Multibeam laser heterodyne measurement with ultraprecision for electrostriction coefficient based on an oscillating mirror modulation

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Received September 25, 2013; accepted October 9, 2013; posted online February 27, 2014

This paper proposes a novel method of multibeam laser heterodyne measurement for an electrostriction coefficient. Based on the Doppler effect and heterodyne technology, loaded with the information of length variation to the frequency difference of the multibeam laser heterodyne signal by the frequency modulation of the oscillating mirror, this method can obtain many values of length variation caused by different voltages after the multibeam laser heterodyne signal demodulation simultaneously. Processing these values by a weighted-average method, it can obtain length variation accurately, and eventually obtain value of electrostriction coefficient of metal by the calculation. This novel method is used to simulate measurement for electrostriction coefficient of PZT under different voltages by MATLAB, and the obtained result shows that the relative measurement error of this method is just 0.98%.

OCIS codes: 040.2840, 120.4290, 160.3900.

doi: 10.3788/COL201412.S10401.

In all cases involving electrical and mechanical systems and devices that are of automatic control, the drive is often considered as one of the most crucial factors to limit its performance and lifetime, and among many types of drives, piezoelectric/electrostrictive drive for its quick-response high capacity, low energy consumption and low prices has attracted much attention. At present, the piezoelectric/electrostrictive drive has been successfully applied in the laser resonator, precision positioning, precision machining, smart structures, bioengineering, aerospace, telecommunications, automotive, robot joins, medical devices and many other technical fields, and it is forming a potential industry. Therefore, the increasingly widespread attention is paid on the development and application of the new technology. In nature, most of the crystal has piezoelectric effect[1–3]; however, most of them are very weak, so there is no practical value. Quartz is the good performance of piezoelectric crystal material. With the development of science and technology, artificial piezoelectric ceramics, such as barium titanate, lead zirconate titanate (PZT) piezoelectric polycrystalline material, one after another. Piezoelectric crystal electrostriction coefficient reflects the properties of the material itself[4–6].

Both the development of new materials measurement of electrostriction and indicator of selection of materials about coefficients of the material are important. At present, the measurements of electrostriction coefficient are using laser interferometry[7], optical lever method[8], capacitance[9], eddy current method[10] and digital speckle correlation method[11]. But each method has its drawbacks, so accuracy cannot meet the current high precision of measurement.

Among the optical measurements, the domestic and international scholars pay more attentions to the laser heterodyne technique, which inherited many advantages of laser heterodynes technology and laser Doppler technique, and this is one of the ultraprecision measurement methods[12–20].This method has advantages of high spatial and temporal resolution, high accuracy, good linearity, fast dynamic response, good reproducibility and wide measurement range, which has become one of the representational techniques of modern ultraprecise detections and measuring instruments, and is widely used in ultraprecision measuring, detecting, processing unit, lidar systems, etc.

The principle of traditional heterodyne detection is all-dual-beam interference, and the spectrum of the heterodyne signal contains only a single-frequency information, and one can obtain a single value of parameter under test after demodulation. Based on the laser heterodyne measurement technique, we propose an experimental scheme of multibeam laser heterodyne measurement for electrostriction coefficient in this article. Based on this scheme, we propose a novel method for multibeam laser heterodyne measurement, which can improve the accuracy of laser heterodyne measurement. That is, based on the Doppler effect and heterodyne technology, we modulate the frequency of different time-incident light by the an oscillating mirror in the optical path, and obtain the multibeam laser heterodyne signal, whose spectrum of signal contains multiple frequencies simultaneously, each frequency contains information of parameter under test, and we can obtain multiple values of parameter that is under test after the heterodyne signal demodulation simultaneously. Processing multiple values of parameter by the weighted average, we increase the accuracy of parameter that is under test.

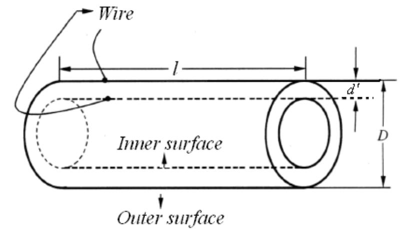

Fig. 1. Piezoelectric ceramic tubular.

A detailed theoretical analysis of this method is taken in this article, there is a simulation test at last, and the relative measurement error is just 0.98%.

