Research on error compensation method for dual-beam measurement of roll angle based on rhombic prism
(Invited Paper)

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The fabrication deviation of prisms and the error crosstalk are two major factors that produce serious systematic errors in dual-beam roll measurement based on a rhombic prism, and an error compensation method is put forward in this letter to reduce these systematic errors. The rotation matrix, reflection matrix, and refraction matrix are used to calculate and obtain the mathematical relationship model of the roll angle as well as the fabrication deviation of prism and error crosstalk. The fabrication deviation can be obtained through comparison experiments and using the least square method. In this way, the systematic error of the roll measurement caused by fabrication deviation of prism and error crosstalk can be eliminated theoretically. The experimental results show that the maximum error of the roll angle measurement reduces evidently to 3.5″ from the previous 347.2″ after compensation.

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There are six error components that need to be measured step by step for a linear guide in modern machining and measuring equipment such as computer numerical control (CNC) machines. Among all six error components, roll measurement is the most difficult[1,2]. Currently, a number of methods are available for roll measurements[3−16]. Interferometric method[3−7], light power detection method based on polarization-splitting[8,9], multi-beam position detection method[1,2,10−13], and pose measurement method[14−16], etc., are typical representatives. Generally speaking, interferometric method has a high measurement resolution with a complex optical configuration; light power detection method has a poor resolution with a simple optical configuration; posemeasurement method has a large measurement range with lowest resolution. The measurement method based on multi-beam, especially dual-beam, is frequently adopted because it has advantages of both simple configuration and relatively high measurement resolution. However, the fabrication deviation and the installation error of prism result in unparallel dual beams, and thence cause a relatively serious systematic error in the roll measurements. Moreover, error crosstalk among the straightness errors and the angular errors of pitch and yaw also influence the roll measurement accuracy. Using dual-beam roll measurement based on a rhombic prism as an example in this letter, the systematic errors caused by the fabrication deviation of prism and the error crosstalk can be greatly reduced by using a mathematic compensation model based on the error analysis.

The schematic diagram of dual-beam roll measurement using a rhombic prism is shown in Fig. 1[6]. Supposing that the distance from the rhombic prism (BSP) to QD_2 is Z_0, the distance between the centers of two parallel collimated beams $\mathbf{I}_1$ and $\mathbf{I}_2$ that shoot at the centers of QD_1 and QD_2, respectively, is H, the direction of the incident beam is $\mathbf{I}_e = [\ 0 \quad 0 \quad -1\ ]^{\mathrm{T}}$, the refraction and the reflectionsurfaces of the retro-reflector prism (RR) in the moving unit are $\mathbf{M}_0$, $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$, respectively, and the refraction and the reflection surfaces of the rhombic prism are $\mathbf{M}_4$, $\mathbf{M}_5$, $\mathbf{M}_6$, and $\mathbf{M}_7$, respectively, the normal directions $\mathbf{N}'_0 - \mathbf{N}'_7$ of the surfaces $\mathbf{M}_0 - \mathbf{M}_7$ can be obtained by

$$\begin{array}{l} \mathbf{N}'_0 = \mathbf{N}'_6 = \mathbf{N}'_7 = [\ 0 \quad 0 \quad 1\ ]^{\mathrm{T}} \\ \mathbf{N}'_1 = -\mathbf{N}'_2 = [\ \sqrt{6}/6 \quad \sqrt{2}/2 \quad \sqrt{3}/3\ ]^{\mathrm{T}} \\ \mathbf{N}'_3 = [\ -\sqrt{6}/3 \quad 0 \quad \sqrt{3}/3\ ]^{\mathrm{T}} \\ \mathbf{N}'_4 = -\mathbf{N}'_5 = [\ -\sqrt{2}/2 \quad 0 \quad -\sqrt{2}/2\ ]^{\mathrm{T}} \end{array}. \quad (1)$$

When the moving unit goes along a linear guide, rotations along X, Y, and Z axes are possible, that is the angular errors of the linear guide, namely, the pitch α, yaw β, and roll γ respectively. In the case of small rotation, the rotation matrix $\mathbf{R}$ can be approximately expressed as

$$\mathbf{R} = \begin{bmatrix} 1 & -\gamma & \beta \\ \gamma & 1 & -\alpha \\ -\beta & \alpha & 1 \end{bmatrix}. \quad (2)$$

