Packet error rate analysis of DPIM for free-space optical links with turbulence and pointing errors

Chao Liu (刘 超)*, Yong Yao (姚 勇), Jiajun Tian (田佳峻), Yijun Yuan (袁易君), Yufeng Zhao (赵宇峰), and Benshuang Yu (于本双)

Department of Electronic and Information Engineering, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, Guangdong 518055, China

**Corresponding author: angel4296@msn.com*

Received October 5, 2013; accepted October 17, 2013; posted online March 1, 2014

In this Letter, we investigate the packet error rate (PER) performance of digital pulse interval modulation (DPIM) for free-space optical (FSO) links under the combined effect of turbulence and pointing errors. The theoretical model is developed by considering the effect of some important parameters, including turbulence condition, beamwidth, receiver aperture size, jitter variance, data rate, transmitted optical power, etc. A closed-form average PER expression for DPIM is derived for this fading channel. The results of numerical simulation are further provided to verify the validation of our model. This work can be helpful for selecting DPIM in the FSO system design.

OCIS codes: 010.1300, 010.1330, 060.2605, 060.4080.

doi: 10.3788/COL201412.S10101.

Free-space optical (FSO) communication systems can realize point-to-point communication links in the atmosphere. FSO systems have many virtues; for example, it can provide low-cost, low-power, high-security, and high rates. However, the systems always suffer from atmospheric turbulence and pointing errors (misalignment). Meanwhile, the laser power is always attenuated as the communication distance increases[1,2]. Atmospheric turbulence can cause severe degradation in the received signals, known as fading or scintillation. Pointing errors due to building sway is another concern in outdoor FSO links. Thermal expansion, dynamic wind loads, and weak earthquakes result in the sway of high-rise buildings, which causes vibrations of the transmitted beam, so the effect of misalignment (pointing errors) occurs between the transmitter and receiver[1]. Many earlier works have been developed to study the performance of FSO communication systems by considering the combined effect of turbulence and pointing errors[3–6]. The majority of these studies refer to the on-off keying (OOK) modulation. Besides, Gappmair and Hranilovic consider pulse position modulation (PPM) and present the average symbol error probability (ASEP) of PPM for FSO links with this combined effect[7]. Comparing with OOK, PPM belongs to the pulse modulation schemes and has higher power efficiency; however, it needs higher bandwidth and symbol-level synchronization in the receiver and increases the system complexity[8,9]. Recently, digital pulse interval modulation (DPIM) is studied as another pulse modulation scheme for FSO links by Ghassemlooy *et al.*[9]. In J. Ma *et al*[10]. pointed out DPIM was a compromise between OOK and PPM for FSO links, which has less bandwidth requirement and needs no symbol synchronization. However, the effect of pointing errors has not been taken into account for analyzing the performance of DPIM FSO communication systems. On the other hand, the former works always firstly analyze the performance of different modulation schemes in FSO communications with bit error rate and symbol errors rates by considering the intensity fluctuation and the process of ensemble average of intensity, and then packet error rate (PER) for different modulation schemes are calculated by the corresponding equations. In fact, for all the modulations, time duration of the slot or even the packer is much smaller than the coherent time of atmosphere, so the PER should be calculated before the ensemble average process is carried out[10].

In this Letter, we present the average PER in closed-form solution for this DPIM FSO communication system. A DPIM–FSO communication system is considered by the combined effect of atmospheric turbulence and pointing errors. A Gamma–Gamma (GG) distribution is used to model the atmospheric turbulence fading. The theoretical model can be used to analyze the PER performance of DPIM FSO systems with the effect of turbulence condition, beamwidth, receiver aperture size, jitter variance, data rate and transmitted optical power.

For an M-ary DPIM system, information is encoded by varying the number of empty slots between adjacent pulses. Symbol durations are variable and each symbol is initiated with a pulse. Sometimes, a guard band may be added to each symbol immediately following the pulse, to avoid the symbols, which have no slots between adjacent pulses. A DPIM pulse signal x can be expressed as[9]

$$x(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_s), \tag{1}$$

where $p(t)$ is the rectangular pulse shape, T_s is the slot duration, and a_n is a set of random variables that represent the presence or absence of a pulse in the nth time slot.