The objects with piezoelectric effect are considered to be piezoelectric objects, including single crystal, polycrystalline (polycrystalline ceramics) and some amorphous solids. In this study, the sample selected is a tube-shaped piezoelectric ceramic. Its shape and structure are shown in Fig. 1. It is made of lead zirconate titanate (PZT). With silver plated inner- and outer-tube surfaces, as the electrode connected to wire leads, the applied voltage can be applied to it. Experiments show that when coupled with its outer-surface voltage (inner surface of the earth), the tube elongation, and vice versa, plus a negative voltage, the tube shorter.

Internal and external surfaces of tube voltage, formed between the inner and outer surfaces of the radial electric field of the electric field strength called E. ε is the tube with axial strain ε. α is the electrostriction coefficient of piezoelectric ceramic in the quasi-line region.

$$\varepsilon = \alpha E. \tag{1}$$

If the length of the piezoelectric ceramics is I, the voltage between the inside and outside surfaces of the piezoelectric ceramic is U, the length is Δl after the voltage putting on, tube wall thickness is d (units are mm), according the above equation are

$$\frac{\Delta l}{l} = \alpha \frac{U}{d'}. \tag{2}$$

Then, we get

$$\alpha = \frac{\Delta l d'}{lU}. \tag{3}$$

In the expression of electrostriction coefficient, d' and l can be directly measured with a vernier caliper, and the voltage U can be read out by the digital voltmeter. As the applied voltage changes, the change in length Δl is small, the length cannot be measured using conventional methods, so it is necessary to use high-precision measurements of the measurement of electrostriction coefficient of small volume.

As it is shown in Fig. 2, by comparison with the distance between thin glass plate and mirror, the thickness of thin glass plate can be ignored, so the reflected light from the back surface of the thin glass plate can be ignored, either. Because the beam reflects and refracts continuously from thin glass plate and mirror, and the reflection and refraction have a contribution to the interference between reflected and transmitted lights at the infinite distance or the focal plane of the lens, and so when discussing interference, we must consider multiple reflection and refraction effects, and hence should discuss the multibeam laser interference.

When the beam makes an oblique incidence at the incident angle θ_0, suppose incident field is $E(t) = E_0 \exp(i\,\omega_0 t)$. The uniform acceleration motion equation and rate equation of the oscillating mirror are $x(t) = \alpha(t^2/2)$ and $v(t) = at$, respectively. Because of the movement of oscillating mirror, based on Doppler effect, frequency of reflected light changes to $\omega = \omega_0\,(1 + \alpha\, t/c)$. In this expression, ω_0 stands for angular frequency of laser, a is vibration acceleration and c is light speed. So, the optical field arrived at the surface of thin glass plate at $t - l/c$ time is

$$E_1(t) = \alpha_1 E_0 \exp\left\{\left[i\,\omega_0\left(1 + \frac{a\left(t - l/c\right)}{c}\right)t + \omega_0 \frac{\frac{a\left(t-l/c\right)^2}{2}}{c}\right]\right\}. \tag{4}$$

As the beam propagated through the thin glass plate is reflected by mirror repeatedly at different moments, the reflected light can be described as

$$\begin{cases} E_2(t) = \alpha_2 E_0 \exp\{i[\omega_0(1 + a\dfrac{t - \frac{l}{c} - \frac{2nd\cos\theta}{c}}{c}) \\ t + \omega_0 \dfrac{(a\dfrac{\left(t - \frac{l}{c} - \frac{2nd\cos\theta}{c}\right)^2}{2} + 2nd\cos\theta)}{c}]\} \\ \quad\vdots \\ E_m(t) = \alpha_m E_0 \\ \exp\{i[\,\omega_0(1 + a\dfrac{t - \frac{l}{c} - \frac{2(m-1)nd\cos\theta}{c}}{c})t + \omega_0 \\ \dfrac{(a\dfrac{\left(t - \frac{l}{c} - \frac{2(m-1)nd\cos\theta}{c}\right)^2}{2} + 2(m-1)nd\cos\theta)}{c}], \end{cases} \tag{5}$$

where, $\alpha_1 = r$, $\alpha_2 = \beta\beta'\ r'$,..., $\alpha_m = \beta\beta'\ r'^{(2m-3)}$, r is the reflectivity of thin glass plate, β is the transmissivity, r' is the reflectivity of mirror, β' is the transmissivity when the beam reflected between thin glass plate and plane mirror propagates out from the thin glass plate and d is the distance between the thin glass plate and plane mirror.

