlenses<sup>[1-5]</sup>, in which the evanescent waves can propagate to the image plane and then attend to the imaging process. Of all the schemes, the most eye-catching one is the hyperbolic lens based on the cylindrical or spherical surface of the multilayer metal structure<sup>[6-13]</sup>. The</sup> hyperbolic lens or hyperlens, named from its material satisfying hyperbolic dispersion curves, can deliver the object details to far field and realize a super-resolution magnifying imaging. Therefore, that kind of lens is expected to be applied in some fields such as biomedical real-time observation. However, the object and image plane of the hyperbolic lens are both curved surface, which set obstacle to object surface locating and image surface observation. Around this problem, a planar hyperlens that can perform a plane to plane imaging was put forward by Wang and others using coordinate transformation method in 2008<sup>[14]</sup>. However, for its nonuniform thickness of its metal and dielectric layers, this kind of planar hyperlens has difficulties both in design and fabrication. In this letter, we propose an alternative planar hyperlens that consists of uniform thickness and spherical-structure multilayers. We prove both in theoretical analysis and numerical simulations that this planar-hyperlens-based device can amplify the super-resolution details of a planar object with constant imaging magnification ratio at any imaging fields. If this device can be manufactured well, we hope it can be applied in real-time biomedical imaging and other potential fields.

The optical properties of a material should be considered firstly in a hyperlens design. A planar lens consisting of isotropic homogeneous material cannot amplify an object, even for lens with negative refractive index material<sup>[1]</sup>. Like traditional imaging system with a positive or negative lens, the chief rays in a planar hyperlens determine the imaging positions, and then the system magnification ratio, as seen in Fig. 1. In Fig. 1, if the chief ray from a source point like A or B in a plane can follow a propagating way like AA' or BB', the object then can be amplified from plane ABto plane A'B'. Traditional isotropic material cannot do this obviously; however, a properly designed anisotropic material can support the chief rays follow this way in a spherical coordinate system.

In a spherical coordinate system, the anisotropic material meeting the dispersion equation for transverse magnetic (TM) wave:

$$\frac{k_r^2}{\varepsilon_{\theta}} + \frac{k_{\theta}^2}{\varepsilon_r} + \frac{k_{\phi}^2}{\varepsilon_r} = k_0^2, \qquad (1)$$



Fig. 1. Diagram of a planar hyperlens combined with a traditional microscopic objective achieving a far-field magnification imaging beyond diffraction-limit.

where  $(r, \theta, \phi)$  is the spherical coordinate triplet; meanwhile  $\mathcal{E}_{r}, \mathcal{E}_{\theta} = \mathcal{E}_{\varphi}$  are defined as the permittivity in radial and tangential directions. If the permittivities satisfy  $\varepsilon_r \cdot \varepsilon_{\theta} < 0$ , hyperbolical dispersion relations are built, as seen in Fig. 2. In the ideal case that  $\mathcal{E}_{\theta}$  is close to zero, the hyperbolical curves are extremely flat and all Fourier components' Poynting vectors S (direction of rays) will travel along the  $k_{\rm c}$  direction, which can be seen from formula (1) and Fig. 2. In such a case, rays emitted from a source point A will follow only AA' direction as mentioned above, and the same to the source point B. On the other hand, because of the magnification of the planar lens, the evanescent waves with tangential wave vectors exceeding  $k_0$  can be compressed smaller than  $k_0$  (wave vector in the vacuum) during the propagation in the lens, so the detail of the object could be delivered to the far field and resolved by traditional microscopic objective shown in Fig. 1.

Such an anisotropic material can be realized by assigning alternating spherical in and uniform-thickness layers of metal with permittivity  $\varepsilon_m$  and dielectric with permittivity  $\varepsilon_d^{[4-13]}$ , which satisfy

$$\varepsilon_{\theta} = \frac{\varepsilon_m + \varepsilon_d}{2}, \text{ and } \varepsilon_r = \frac{2\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}.$$
(2)

Some careful chosen noble metal and transparent dielectric material can satisfy  $\varepsilon_r$ .  $\varepsilon_{\theta} < 0$  and  $\varepsilon_{\theta} \rightarrow 0$ . For example, silver (Ag) ( $\varepsilon_m = -3.12 + 0.21i^{[15]}$ ) and aluminum oxide (Al<sub>2</sub>O<sub>3</sub>) ( $\varepsilon_d = 3.21^{[9]}$ ) at the wavelength of 375 nm can form an anisotropic material with real part of the permittivity  $\varepsilon_{\theta}$  being only 0.045 and  $\varepsilon_r$  being around -29, calculated by formula (2).

The resolution of the device is related to the magnification of the planar hyperlens. According to Fig. 1, the magnification can be expressed by a simple formula as

$$M_1 = (L_1 + L_2)/L_1, (3)$$

where  $L_1$  and  $L_2$  are defined in Fig. 1. Firstly, we assume that the maximum transverse wave vector, which can spread from the object surface of the planar hyperlens to the image surface, is  $k_{\text{max}}$ . According to the law of conservation of angular momentum<sup>[6]</sup>,  $\beta_1 k_{\text{max}} = \beta_2 k_0 NA$  is satisfied, where NA is the numerical aperture of the



Fig. 2. A hyperbolic dispersion curve can be built when  $\mathcal{E}_r$  .  $\mathcal{E}_\theta \! < \! 0$  is satisfied.

microscopic objective and  $\beta_1$  and  $\beta_2$  are the resolutions in an object and image plane, respectively. Accordingly, the maximum transverse wave vector delivered from object surface to image surface is  $k_{\text{max}} = (\beta_2 N A / \beta_1) k_0$  $= M_1 k_0 N A$ , and the far-field imaging resolution of the device is

$$\boldsymbol{\beta}_{1} = \frac{2\pi}{2k_{\max}} = \frac{\lambda_{0}}{2M_{1}NA}, \qquad (4)$$

which exceeds the diffraction limit by factor  $1/M_1$ .