We consider an FSO communication system using M-ary DPIM. The laser beams propagate along a horizontal path through GG turbulence channel with additive white Gaussian noise (AWGN) in the presence of pointing errors. The channel is assumed to be memoryless, stationary and ergodic, with independent and identically distributed intensity fast-fading statistics. We also consider that the channel state information is available at both transmitter and receiver. The received signal y is given by

$$y = hx + n, \tag{2}$$

where h is normalized channel fading coefficient considered to be constant over a large number of transmitted bits, and n is AWGN with variance σ_n^2. Since the atmospheric turbulence and pointing errors are random factors that cause the channel fading, h can be expressed as $h = h_a h_p$, where h_a is the attenuation due to atmospheric turbulence and h_p is the attenuation due to pointing errors. For the GG turbulence channel, the probability density function (PDF) of h_a is given by[1]

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)}(h_a)^{\frac{(\alpha+\beta)}{2}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right), \tag{3}$$

where$K_n[\cdot]$ is the modified Bessel function of the second kind of order. Positive parameters α and β represent the effective number of large-scale and small-scale cells of the scattering process, which are defined for the case of a spherical wave by[1]

$$\alpha = \left\{\exp\left[\frac{0.49\sigma_I^2}{\left(1+0.18d^2+0.56\sigma_I^{12/5}\right)^{7/6}}\right]-1\right\}^{-1}, \tag{4}$$

$$\beta = \left\{\exp\left[\frac{0.51\sigma_I^2(1+0.69\sigma_I^{12/5})^{-5/6}}{1+0.9d^2+0.62d^2\sigma_I^{12/5}}\right]-1\right\}^{-1}, \tag{5}$$

where $\sigma_I^2 = 0.5C_n^2(h)k^{7/6}L^{11/6}$ is Rytov variance for spherical wave, $d = \sqrt{kD^2/4L}$ is defined as receiver aperture, $k = 2\pi/\lambda$ is optical wave number, D is the receiver aperture diameter, λ is communication wavelength, and L is link length. Here, $C_n^2(h)$ is the refractive-index structure parameter at the altitude of h. The most commonly used model for $C_n^2(h)$ is the Hufnagle–Valley model[1]

$$C_n^2(h) = 0.00594(v/27)^2(10^{-5}h)^{10}\exp(h/1000) + 2.7\times10^{-16}\exp\left(-h/1500\right) + A\exp(-h/100), \tag{6}$$

where v is the rms wind speed in meters per second, A is the nominal value of $C_n^2(0)$ at the ground in $m^{-2/3}$. In general, $C_n^2(h)$ varies from $10^{-13}m^{-2/3}$ for strong turbulence to $10^{-17}m^{-2/3}$ for weak turbulence with $10^{-15}m^{-2/3}$ that often is defined as a typical average value.

Independent identical Gaussian distributions for elevation and horizontal displacements are considered. Assuming a circular detection aperture of radius r and a Gaussian beam, the PDF of h_p can be derived[4] as

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_0^{\gamma^2}}h_p^{\gamma^2-1},\ \ 0 \le h_p \le A_0, \tag{7}$$

where $\gamma = W_{Zeq}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard derivation (jitter) at the receiver, A_0 is the fraction of the collected power at $r = 0$ and W_{Zeq} is the equivalent beam width, and

$$A_0 = \left[\mathrm{efr}(\upsilon)\right]^2,\ \ W_{Zeq}^2 = \frac{W_z^2\sqrt{\pi}erf(\upsilon)}{2\upsilon\exp(-\upsilon^2)}, \tag{8}$$

where efr$(\cdot)$ is the error function, W_z is the beam waist at distance z, and $\upsilon = \sqrt{\pi}r/\sqrt{2}W_z$. The channel fading will vary by the combined effect of turbulence and pointing errors, the combined PDF of h is given as[4]

$$f_h\left(h\right) = \int f_{h|h_a}\left(h\middle|h_a\right)f_{h_a}\left(h_a\right)dh_a, \tag{9}$$

where $f_{h|h_a}\left(h\middle|h_a\right)$ is the conditional probability and is expressed by[4]

$$f_{h|h_a}\left(h\middle|h_a\right) = \frac{1}{h_a}f_{h_p}\left(\frac{h}{h_a}\right) = \frac{\gamma^2}{A_0^{\gamma^2}h_a}\left(\frac{h}{h_a}\right)^{\gamma^2-1},0 \le h \le A_0h_a. \tag{10}$$