From Eqs. (1) and (2), the normal directions $\mathbf{N}_0 - \mathbf{N}_7$ of surfaces $\mathbf{M}_0 - \mathbf{M}_7$ after rotation can be obtained by

$$\mathbf{N}_i = \mathbf{R}\mathbf{N}'_i \, (i = 0 - 7). \quad (3)$$

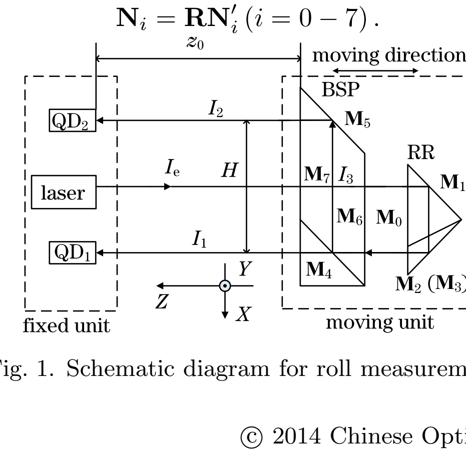

Fig. 1. Schematic diagram for roll measurement.

In order to calculate the transmission direction of beams, the reflection matrix $\mathbf{B}_i$ for surfaces $\mathbf{M}_1-\mathbf{M}_5$ and the refraction matrix $\mathbf{T}_i$ for surfaces $\mathbf{M}_0$, $\mathbf{M}_6$, and $\mathbf{M}_7$ are introduced by

$$\mathbf{B}_i=\begin{bmatrix} 1-2N_{ix}^2 & -2N_{ix}N_{iy} & -2N_{ix}N_{iz} \\ -2N_{ix}N_{iy} & 1-2N_{ix}^2 & -2N_{iy}N_{iz} \\ -2N_{ix}N_{iz} & -2N_{iy}N_{iz} & 1-2N_{ix}^2 \end{bmatrix}(i=1-5)\,, \tag{4}$$

$$\begin{aligned} \mathbf{T}_i &= \begin{bmatrix} 1/n & 0 & (1-1/n)\,N_{ix} \\ 0 & 1/n & (1-1/n)\,N_{iy} \\ 0 & 0 & N_{iz} \end{bmatrix}(i=0,7) \\ \mathbf{T}_6 &= \begin{bmatrix} n & 0 & (1-n)\,N_{6x} \\ 0 & n & (1-n)\,N_{6y} \\ 0 & 0 & N_{6z} \end{bmatrix} \end{aligned}\,, \tag{5}$$

where N_{ix}, N_{iy}, and N_{iz} are the X, Y, and Z components of the normal direction at the ith surface, respectively, and n is the refractive index of the rhombic prism. From Eqs. (3)–(5), the directions of beams $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ can be obtained by

$$\mathbf{I}_1=\mathbf{T}_7\mathbf{T}_6\mathbf{T}_0\mathbf{B}_3\mathbf{B}_2\mathbf{B}_1\mathbf{T}_0\mathbf{T}_6\mathbf{T}_7\mathbf{I}_e=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}, \tag{6}$$

$$\begin{aligned} \mathbf{I}_3 =& \mathbf{B}_4\mathbf{T}_6\mathbf{T}_0\mathbf{B}_3\mathbf{B}_2\mathbf{B}_1\mathbf{T}_0\mathbf{T}_6\mathbf{T}_7\mathbf{I}_e \\ =& \begin{bmatrix} -1 & (1/n)\,\alpha-\gamma & (1+1/n)\,\beta \end{bmatrix}^{\mathrm{T}}, \end{aligned} \tag{7}$$

$$\mathbf{I}_2=\mathbf{T}_7\mathbf{B}_5\mathbf{B}_4\mathbf{T}_6\mathbf{B}_3\mathbf{B}_2\mathbf{B}_1\mathbf{T}_0\mathbf{T}_6\mathbf{T}_7\mathbf{I}_e=\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}. \tag{8}$$

Supposing y_{QD_1} is the reading of beam $\mathbf{I}_i\,(i=1,2)$ on the $\mathrm{QD}_i\,(i=1,2)$ along y axis; y_1 is the reading of the intersection of $\mathbf{I}_1$ and $\mathbf{M}_4$ along y axis; I_{iy} is the component of $\mathbf{I}_i(i=1-3)$ along y axis. From Fig. 1, we can obtain

$$\begin{aligned} y_{\mathrm{QD}_1} &= y_1+I_{1y}z_0 \\ y_{\mathrm{QD}_2} &= y_1+I_{3y}H+I_{2y}z_0 \end{aligned}\,. \tag{9}$$

From Eqs. (6)–(9), the roll can be measured by

$$\gamma_{\mathrm{c}}=\left(\Delta y_{\mathrm{QD}_1}-\Delta y_{\mathrm{QD}_2}\right)/H=\gamma-(1/n)\alpha, \tag{10}$$

where α and γ are the true values of pitch and roll, respectively.

