Blocking probability analysis model for flexible spectrum optical networks

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Received January 17, 2014; accepted April 4, 2014; posted online June 20, 2014

Compared to the traditional wavelength division multiplexing (WDM) optical networks with rigid and coarse granularities, flexible spectrum optical networks have high spectrum efficiency, which can support the service with various bandwidth requirements, such as sub and super channel. Among all network performance parameters, blocking probability is an important parameter for the performance evaluation and network planning in circuit-based optical networks including flexible spectrum optical networks. We propose an analytical method of blocking probability computation for flexible spectrum optical networks in this letter through mathematical analysis and theoretical derivation. Two blocking probability models are built respectively based on whether considering spectrum consecutiveness or not. Numerical results validate our proposed blocking probability models under different link capacity and traffic loads.

OCIS codes: 060.4510, 060.4250, 060.4254, 060.4256.

doi: 10.3788/COL201412.070601.

In order to meet the service requirement of high burstiness and large bandwidth, flexible spectrum optical networks architecture is proposed which can eliminate the bandwidth issues in current wavelength routed optical network[1]. There are some issues in the flexible spectrum optical networks, such as spectrum resource allocation and management[2−5]. But no blocking probability model has been built for flexible spectrum optical networks up to now, which will be very important for the performance evaluation, network planning and optimization. Based on this idea, we propose an analytical method of blocking probability computation for flexible spectrum optical networks in this letter.

Insufficient capacity and outdated information are two main reasons for the lightpath blocking in wavelength division multiplexing (WDM) optical networks[6], and a lot of works have been done to improve the blocking performance of WDM optical networks[7−12]. For example, the layered graph model was introduced to compute the blocking probability of WDM optical networks, which was implemented based on wavelength decomposition approach[13−15]. An iterative model was derived to compute the blocking probability of WDM optical networks, which divided the network according to wavelength continuity requirement[16]. The overflow traffic from one layer to another is characterized by a moment matching method. In this letter, an analytical method is proposed to compute the blocking probability of orthogonal frequency division multiplexing (OFDM)-based flexible spectrum optical networks. Two blocking probability models are built respectively based on whether considering spectrum consecutiveness or not. Analytical and simulation work has been done for these two blocking probability models.

OFDM-based flexible spectrum optical networks are considered as the application scenario of the proposed blocking probability model, the concept of which has been defined in Ref. [2]. In OFDM-based flexible spectrum optical networks, spectrum slot is the minimal unit of spectrum resources. More than one single spectrum slots can be allocated for each service request. Spectrum slot continuity needs to be assured similar with the wavelength continuity in WDM optical networks. Then, some assumptions have been made.

(1) Spectrum resource is assumed to be used in consecutive and inconsecutive from the point of spectrum domain[16]. These two kinds of cases, i.e. consecutive and inconsecutive scenarios, are both considered in the letter.

(2) The transponders in flexible spectrum are assumed to be sliceable and can be used with different spectrum slots clusters for different services.

(3) The guard bands are assumed to be allocated with the service spectrum resource together.

(4) The service arrival rate follows Poisson distribution, and the rejected connection number will be increased by 1 if there is not enough spectrum resource.

(5) The latency of signaling transmission and protocol interaction is not considered in the blocking probability model.

(6) The layered graph model is used for the blocking probability model. In this letter, each spectrum slot can be considered as a layer, and all the same spectrum slot resources in the network form a graph. The traffic can be overflowed from one layer to another characterized by a moment matching method[16].

All the symbols used in the letter are listed in Table 1.

To analyze the blocking probability of each layer, we cite some equations from wavelength routed optical network (WRON)[16], in which time-reversible Markov process can be adopted to describe the status of an n hop lightpath for wavelength w at time t. Consider the lightpath with $k-1$ hops. Then we can use the $k(k-1)/2$ dimensional process to describe the status of wavelength (layer) w at time t, which is time-reversible Markov process.

Table 1. Symbols

Symbol	Meaning
Y	Y is source and destination (SD) node pair set, y is one of them and represents the service request (s, d, *slots*), in which s and d are the SD nodes, and *slots* are the required spectrum slots.
$r(s,d)$ or $r(y)$	Route for SD denoted by $r(s,d)$ or $r(y)$ for $y \in Y$
λy	The service arrival rate of request y following Poisson distribution
$\mu = 1$	Average call duration
A_y^w	The equivalent Poisson load to wavelength w for $r(y)$, clearly $A_y^1 = \lambda y$
$a_{i,j}^w$	The total equivalent Poisson load to wavelength w passing through link (i,j)
$n_{i,j}^w$	$n_{i,j}^w$ is the requests number through segment $r(i, j)$ including link (i,j) and subpath (i,j). It is boolean because there is only one request through $r(i,j)$ for layer w at most.
$\overline{N}_y^w$	The equivalent capacity of wavelength w for $r(y)$
$\overline{A}_y^w$	The average value of traffic overflowed to the wth layer
$\overline{V}_y^w$	The variance value of traffic overflowed to the wth layer
$\overline{Z}_y^w$	The peak value of the traffic overflowed to the wth layer, can be computed as $\overline{V}_y^w / \overline{A}_y^w$
$\overline{PASS}_y^{w+1}(k)$	The average value of the kth kind of traffic being able to pass through the wth layer to the $(w+1)$th layer, also means the traffic overflowed from layer w to layer $w+1$
$\overline{PA}_y^w(k)$	the kth largest value in set $\left\{\overline{PASS}_y^1(k), \overline{PASS}_y^2(k), \cdots, \overline{PASS}_y^w(k)\right\}$
P_y^w or $P_{(s,d)}^w$	The blocking probability for $r(y)$ or $r(s,d)$ on wavelength w.
P_y or $P_{(s,d)}$	The blocking probability for $r(y)$ or $r(s,d)$
P	Blocking probability of the entire network

$$X_{r(1,k)}^w(t) = [n_{1,2}^w(t), n_{1,3}^w(t), \cdots, n_{k-1,k}^w(t)]. \quad (1)$$