Using Eqs. (7) and (10), (9) results in

$$f_h\left(h\right) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}\gamma^2h^{\gamma^2-1}}{\Gamma(\alpha)\Gamma(\beta)A_0^{\gamma^2}}\int_{h/A_0}^{\infty}(h_a)^{\frac{(\alpha+\beta)}{2}-\gamma^2-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right)dh_a. \tag{11}$$

Using the following Meijer G function in[11]

$$K_\nu(x) = \frac{1}{2}G_{0,2}^{2,0}\left[\frac{x^2}{4}\middle|\frac{\nu^2}{2},-\frac{\nu^2}{2}\right]. \tag{12}$$

Then, using [12, Eq.07.34.21.0085.01], a closed-form expression for the combined effects is presented as

$$f_h\left(h\right) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}\gamma^2h^{(\alpha+\beta)/2-1}}{\Gamma(\alpha)\Gamma(\beta)A_0^{(\alpha+\beta)/2}}\times G_{1,3}^{3,0}\left[\frac{\alpha\beta h}{A_0}\middle|\begin{matrix}\gamma^2\\ \gamma^2-1,\alpha-1,\beta-1\end{matrix}\right], \tag{13}$$

where $\gamma = W_{Zeq}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard derivation (jitter) at the receiver, $A_0 = \left[\mathrm{erf}(\upsilon)\right]^2$ is the fraction of the collected power at pointing deviation $a = 0$ and W_{Zeq} is the equivalent beam width, and $W_{Zeq}^2 = W_z^2\sqrt{\pi}\mathrm{erf}(\upsilon)/(2\upsilon\exp(-\upsilon^2))$, where erf$(\cdot)$ is the error function, W_z is the beam waist at distance z, and $\upsilon = \sqrt{\pi}r/\sqrt{2}W_z$.

For M-ary DPIM, the average slot number per symbol can be given as $\overline{n} = \left(2^m + 2n_g + 1\right)/2$, where $m = \log_2 M$

and n_g is the number of guard band. The number of the slot in a packet is given as $n_s = \bar{n}l/m$, where l is the length of the packet. We assume an erasure error $P_{0/1}$ is equal to a false alarm error $P_{1/0}$, and the detection threshold level is half the amplitude of the received DPIM pulses at the sampling instant. With the fading coefficient h, the PER for M-ary DPIM can be given as[9]

$$\mathrm{PER}_{M\text{-DPIM}}(h)=\frac{1}{2}\left(n_s-\frac{l}{m}\right)\mathrm{erfc}\left(\frac{\bar{n}Rh<I>}{2\sqrt{2qI_bR_s}}\right), \quad (14)$$

where erfc(·) is the complementary error function, R is the photo-detector responsivity, q is the charge of the electron, and I_b is the average background photocurrent which can be calculated by $I_b = \pi I_B B_W r^2$, where I_B is the spectral density of the background light (near the communication wavelength), B_W is the bandwidth of the optical filter, R_S is the data rate, and $<I>=4P_t r^2/W_z^2$ is the average received irradiance[10], where P_t is the average transmitted optical power.

Considering the combined effect of turbulence and pointing errors, the average PER for M-ary DPIM, $<\mathrm{PER}_{M\text{-DPIM}}>$ can be obtained as

$$<\mathrm{PER}_{M\text{-DPIM}}>=\int_0^\infty \mathrm{PER}_{M\text{-DPIM}}(h)\cdot f(h)dh. \quad (15)$$

By substituting Eqs. (13) and (14) in Eq. (15), using the Meijer G function of $\mathrm{erfc}(x)=\frac{1}{\sqrt{\pi}}G_{1,2}^{2,0}\left(x^2\middle|\begin{matrix}1\\0,\ 1/2\end{matrix}\right)$ in[11], and utilizing [12, Eq.07.34.21.0013.01] and [13, eq. (9.31.1)], a closed-form expression for $<\mathrm{PER}_{M\text{-DPIM}}>$ will be derived as