There are a lot of factors that influence the roll measurement accuracy. However, there are mainly two kinds of factors that produce systematic errors for roll measurements. One is the error crosstalk among the straightness errors and the angular errors of pitch and yaw for the linear guide; the other is the fabrication deviation and installation errors of optical components.

As shown in Fig. 1, the error crosstalk of moving unit includes straightness errors and angular errors of pitch and yaw in the linear guide. As the translation caused by the straightness errors of the linear guide does not change normal directions of surfaces $\mathbf{M}_0-\mathbf{M}_7$, from Eqs. (6)–(8), it can be seen that the directions of beams $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ do not change, which means that straightness errors of the linear guide do not influence the roll measurement results. From Eq. (10), it can be also seen that angular error of pitch is the primary factor of influencing the roll measurement, while the angular error of yaw has little influence on the roll measurement result.

As shown in Fig. 1, the moving unit comprises only two optical components, namely, the retro-reflector prism and the rhombic prism. If only the fabrication deviation of the retro-reflector prism is considered, taking $\mathbf{M}_1$ as the datum surface; recording the included angles of $\mathbf{M}_2$ and $\mathbf{M}_3$ with $\mathbf{M}_1$ as $\pi/2+\sigma_{12}$, $\pi/2+\sigma_{13}$, and the included angles of $\mathbf{M}_3$ and $\mathbf{M}_2$ as $\pi/2+\sigma_{23}$, the normal direction $\mathbf{N}_2''$ and $\mathbf{N}_3''$ of the surface $\mathbf{M}_2$ and $\mathbf{M}_3$ can be corrected as

$$\begin{aligned} \mathbf{N}_2''&=\begin{bmatrix} \frac{\sqrt{6}}{6}+\frac{\sqrt{6}}{6}\sigma_{12} & -\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\sigma_{12} & \frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{3}\sigma_{12} \end{bmatrix}^{\mathrm{T}} \\ \mathbf{N}_3''&=\begin{bmatrix} -\frac{\sqrt{6}}{3}+\frac{\sqrt{6}}{6}\sigma_{13}+\frac{\sqrt{6}}{6}\sigma_{23} & -\frac{\sqrt{2}}{2}\sigma_{13}+\frac{\sqrt{2}}{2}\sigma_{23} \\ \frac{\sqrt{3}}{3}+\frac{\sqrt{3}}{3}\sigma_{13}+\frac{\sqrt{3}}{3}\sigma_{23} & \end{bmatrix}^{\mathrm{T}} \end{aligned}. \tag{11}$$

Replacing $\mathbf{N}_2'$ and $\mathbf{N}_3'$ in Eq. (1) with $\mathbf{N}_2''$ and $\mathbf{N}_3''$ in Eq. (11), and from Eqs. (3)–(10), the roll measurement result can be modified by

$$\begin{aligned} \gamma_{\mathrm{c}} &= \left(\Delta y_{\mathrm{QD}_1}-\Delta y_{\mathrm{QD}_2}\right)/H \\ &= \gamma-(1/n)\,\alpha-(1/n)\left(-\frac{\sqrt{6}}{3}\sigma_{12}+\frac{\sqrt{6}}{3}\sigma_{23}\right). \end{aligned} \tag{12}$$

If the pyramidal error of the rhombic prism (the second optical parallelism error) is considered only, surfaces $\mathbf{M}_5$ and $\mathbf{M}_4$ form an included angle θ at the Y axis. Taking $\mathbf{M}_4$ as the datum surface, the normal direction $\mathbf{N}_5''$ of the surface $\mathbf{M}_5$ can be modified as

$$\mathbf{N}_5''=\begin{bmatrix} \sqrt{2}/2 & \theta & \sqrt{2}/2 \end{bmatrix}^{\mathrm{T}}. \tag{13}$$

Similarly, we can get

$$\gamma_{\mathrm{c}}=\gamma-z_0\sqrt{2}n\theta/H. \tag{14}$$

If the retro-reflector prism and the rhombic prism have installation errors, taking the rhombic prism as the datum, the retro-reflector prism will have some tiny angular changes relative to the location of the rhombic prism. According to the characteristics of the retro-reflector prism, the outgoing beam does not change with its angular change. Thus, all the beams in the measurement system do not change their directions, and the roll measurement result keeps constant.

