Modified local mean decomposition algorithm for adaptive analysis of fringe pattern

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We propose modified local mean decomposition to analyze signals of fringe pattern adaptively. It decomposes the signals into a set of functions, each of which is the product of amplitude signal and frequency signal. Then, the physical components of noise and background are extracted according to the corresponding product functions. Moreover, to solve the most likely mode mixing problem, a high-frequency signal is constructed according to the amplitude and frequency characteristics of the intrinsic noise. Using the presented method, carrier signals are recovered accurately as well as the wrapped phase. Compared experiments illustrate the validity of this method.

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Fringe analysis is a critical issue that is worthy of study in optical three-dimensional measurement. If noise, carrier signals or background of fringe pattern can be recovered accurately with fringe analysis, phase retrieval can be achieved accurately and easily. Traditional Fourier-based methods of fringe analysis include S transform[1], windowed Fourier transform[2], etc. These methods are constrained by Heisenberg uncertainty theory and limited to process the fringe pattern with slow change in phase. Empirical mode decomposition (EMD) is another kind of fringe analysis method unconstrained by the above limitation. It has been testified to be effective in detrending[3], denoising[4] or other such fringe analyses[5]. However, EMD subjects to the restriction of sampling or interpolation, which leads to over-decomposition if selected improperly. In addition, mode mixing is another tricky problem of EMD, which is caused by the intermittent noise. The existing of mode mixing may confuse the physical meaning of the obtained intrinsic mode function (IMF). To address this problem, the ensemble EMD (EEMD) method is introduced and even being expanded in bi-dimensional space to analyze fringe pattern[6]. EEMD shifts the ensemble of white noise-added signal and treats the mean as final result[7]. However, this method is empirical, as the ensemble number and the white noise have to be selected manually. Moreover, a large number of iterations make EEMD very time-consuming and unsuitable to use practically[6].

Local mean decomposition (LMD) is a novel algorithm, which owns good demodulation capability in analyzing complicated signals[8]. But, LMD is also restricted by the mode mixing problem. In this paper, we introduce and modify the LMD method for adaptive fringe analysis. The smoothed local mean of LMD outperforms the cubic spline interpolation of EMD in reducing iterations and restraining border effect. A high-frequency signal is designed and added into the original signals to form an envelope of continuous extrema during the iteration, and then the original noise and the added signal both may be separated out without other mixed modes. With the present modified LMD (MLMD), the physical meaning of each separated product function (PF) is much clear. The carrier signals can be extracted accurately, which contribute further to accurate phase retrieval.

One row signal of fringe pattern is expressed as

$$I(x) = A(x) + B(x)\cos[2\pi f_0 x + f(x)] + n(x), \tag{1}$$

where $A(x)$ represents the background, $B(x)$ the amplitude modulation, f_0 and $\phi(x)$ are the modulated frequency and phase, respectively, and $n(x)$ is noise. Equation (1) shows that the signal is amplitude–frequency modulated, and this characteristic exactly conforms to the decomposition mechanism of LMD. The details of LMD can be obtained from a study conducted by Smith[8]. After LMD, the signal can be decomposed into a set of PFs which are sorted from high frequency to low, representing the noise, carrier signals and background orderly. LMD is an iterating process based on analysis of extrema of signals. The mode mixing problem appears when the extrema of intermittent noise and fundamental components are mixed[7]. Moreover, mode mixing in the first two PFs gradually affects the following PFs. To overcome this problem, we have designed a high-frequency signal and added it as "noise" into original signals in order to uniformize all the noise. The high-frequency "noise" $n'(x)$ is designed as

$$n'(x) = \bar{a}_1 \cos 2\pi(2\times \bar{f}_1)x,$$
$$\left(n\bar{a}_1 = \frac{\sum a_1^2(x)}{\sum a_1(x)} n\bar{f}_1 = \frac{\sum a_1(x) f_1^2(x)}{\sum a_1(x) f_1(x)} n\right), \tag{2}$$

where $a_1(x)$ and $f_1(x)$ are the amplitude-modulated (AM) function and frequency-modulated (FM) function of PF_1, respectively. $\bar{a}_1$ is the weighted mean of $a_1(x)$, $\bar{f}_1$ is the approximation of $f_1(x)$ got by energy-weighted mean. $\bar{f}_1$ is multiplied by 2 to ensure that the frequency of $n'(x)$ is at least twice larger than the frequency of fundamental components, because in a study conducted by Rilling and Flandrin[9], a cutoff frequency ratio of two-tones signal is analyzed to ensure the two tones being separated thoroughly by EMD, i.e., the frequency ratio cannot be between [0.5] and [2]. The conclusion is also suitable for LMD because it is still an extrema-based iterative approach.