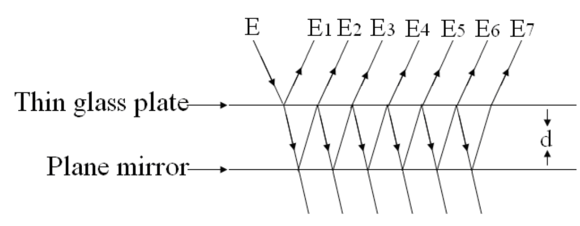

Fig. 2. Multibeam interference principle.

So, if the total optical field received by the photodetector is

$$E(t) = E_1(t) + E_2(t) + \cdots + E_m(t), \tag{6}$$

then the output photocurrent of the photodetector is

$$\begin{aligned} I &= \frac{\eta e}{h\nu}\frac{1}{Z}\iint_S \frac{1}{2}[E_1(t) + E_2(t) + \cdots \\ &\quad + E_m(t)][E_1(t) + E_2(t) + \cdots + E_m(t)]^* ds \\ &= \frac{\eta e}{2h\nu}\frac{1}{Z}\iint_S [\sum_{j=1}^{m} E_j^{\,2}(t) \\ &\quad + \sum_{p=1}^{m-1}\sum_{j=1}^{m-p}(E_j(t)E_{j+p}^{\;*}(t) + E_j^{\,*}(t)E_{j+p}(t))]ds, \end{aligned} \tag{7}$$

where e is electronic charge, Z is intrinsic impedance of the medium in the surface of photodetector, η is quantum efficiency, S is sensitive surface area of photodetector, h is Planck constant and v is laser frequency. As the DC component can be filtered after passing through the low-pass filter, therefore, considering AC component only and calling this AC component as intermediate frequency current, we can coordinate and obtain the intermediate frequency current as

$$\begin{aligned} I_{if} &= \frac{\eta e}{2h\nu} \\ &\quad \frac{1}{Z}\iint_S \sum_{p=1}^{m-1}\sum_{j=1}^{m-p}(E_j(t)E_{j+p}^{\;*}(t) + E_j^{\,*}(t)E_{j+p}(t))ds. \end{aligned} \tag{8}$$

Substituting Eqs (4) and (5) into the Eq. (8), calculate integral by software and obtain the results as

$$\begin{aligned} I_{if} &= \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p} \alpha_j\alpha_{j+p}E_0^{\,2}\cos\left[\frac{4\omega_0 anpd\cos\theta}{c^2}\right. \\ &\quad \left. t + \omega_0\frac{\frac{an^2(pd\cos\theta)^2}{c^2} - 2npd\cos\theta}{c}\right]. \end{aligned} \tag{9}$$

After ignoring the minor term of $1/c^3$, Eq. (9) can be simplified as

$$\begin{aligned} I_{if} &= \frac{\eta e}{h\nu}\frac{\pi}{Z}\sum_{p=1}^{m-1}\sum_{j=1}^{m-p} \alpha_j\alpha_{j+p} \\ &\quad E_0^{\,2}\cos\left(\frac{4\omega_0 anpd\cos\theta}{c^2}t - \omega_0\frac{2npd\cos\theta}{c}\right). \end{aligned} \tag{10}$$

Eq. (11) can be written as

$$I_{if} = \frac{\eta e}{h\nu}\frac{\pi}{Z}E_0^{\,2}\sum_{p=1}^{m-1}\cos[\Omega(p)t + \Phi(p)](\sum_{j=1}^{m-p}\alpha_j\alpha_{j+p}), \tag{11}$$

where

$$\Omega(p) = \frac{4\omega_0 anpd\cos\theta}{c^2}, \tag{12}$$

$$\Phi(p) = \frac{2\omega_0 npd\cos\theta}{c}, \tag{13}$$

where, p is defined as positive integer.

By Eq. (11), it can be seen that there are informations of the distance d between thin glass plate and plane mirror for frequency difference of intermediate frequency (IF) component and phase retardation obtained by the multibeam heterodyne measurement. Analyse frequency difference of IF component primarily, since the use of Fast Fourier Transform (FFT) is easy to implement a frequency measurement. According to Eq. (12), the frequency of interference signal can be recorded as

$$f_p = \Omega(p) = K_p d. \tag{14}$$

According to Eq. (14), the frequency of interference signal is in a direct proportion to the distance that is under test, and the scale factors is

$$K_p = \frac{4\omega_0 anp\cos\theta}{c^2}, \tag{15}$$

which depends on angular frequency of light source ω_0, refractive index n of the medium between thin glass plate and plane mirror, refractive angle θ and vibration acceleration a.