Generally speaking, there simultaneously exist error crosstalk, fabrication deviation and installation errors of the prisms in the roll measurement method shown in Fig. 1. The roll can be obtained as

$$\begin{aligned} \gamma_{\mathrm{c}} =& \gamma-(1/n)\alpha \\ & -(1/n)\Big(-\frac{\sqrt{6}}{3}\sigma_{12}+\frac{\sqrt{6}}{3}\sigma_{23}\Big)-z_0\sqrt{2}n\theta/H. \end{aligned} \tag{15}$$

The roll measurement experimental set-up based on the above method is shown in Fig. 2. In this experimental set-up, a single-mode fiber coupled laser diode was used as a laser source, whose laser power was 3 mW and wave length was 635 nm; two quadrant detectors (SOOT-9D,

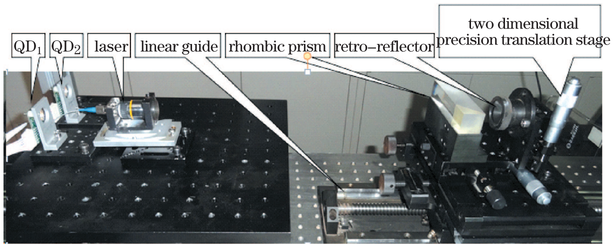

Fig. 2. (Color online) Roll measurement experimental set-up.

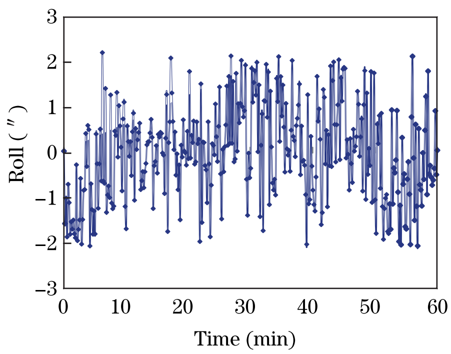

Fig. 3. (Color online) Stability of roll measurements.

UDT Company, USA), were adopted as the QD_1 and QD_2 with the position resolution of 0.1 μm. Theoretically, the resolution of 0.25″ for the roll measurement can be obtained by Eq. (15) for H being 80 mm; a linear guide is placed and fixed in an optical platform.

For the linear guide, the measurement value of roll $\Delta\gamma_c$ at any position should be the difference between the reading γ_{c1} of roll at this position and the reading γ_{c0} at the initial position, namely:

$$\Delta\gamma_c=\gamma_{c1}-\gamma_{c0}=\Delta\gamma-(1/n)\,\Delta\alpha-\Delta z\sqrt{2}n\theta\Big/H, \quad (16)$$

where $\Delta\gamma$ and $\Delta\alpha$ are the true value variations of roll and pitch respectively; Δz is the displacement distance.

Compared with Eq. (15), it shows that roll measurement result obtained from Eq. (16) is not influenced by the fabrication deviation of the retro-reflector prism.

After our experimental system is built, a grating ruler with a measurement resolution of 0.01 μm is used as a standard meter to make calibrations. As shown in Fig. 2, when the retro-reflector moves along the X and Y directions of the two dimensional precision translation stage, its straightness errors in the X and Y axes can be measured and calibrated point by point at intervals of 10 μm with the grating ruler.

The stability experiment results within 1 h are shown in Fig. 3 and the maximum fluctuation for roll measurements is ±2.1″.

As shown in Fig. 2, the moving unit stops every 12.5 mm along the linear guide and the reading of roll measurements are taken by our experimental set-up and an electronic level whose resolution is 0.2″. Figure 4 shows the experimental comparison results.

From Fig. 4, it can be seen that there exist some deviations between readings from the experimental set-up and those from the electronic level owing to the angular error of the pitch and the pyramidal error of the rhombic prism, and these deviations become larger with the increase of the moving distance, and the maximum deviation is 347.2″. Meanwhile the angular errors of pitch for the linear guide are measured by a autocollimator (Collapex EXP, AcroBeam, China) with its angular resolution of 0.008″ and measurement accuracy of 0.3″, and the results are shown in Fig. 5.