To design such a device, we firstly should set some technical parameters such as the working wavelength  $\lambda_0$ , resolution  $\beta_1$ , planar hyperlens thickness  $L_2$  and observing range  $S_1$  (object height in a one-dimension condition). Secondly, we should choose a proper microscopic objective with numerical aperture NA and resolution  $\beta_2 = \lambda_0/2NA$ . With above parameters, we then can determine the planar hyperlens' parameters, such as the magnification  $M_1 = \beta_2/\beta_1$ , imaging height  $S_2 = S_1M_1$  and distance  $L_1 = L_2/(M_1-1)$ . The working wavelength  $\lambda_0$  also determines the metal and dielectric materials used in the planar hyperlens. In addition,  $S_2$  determines the view angle of the microscopic objective. We summarize the design diagram of the proposed device as Fig. 3.

According to the above steps, a full-wave two-dimensional electromagnetic simulation with Finite Element Analysis software COMSOL 4.2 was performed to validate these conclusions. Corresponding to the two points A and B shown in Fig. 1, the sources with dimension of 50 nm are two holes separated by 100 nm (desired resolution of the device), which is smaller than the diffraction limit at the wavelength of 375 nm. The planar hyperlens, which consists of alternating silver and aluminum oxide mentioned above with each layer's thickness of 10 nm, is designed with parameters  $L_1$  of 50 nm,  $L_{\rm 2}$  of 800 nm, and the corresponding magnification of 17. Theoretically, the height of source image at the exit port of the planner hyperlens is 850 nm. A microscopic objective was designed in the simulation test with numerical aperture of 0.1, focal length of 4 µm and magnification of 1.4, which was used to deliver the super-resolution details to a far field.



Fig. 3. Design diagram of a planar-hyperlens-based imaging device with input parameters.



Fig. 4. Images at the entrance and exit port of the planar hyperlens and image plane of microscopic objective.

We simulate the power flow distributions at the entrance of the planar hyperlens (position of plane ABof Fig. 1), the exit port of the planar hyperlens (position of plane A'B' of Fig. 1), and the image plane in the cross-section direction (position of plane A''B'' of Fig. 1). The simulation results are shown in Fig. 4. We can see that an evanescent planar detail with characteristic dimension of 100 nm  $(\lambda_0/3.75)$  can be magnified by hyperlens, and these "second images" are in propagating modes because they have be delivered to the far-field seen in the triangle curve in Fig. 4, and thus the super-resolution far-field magnification imaging is realized. The device could also be used conversely to perform de-magnification super-resolution imaging, which can be used in nanolithography and optical data storage.

In conclusion, we report an innovative device consisting of the above planar hyperlens and microscopic objective are proposed to magnify the super-resolution object details and deliver them to the far field in a plane-to-plane mode instead of traditional sphere-to-sphere mode. This planar hyperlens is with uniform thickness and spherical structure multilayers. If it can be fabricated well, we hope the proposed device can be used in real-time biomicroscope imaging and other potential fields.

This work is supported by the National Natural Science Foundation of China under Grant Nos 10904118, 11374235, 61007024, and 61271150.

## References

- 1. J. B. Pendry, Phys. Rev. Lett. 85, 3966 (2000).
- X. Z. Chen, L. L. Huang, H. Mühlenbernd, G. X. Li, B. F. Bai, Q. F. Tan, G. F. Jin, C. W. Qiu, S. Zhang, and T. Zentgraf, Nat. Commun. 3, 1198 (2012).
- 3. X. Zhang and Z. W. Liu, Nature 7, 435 (2008).
- C. T. Wang, Y. H. Zhao, D. C. Gan, C. L. Du, and X. G. Luo, Opt. Express 16, 4217 (2008).
- N. Yao, Z. Lai, L. Fang, C. T. Wang, Q. Feng, Z. Y. Zhao, and X. G. Luo, Opt. Express **19**, 15982 (2011).
- Z. Jacob, L. V. Alekseyev, and E. Narimanov, J. Opt. Soc. Am. A 24, 52 (2007).
- 7. A. Salandrino, and N. Engheta, Phys. Rev. B 74, 075103 (2006).
- Z. Liu, H. Lee, Y. Xiong, C. Sun, and X. Zhang, Science 315, 1686 (2007).
- G. X. Zheng, R. Y. Zhang, S. Li, P. A. He, H. Zhou, and Y. Shi, IEEE Photonic. Tech. Lett. 23, 1234 (2011).
- J. N. Zhao, G. X. Zheng, S. Li, H. Zhou, Y. Ma, R. Y. Zhang, Y. Shi, and P. A. He, Chin. Opt. Lett. **10**, 042301 (2012).
- W. Wang, H. Xing, L. Fang, Y. Liu, J. X. Ma, L. Lin, C. T. Wang, and X. G. Luo, Opt. Express 16, 21142 (2008).
- M. B. Pu, C. G. Hu, C. Huang, C. T. Wang, Z. Y. Zhao, Y. Q. Wang, and X. G. Luo, Opt. Express 21, 992 (2013).
- 13. G. X. Zheng, R. Y. Zhang, S. Li, P. A. H, and H. Zhou, Chin. Phys. B 20, 117802 (2011).
- 14. W. Wang, H. Xing, L. Fang, Y. Liu, J. X. Ma, L. Lin, C. T. Wang, and X. G. Luo, Opt. Express 16, 21142 (2008).
- 15. P. B. Johnson and R. W. Christy, Phys. Rev. B 6, 4370 (1972).