$$<\mathrm{PER}_{M\text{-DPIM}}>=\frac{2^{\alpha+\beta-4}\gamma^2\left(n_s m-l\right)}{\pi^{3/2}\Gamma\left(\alpha\right)\Gamma\left(\beta\right)m}\times$$
$$G_{6,3}^{2,5}\left[\frac{4\bar{n}^2A_0^2R^2P_t^2r^4}{I_b qR_s\alpha^2\beta^2W_z^4}\middle|\begin{matrix}\frac{2-\gamma^2}{2},\frac{1-\alpha}{2},\frac{2-\alpha}{2},\frac{1-\beta}{2},\frac{2-\beta}{2},1\\0,\frac{1}{2},\frac{-\gamma^2}{2}\end{matrix}\right]. \quad (7)$$

In this section, the average PER for an FSO link with DPIM is investigated based on the following parameters: the number of guard band $n_g = 1$, the length of the packet l = 1024bits, photo-detector responsivity R = 1, the spectral density of the background light $I_B = 10^{-9}\,\mathrm{Wm^{-2}nm^{-1}}$, the bandwidth of the optical filter B_W = 10nm, the diameter of the receiver aperture D = 20cm, and the data rate R_s = 1Gbps. These parameters are acceptable values in[9,10–14]. In Fig. 1, it shows that the average PER as the function of transmitted optical power in dBm for 2-DPIM, 4-DPIM and 8-DPIM with different values of parameter β. These results are based on parameters of normalized beamwidth (i.e., W_z/r = 6) and normalized jitter (i.e., σ_s/r = 0.1), which are acceptable values for the system[4]. The obtained results indicate that the average PER for DPIM can be improved by increasing the parameter β and M for a given transmitted optical power.

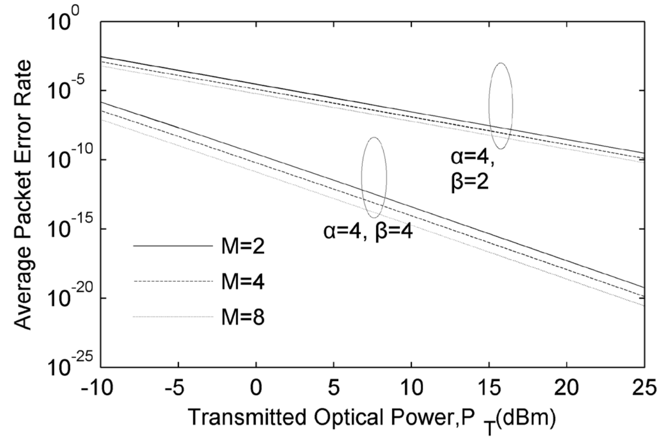

Fig. 1. Average packet error rate for 2-DPIM, 4-DPIM, 8-DPIM with (α, β) = (4, 2), (4, 4), as a function of transmitted optical power, assuming W_z/r = 6 and σ_s/r = 0.1.

In Fig. 2, the average PER for 2-DPIM, 4-DPIM and 8-DPIM is plotted against the normalized beamwidth for the different values of parameter α and β assuming a constant value of transmitted optical power P_t = 10dBm. The normalized beam width is assumed to vary between 6 and 16 with the normalized jitter, σ_s/r = 0.1. It is showed that the average PER increases with the increment of the beamwidth for a given transmitted optical power.

Next, we increase the normalized jitter for 4-DPIM with α = 4, β = 4 assuming the normalized beam width W_z/r = 6. The results are shown in Fig. 3. It indicates that the average PER falls with the decrement in the normalized jitter. It is shown that the combine effects of turbulence and pointing errors degrade the PER performance of FSO links and for a constant transmitted optical power the average PER for DPIM can be affected by the pointing errors.