It should be noted that through Eqs. (11) and (15), we can infer that the output photocurrent of detector is composed of different harmonics, and each of them corresponds to the positive integer multiple of the frequency, and also it means that adjacent frequency difference of arrival is a fixed value, after FFT, wave peak of different harmonic frequencies can be seen on the frequency spectrum, and the distance d between thin glass plate and plane mirror can be measured by measuring the different harmonic frequencies. When d changes, the variation Δd of d can be measured according to Eq. (14), and then weighted average to Δd, so that the measurement accuracy of measuring Δd is improved. According to Eq. (3), electrostriction coefficient of sample under test can be calculated when Δd is known.

The measurement scheme of electrostriction coefficient measurement by multibeam laser heterodyne is shown in Fig. 3. The device consists of solid-state laser, a quarter-wave plate, oscillating mirror, plane mirror 1, the polarization beam splitter PBS, converging lens, thin glass, mirrors 2, piezoelectric ceramic, two-dimensional adjustment frame, high-voltage power supply, photodetector and signal processing systems (the device by the filter, preamplifier, A/D and DSP components). The oscillating mirror moves as uniformly accelerated motion under the action of the signal source, and the advantage of the oscillating mirror is the fact that the

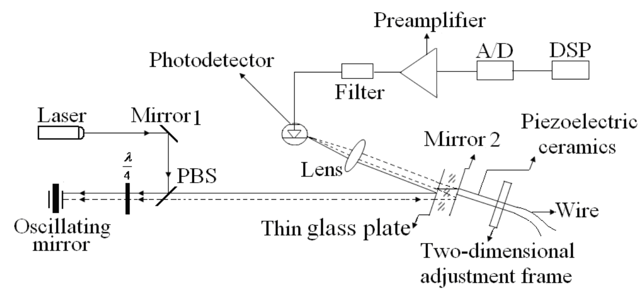

Fig. 3. Scheme of multibeam laser heterodyne measurement electrostriction coefficient.

laser incidence on the oscillating mirror in different moment can be modulated in frequency.

First, install cable to connect the special high-voltage supply output voltage and the piezoelectric ceramic input cable, and adjust the power output. At the same time, open the laser, in order to linearly polarize light through the plane mirror 1, the polarization beam splitter PBS and a quarter wave plate and then exposure to the surface of galvanometer, while the reflected light prepared by galvanometer through a quarter wave plate and a polarizing beam splitter PBS and then incidence into the thin glass. The reflected light by mirrors 2 which transmission through a thin glass and the light reflected by the thin glass plate in its front surface converged by converging lens to the photosensitive surface of detector. Finally, the tested parameter information at different times can be obtained by photoelectric conversion of the signal after the detector amplifier, A/D converter and digital signal processor DSP.

It should be noted that, when using the device to measure, mirrors affixed to the piezoelectric ceramic tube 2 fixed shelves of the two-dimensional adjustment, and the thin glass plate placed before the plane mirror 2 at 20 mm. Adjusting carefully by a two-dimensional frame to make the thin glass and the mirror plane to be parallel and have the same height. Then, fix the two-dimensional adjustment to make sure that the length of the piezoelectric ceramic tube changes in one direction. Using high-precision digital voltmeter to show the output voltage, read and record the voltage displayed value U and the signal processing obtained by Δl value. The value of Δl is just equal to the change of the distance between thin glass and mirror 2, so the values of Δl can be obtained by recording the change of distance Δd between the thin glass plate and the mirror 2.