From Eq. (16), we can get

$$\Delta\gamma_c-\Delta\gamma+(1/n)\,\Delta\alpha=-\Delta z\sqrt{2}n\theta\Big/H, \quad (17)$$

where $\Delta\gamma_c$ is measured by the experimental set-up; $\Delta\gamma$ by the electronic level; $\Delta\alpha$ by the autocollimator. Set:

$$\begin{aligned} y &= \Delta\gamma_c-\Delta\gamma+(1/n)\,\Delta\alpha \\ x &= \Delta z \\ a &= -\sqrt{2}n\theta/H \end{aligned}, \quad (18)$$

Eq. (17) can be simplified as

$$y=ax. \quad (19)$$

Taking the moving distance as the independent variable x, and the number of measurements m in this case is 11. According to the least square method, we can get

$$a=\left[m\sum_{i=1}^{m}x_iy_i-\left(\sum_{i=1}^{m}x_i\sum_{i=1}^{m}y_i\right)\right]\Bigg/\left[m\sum_{i=1}^{m}x_i^2-\left(\sum_{i=1}^{m}x_i\right)^2\right]. \quad (20)$$

In our case, $n=1.5163$, $H=80$ mm, from Eqs. (19) and (20), we can get: $a=-3.0915$, $\theta=1.923'$.

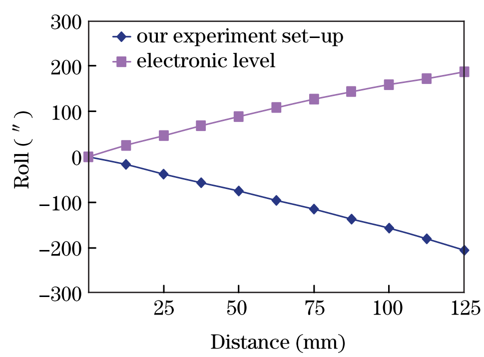

Fig. 4. (Color online) Comparison results between our experimental set-up and electronic level before compensation.

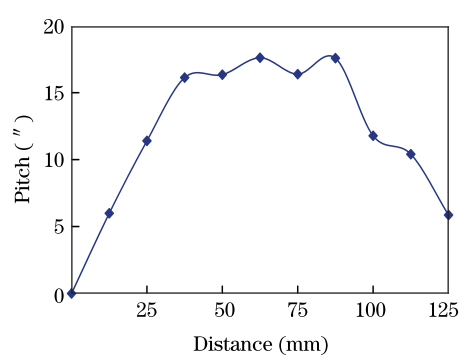

Fig. 5. (Color online) Angular error of pitch for the linear guide.

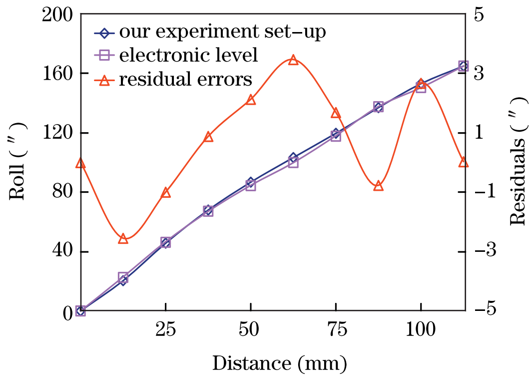

Fig. 6. (Color online) Comparison results between the experimental set-up and electronic level after compensation.

After getting the pyramidal error θ of the rhombic prism and the corresponding angular variation of pitch $\Delta\alpha$ for the linear guide, the roll can be obtained and compensated by Eq. (16). Figure 6 shows the actual compensation and comparison results.

From Fig. 6, it can be seen that the maximum residual error between our set-up and the electronic level after compensation reduces to $3.5''$ from $347.2''$. From Fig. 3, it can be seen that air disturbance can cause beam drifting, which can produce a random roll measurement error of $\pm 2.1''$. Therefore, the primary reasons for the maximum residual error of $3.5''$ include the stability of our measuring system, the measurement error of the electronic level, and the error for measuring the pitch of the linear guide. In other words, the systematic error of the roll measurement caused by the prism angle deviation, particularly the pyramidal error, can be eliminated in theory.

In conclusion, taking the dual-collimated-beam roll measurement method based on the rhombic prism as an example, a compensation method for the angle fabrication deviation of the retro-reflector prism, pyramidal error of the rhombic prism, and the angular error of pitch for the linear guide are presented. Theoretical analysis and experimental results verify that this method can largely reduce the systematic errors caused by the fabrication deviation of prism and error crosstalk of the linear guide in roll measurements, thus providing an error analysis and compensation method for laser collimation measurements.

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