The procedure of MLMD can be summarized as follows:

1. Apply LMD to original signal $I(x)$ to get the PF_1:

① Between each two adjacent extrema of $I(x)$, calculate their mean value denoted as $m_{pk}(x)$ and the average of their peak-to-peak value as $a_{pk}(x)$, where p denotes the number of PFs and k is the number of iterations to get a $\mathrm{PF}(x)$.

② With moving average method, $m_{pk}(x)$ and $a_{pk}(x)$ are smoothed to get $\tilde{m}_{pk}(x)$ and $\tilde{a}_{pk}(x)$, respectively. The length of the moving average is weighted by the largest distance of two adjacent extrema.

③ Let $h_{11}(x) = I(x) - \tilde{m}_{11}(x)$ and $s_{11}(x) = h_{11}(x) / \tilde{a}_{11}(x)$. Compute $\tilde{a}_{12}(x)$ of $s_{11}(x)$ and judge whether $\tilde{a}_{12}(x) = 1$. If so, go to step ④. Otherwise, repeat steps ① – ② k_1 times until $s_{1k_1}(x) \quad h_{1k_1}(x) / a_{1k_1}(x)$ is a pure flat FM signal, i.e., $s_{1k_1}(x)$ fluctuates between [-1] and [1]. And $s_{1k_1}(x)$ can be written as $s_{1k_1}(x) = \cos\omega_1(x)$, where, $\omega_1(x)$ is the instantaneous phase. Also, the instantaneous frequency is obtained by $f_1(x) = d\omega_1(x) / dx$. Then, the instantaneous amplitude can be obtained as $a_1(x) = \tilde{a}_{11}(x)\tilde{a}_{12}(x)\ldots\tilde{a}_{1k_1}(x)$.

④ Multiply the AM function and the FM function, the first PF is obtained as $\mathrm{PF}_1(x) = a_1(x) \times s_{1k_1}(x)$, where k_1 is the iteration number.

2. Construct the high-frequency signal by Eq. (2) and add it to $I(x)$ to get the new signal $I_{new}(x)$.

3. Take $I_{new}(x)$ as $I(x)$ and repeat steps ① – ④, so a new first PF is obtained as $\mathrm{PF}_1^{new}(x)$. Let $r_1(x) = I_{new}(x) - \mathrm{PF}_1^{new}(x)$, and $r_1(x)$ is a smoothed version of $I_{new}(x)$. Repeat the similar procedure using steps ① – ④ on $r_1(x)$ to get $\mathrm{PF}_2(x)$. Let $r_2(x) = r_1(x) - \mathrm{PF}_2(x)$ and repeat steps ① – ④ on $r_2(x)$ to get $\mathrm{PF}_3(x)$, etc. The iteration is continued until the residue $r_p(x)$ is a monotonic function with no more oscillations. Finally, $I_{new}(x)$ can be reconstructed by

$$I_{new}(x) = \mathrm{PF}_1^{new}(x) + \sum_{j=2}^{p} \mathrm{PF}_j(x) + r_p(x).$$

Figure 1 shows the simulation (1020 × 1020) of fringe pattern. The modulated frequency is 0.05 (1/pixel) and the background is simulated as constant 0.5. The fringe is modulated 7 times of the peaks function in order to deform more seriously. Random noise is added into the

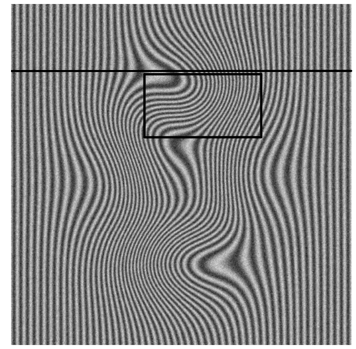

Fig. 1. Simulation of fringe pattern.

fringe pattern and the SNR is 13.2 dB. The straight line in Fig. 1 denotes the 200th row signal and the black box shows a random local area.