The system of multibeam laser heterodyne measurement based on Fig. 1, using MATLAB software to simulate and measure the 15-mm long, 1.5-mm thickness of PZT material electrostriction coefficient, and PZT materials, obtained the theoretical value of coefficient of electrostriction 1.85×10^{-9} m/V to verify the feasibility of multibeam laser heterodyne measurement For example, Ho solid-state laser operates at eye-safe wavelengths above 2050 nm. Usually, the index of medium between the thin glass plate and plane mirror 2 is n = 1, the photosensitive surface aperture of detector R = 1 mm, and sensitivity is 1 A/W. Motion equation of oscillating mirror $x(\mathrm{t}) = a(t^2/2)$, taking $a = 4 \times 10^6$ m/s^2. During the experiment, demand the increase voltage in the piezoelectric ceramic according to a certain step slowly from 0 to about 800 V, while recording the change of length Δl. The simulation can be seen that Fourier transform spectrum of multibeam laser heterodyne signal by the signal processing is shown in Fig. 4; in the case of the laser oblique incidence is the solid lines, the measurement of the length variation Δl, PZT corresponding to multibeam laser heterodyne signal of the Fourier transform spectrum, and the dotted line is the case of normal incidence of laser to measure the change in length Δl, and PZT corresponding to multibeam laser heterodyne signal of the Fourier transform spectrum.

From the solid line in Fig. 4, it can be seen that Multibeam laser heterodyne signal spectrum is equally spaced distribution, which is consistent with the theoretical analysis. The case of normal incidence of the theoretical curve is given in the experiments. The purpose is that in the multibeam laser heterodyne signal spectrum, you can also get multibeam laser at oblique incidence heterodyne signal spectrum of the first peak of the center frequency and the value of the center frequency, so it is easy to get the radio of the two-center frequency.

$$\zeta = \cos\theta. \tag{16}$$

In the case of obtaining center frequency, by Eq. (16), the size of θ, which is the de refraction of laser beam through the thin glass, can be calculated, and then θ_0 can be got according the law of refraction. Finally, the value of K_p can be got. Ultimately, the change of the distance between thin glass plate and plane mirror 2 Δd can be got. Since $\Delta d = \Delta l$, which is according to Eq. (3), the electrostriction coefficient of PZT in any incident angle can be calculated. Meanwhile, the simulation obtained at different voltages, the multibeam laser heterodyne measurement of PZT variation corresponding to the length of the Fourier transform spectrum of multibeam laser heterodyne signal, as shown in Fig. 5. It can be seen from Fig. 5, as the voltage increases, the relative position of the spectrum is shifted to the low voltage that increases as the frequency decreases. The reason is, in the PZT electrostriction coefficient unchanged, the change in voltage and the length of PZT are directly proportional. When the voltage increases, the length of PZT increases,which means the distance between the thin glass and mirror 2 decreases. The relationship between frequency f_p and the distance between the plan mirror 2 and lens is $f_p = K_p d$, so when K_p is stand ,the relationship between f_p and d is linear, and so as the voltage increases, the relative position of the spectrum is shifted to the low-frequency region. Fig. 5 is good to verify that the theoretical analysis is correct. It should be noted that Fig. 5 to illustrate the frequency of the specific relationship with the

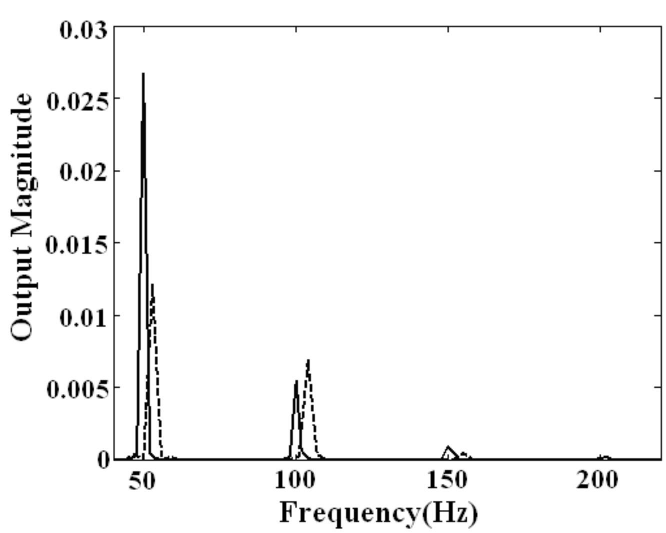

Fig. 4. Fourier transform spectrum of multibeam laser heterodyne signal.

voltage only gave the multibeam heterodyne single peak spectrum after FFT. Expanding the spectrum of Fig. 5 as similar to Fig. 4, the multipeak spectrum Fourier transform will be seen. Since heterodyne detection is the detection of a nearly diffraction limited way, extremely high detection sensitivity, so as per Figs. 4 and 5 of the heterodyne signal-to-noise ratio is very high.