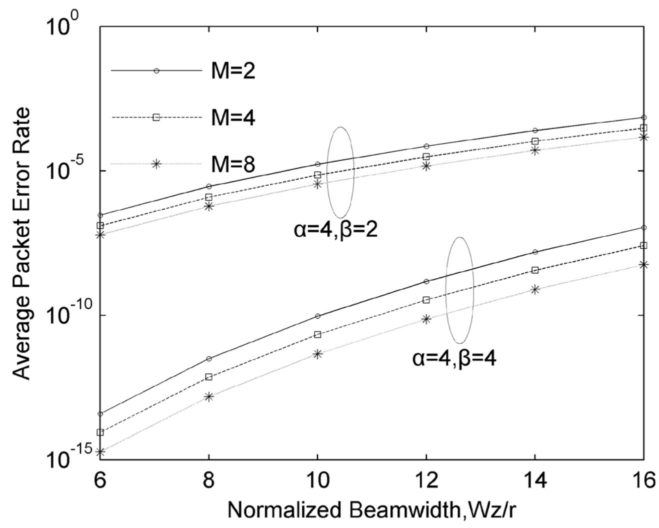

Fig. 2. Average packet error rate for 2-DPIM, 4-DPIM, 8-DPIM with (α, β) =(4, 2), (4, 4), as a function of normalized beamwidth, assuming P_t = 10dBm and σ_s/r = 0.1.

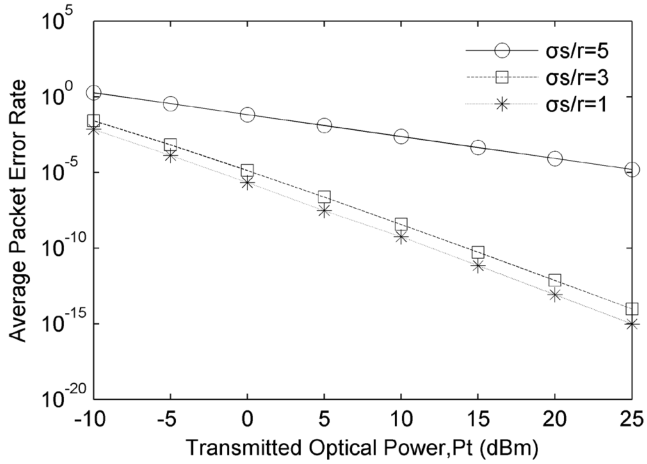

Fig. 3. Average packet error rate for 4-DPIM with (α, β) = (4, 4), as a function of transmitted optical power for three values of normalized jitter (σ_s/r = 1, 3, 5), assuming $W_z/r = 12$.

In this work, we study the average PER of DPIM for an FSO link with pointing errors over GG turbulence channel. A closed-form PER expression for *M*-ary DPIM is derived for this fading channel. It can be used to evaluate the average PER with the effects of some important system parameters, such as turbulence condition, beamwidth, receiver aperture size, jitter variance, data rate and transmitted optical power. This work is helpful for predict the error performance of DPIM for an FSO system design.

This work was supported by the Shenzhen Municipal Science and Technology Plan Project (JC201105160592A, JCYJ20120613143649014 and JC201105160524A), and National Natural Science Foundation of China (NSFC) (61205046, 11274083 and 61107036).

References

1. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications* (Bellingham, New York, 2001).
2. D. Kedar and S. Arnon, IEEE Commun. Mag. **42,** 2 (2003).
3. S. Arnon, Opt. Lett. **28,** 129 (2003).
4. A. A. Farid and S. Hranilovic, J. Lightw. Technol. **25,** 1702 (2007).
5. H. G. Sandalidis, T. A. Tsiftsis, G. K. Karagiannidis, and M. Uysal, IEEE Commun. Lett. **1,** 44 (2008).
6. D. K. Borah and D. G. Voelz, J. Lightwave Technol. **27,** 3965 (2009).
7. W. Gappmair, S. Hranilovic, and E. Leitgeb, IEEE Commun. Lett. **14,** 44 (2010).
8. Z. Ghassemlooy, A. R. Hayes, N. L. Seed, and E. D. Kaluarachchi, IEEE commun. Mag. **36,** 95 (1998).
9. Z. Ghassemlooy and A. R. Hayes, Int. J. Commun. Syst. **13,** 519 (2000).
10. J. Ma, Y. Jiang, S. Yu, L. Tan, and W. Du, Opt. Commun. **283,** 237 (2010).
11. V. S. Adamchik and O. I. Marichev, in Proceedings of International Conference on Symbolic and Algebraic Computation. (ACM Press, 1990), pp. 212.
12. Wolfram, http://functions.wolfram.com (June 9, 2001).
13. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*. (Bellingham, New York, 1994).
14. H. G. Sandalidis, T. A. Tsiftsis, and G. K. Karagiannidis, J. Lightw. Technol. **27,** 4440 (2009).