To prove the performance of MLMD, we process the 200th row signal with EMD, LMD, EEMD and MLMD. Figure 2(a) shows the original signal. Figures 2(b) and (c) are the results of EMD and LMD, respectively. We can observe that mode mixing appears in both the methods, but the number of IMFs in Fig. 2(b) is much more than the number of PFs in Fig. 2(c), which means that the effect of mode mixing is much heavier in EMD than LMD. Figure 2(d) shows the result of EEMD whose ensemble number is 200, and the ratio of standard deviation between the white noise and original signal is 0.25. The result of MLMD is shown in Fig. 2(e). It shows that mode mixing is relieved not only in high-frequency PFs but also in low-frequency PFs.

The carrier signal of the 200th row is recovered by removing the first component and the residue for each result in Fig. 2. The recovered result is compared with the true value, which is shown in Fig. 3. EEMD and MLMD perform better than the other two methods,

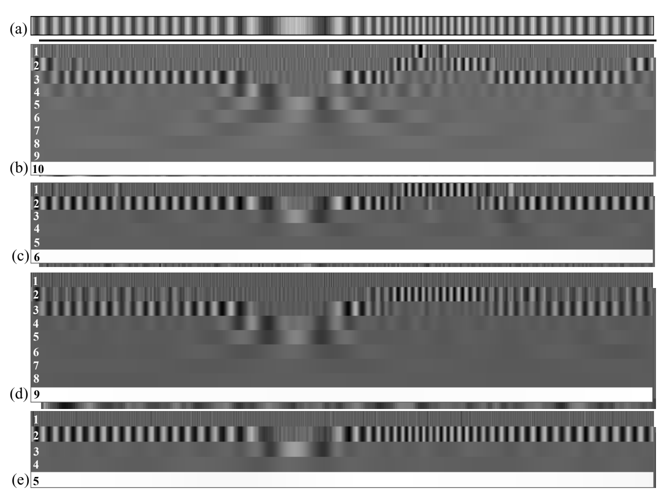

Fig. 2. Image description of (a) the 200th row signals in Fig. 1, (b) IMFs obtained by EMD, (c) PFs obtained by LMD, (d) IMFs obtained by EEMD and (e) PFs obtained using the proposed MLMD.

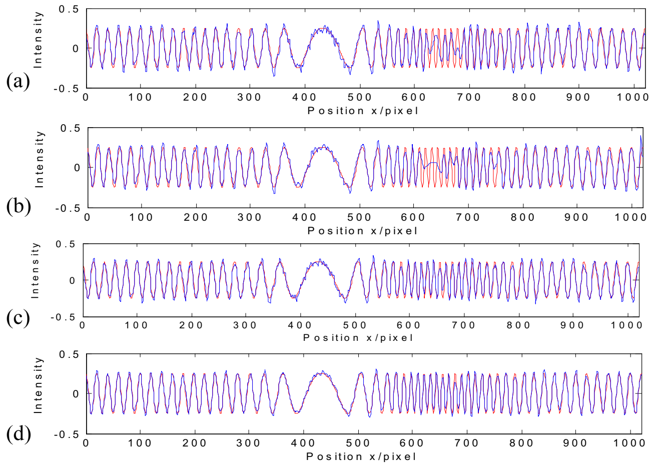

Fig. 3. Comparison of the carrier signals between the true values (red line) and the result obtained (blue line) using (a) EMD, (b) LMD, (c) EEMD and (d) MLMD.

while noise remains high as shown in Fig. 3(c). We also recover the carrier signals of whole fringe pattern. And, the obtained result and the ideal values are subtracted to estimate the errors. The mean error and the standard deviation of EEMD are –0.0015 and 0.0376, while for the MLMD, they are 9.4×10^{-4} and 0.0388.

We retrieve the wrapped phase with Hilbert transform on the recovered carrier signals. Figure 4 shows the wrapped phase of the local area in Fig. 1. For comparison, we add another two common methods in phase demodulation, i.e., the Fourier transform (FT) method and the ridge of wavelet transform (RWT) method. The results of these two methods are inferior to the other four methods because they are weak in processing the fringe pattern which deforms dramatically, as shown in Fig. 1. The wrapped phase is unwrapped by quality-guided method and the errors are computed as the difference of the obtained phase and the true phase. The resulting data prove that EEMD and MLMD are much better among these methods, which fits the result shown in Fig. 4. The mean error and standard deviation of errors are –0.0036 and 0.2383 for EEMD, respectively, and –0.0032 and 0.2388 for MLMD, respectively. Both the methods are accurate, but MLMD is more automatic and time-saving compared with EEMD, where EEMD needs about 4 hours to process the whole pattern (1020×1020) while MLMD just requires 7 minutes with the same computer. The reason is that EEMD needs addition of white noise hundreds of times, which means decomposition using the EMD method hundreds of times, but MLMD just needs only one time adding of the designed "noise".