In the theoretical derivation, ignoring the thickness of thin glass means without considering the reflecting to heterodyne signal by the reflected light of surface, while in fact, the glass is generally less than the thickness of 1 mm. To overcome this effect, by adding a filter in the light path in the experiment to block out the interference of low-frequency heterodyne signal, because of the Eq. (13), which means that the frequency distribution of multibeam heterodyne signal generated by the reflection of glass surface is near zero frequency in the spectrum. Eight sets of data are measured using the measurement of multibeam laser heterodyne method, and the simulated measurement of the length of PZT length variation under tested can be obtained in the case of different voltages, as shown in Table 1.

The explanation is as follows: From the simulation experiment data in Table 1, according to (2) type, the average measured value of PZT electrostriction coefficient can be calculated, which is 1.83178×10^{-9}m/V, so that the measurement of relative error is 0.98%, and we can see that the accuracy of this method is extremely high. Meanwhile, from the data analysis, we can also see, in condition of slowly increasing the voltage, that the system error made by environment and reading error can be ignored when simulating, the error of simulation experiment mainly comes from trueness error after FFT and round-off error in the processing of calculation.

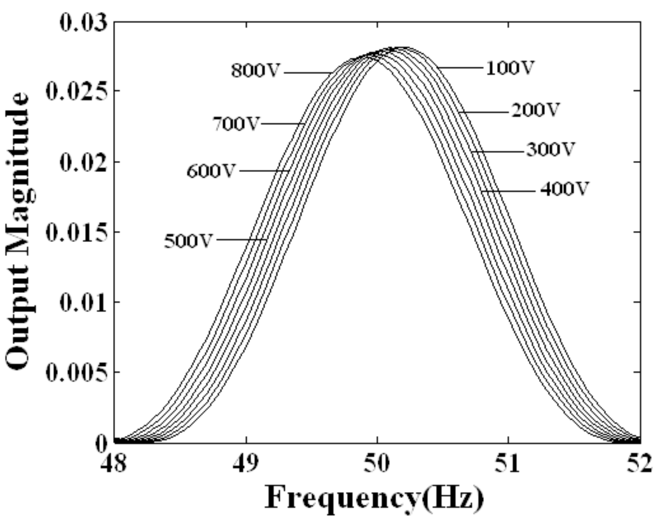

Fig. 5. In the case of different voltages, the spectrum corresponds to the change of PZT length variation.

This paper introduces galvanometer in optical path, which makes different incident light signal attached an optical frequency, in the condition of satisfying interference of reflected light from thin glass and flat mirror, which produced beam heterodyne interference signals, and thus successfully make the measurement information in intermediate frequency. In the process of electrostriction coefficient of samples measurement, this method includes multiple frequency value of metal length changing values in frequency domain, and length variation can be obtained after signal demodulation, and also the precise change relationship of sample length with current can be obtained by weighted. Taking nickel alloy for example, measurement relative error of electrostriction coefficient is only 0.98%, and the measuring precision is obviously improved.

Compared with other measurement methods, using multibeam laser heterodyne method to measure electrostriction coefficient can get high spatial and temporal resolution, high measurement speed, good linearity, anti-jamming, fast dynamic response, good repeatability and measurement range, etc. In addition, the experimental device structure is simple, the power consumption is low, operation is easy; experiments have small error, high accuracy and many other advantages. Meanwhile, the phenomenon of the method is obvious, and the experimental data are reliable. Because the experiment is directly connected with the development of new materials and so this method has practical application value, it can be widely used in coherent wind radar and such engineering design fields.

We gratefully acknowledge the contributions of Prof. Chunhui Wang, as the discussions with him had been very helpful. This work is supported by the National Natural Science Foundation of China (Grant No. 61108018) and 2014 Research Foundation of Education Bureau of Heilongjiang Province.

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Table 1. Simulation Measurement Result of Length Variation of PZT in Different Voltage Conditions

Time	1	2	3	4	5	6	7	8
Voltage (V)	100.0	200.0	300.0	400.0	500.0	600.0	700.0	800.0
Variation of $\Delta l_i(\times10^{-6}$m)	1.98916	3.97828	5.96738	7.95651	7.96150	9.95076	11.94014	13.92959
Electrostriction coefficient $E(\times10^{-9}$m/V)	1.98916	1.98914	1.98913	1.98912	1.59230	1.65846	1.70573	1.74120

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