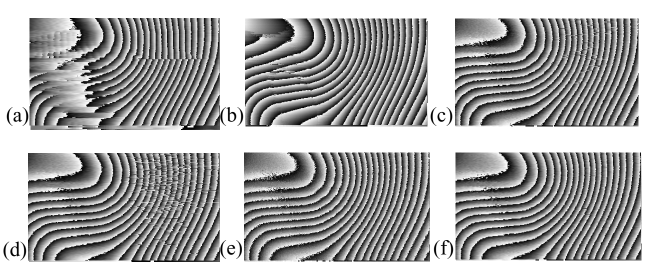

Fig. 4. The wrapped phase of local area obtained by (a) FT, (b) RWT, (c) EMD, (d) LMD, (e) EEMD and (f) MLMD.

Real experiment is performed on a fringe pattern of foam board in Fig. 5(a), where the straight line denotes the 200th row. The carrier signals of the whole map are recovered adaptively, which is shown in Fig. 5(b). It shows that the detailed information is maintained well. The 200th row of original signal and the denoised signal obtained by MLMD are compared in Fig. 5(c), which indicates the good denoising ability of the method. Figure 5(d) is the carrier signal with noise and background removed. A comparison of the wrapped phase is also given in Fig. 6. The presented MLMD

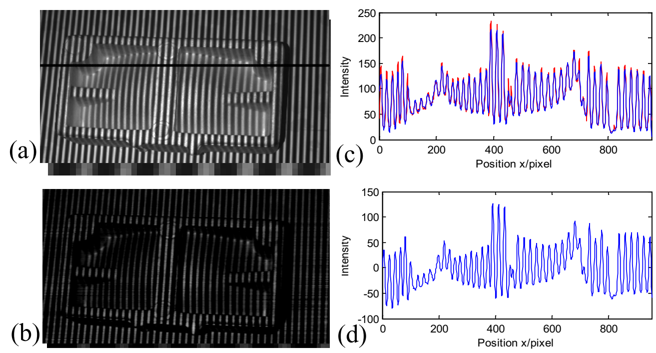

Fig. 5. (a) The fringe pattern of foam board, (b) the whole map of recovered signal, (c) comparison of the 200th original signal (red line) and the denoised signal (blue line) by MLMD and (d) the recovered signal of the 200th row.

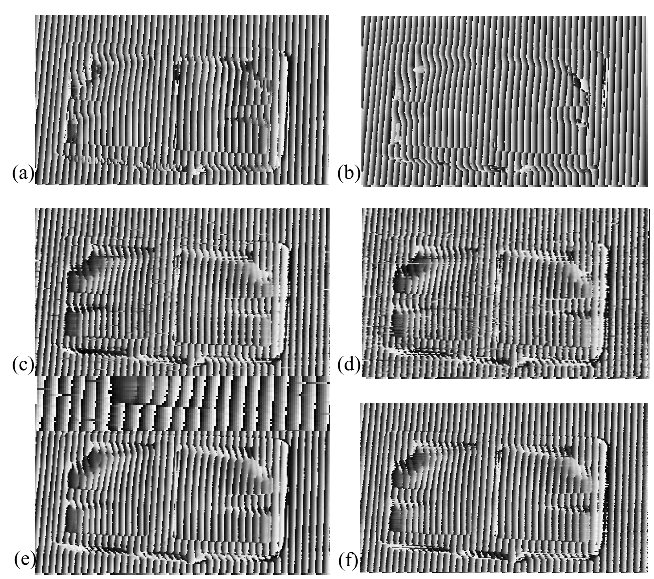

Fig. 6. The wrapped phase obtained by (a) FT, (b) RWT, (c) EMD, (d) LMD, (e) EEMD and (f) MLMD.

exhibits the good performance on accuracy, processing speed, etc. In the present method, the definition of instantaneous frequency and amplitude are much reasonable. The designed signal results in the similar character with the original noise, so all the "noise" is collected together to be completely separated as $\mathrm{PF}_1^{new}(x)$. But, it is noteworthy that if noise is so small to be ignored, step 2 is not needed. This case is rare. But if so, the PF_1 represents the fundamental component purely and carrier signals can be recovered only by removing the background, namely the residue.

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