The status of the $k-1$ hop path $r(1,k)$ can be denoted by the calls number in progress for each segment $r(i,j)$ $1 \leqslant i < k$, $1 < j \leqslant k$, $i < j$, where $n_{i,j}^w + n_{l,m}^w \leqslant 1 \forall r(i,j) \cap r(l,m) \neq \emptyset$ and $1 \leqslant l < k, 1 < m \leqslant k, l < m$. Then the stationary vector π can be given by $\pi(n_{1,2}^w, n_{1,3}^w, \cdots, n_{k-1,k}^w) = \frac{1}{G_{r(1,k)}^w} \times [(a_{1,2}^w)^{n_{1,2}^w} \times (a_{1,3}^w)^{n_{1,3}^w} \times \cdots \times (a_{k-1,k}^w)^{n_{k-1,k}^w}]$, where $G_{r(1,k)}^w$ is the normalization constant for wavelength w on path $r(1,k)$, which is given as

$$G_{r(1,k)}^w = \sum_{\substack{n_{i,j}^w + n_{l,m}^w \leqslant 1 \\ \forall r(i,j) \cap r(l,m) \neq \emptyset \\ r(i,j) \subseteq r(1,k), \\ r(l,m) \subseteq r(1,k)}} \prod_{r(i,j) \subseteq r(1,k)} (a_{i,j}^w)^{n_{i,j}^w}. \quad (2)$$

Then the normalization constant $G_{r(1,k)}^w$ can be computed recursively as[16]

$$G_{r(1,k)}^w = G_{r(1,k-1)}^w + \sum_{i=1}^{k-1} G_{r(1,i)}^w a_{i,k}^w. \quad (3)$$

The sum of all equivalent traffic from all source and destinations (SDs) through segment $r(i,j)$ at wavelength w can also be calculated as[16]

$$a_{i,j}^w = \sum_{\substack{(s,d) : r(i,j) \subseteq r(s,d) \\ A_{s,d}^w \text{ assigned uniquely to } (i,j)}} \frac{A_{s,d}^w \cdot (1 - P_{s,d}^w)}{1 - P_{i,j}^w}. \quad (4)$$

The blocking probability of path $r(1,k)$ can be given

$$P_{r(1,k)}^w = 1 - \pi(0,0,\cdots,0) = 1 - \frac{1}{G_{r(1,k)}^w}. \quad (5)$$

Equation (5) can compute the blocking probability of each layer, where the traffic load is assumed to follow Poisson distribution ($\overline{A}_y^w = \overline{V}_y^w$, or $\overline{Z}_y^w = 1$). However, the overflow from the up to low layer (from layer w to layer $w+1$) is usually burst, not Poisson flow ($\overline{A}_y^{w+1} \neq \overline{V}_y^{w+1}$, or $\overline{Z}_y^{w+1} \neq 1$). Then, an equivalent Poisson flow is needed to match the overflow.

By the method of Fredericks and Hayward[17], the blocking probability of a single link system which has a $\overline{N}_y^w$ trunk capacity and non-Poisson flow traffic load (the average value is $\overline{A}_y^w$ and the peak value is $\overline{Z}_y^w \neq 1$) is the same as that of a system which has a $\overline{N}_y^w / \overline{Z}_y^w$ trunk capacity and Poisson flow traffic load (the average value is $\overline{A}_y^w / \overline{Z}_y^w$ and the peak value is $Z_y^w = 1$). Therefore, for an end-to-end overflow, we construct an equivalent link model. This single link system is an overflow equivalent system, which means the actual end-to-end system has the same overflow stream—parameter matching.

$$A_y^w \cdot Er\left(A_y^w, \overline{N}_y^w\right) \equiv \overline{A}_y^w \cdot Er\left(\frac{\overline{A}_y^w}{\overline{Z}_y^w}, \frac{\overline{N}_y^w}{\overline{Z}_y^w}\right), \quad (6)$$

where $Er(\lambda y, N_y^w)$ is the formulation of Erlang-B, and the trunk capacity can be obtained by

$$Er\left(A_y^w, \overline{N}_y^w\right) = P_y^w. \quad (7)$$

Then, we can get the final blocking probability P of the entire optical network iterative from Eqs. (5)−(7).

In the wth spectrum layer, the overflow stream on the path is the blocked traffic flow on this path. Then, the total path blocking probability is

$$Py = \frac{A_y^w \cdot P_y^w}{\lambda y} = \frac{\overline{A}_y^{w+1}}{\lambda y}. \quad (8)$$

Counting all the blocking probability between SD nodes, the blocking probability of the entire network can be computed as

$$P = \frac{\sum_{y\in Y} \overline{A}_y^{w+1}}{\sum_{y\in Y} \lambda y}. \quad (9)$$

In the layered graph model, the unbeard traffic load in the wth layer will overflow to the $(w+1)$th spectrum layer. We use average value $\overline{A}_y^{w+1}$, variance $\overline{V}_y^{w+1}$, and peak value $\overline{Z}_y^{w+1}$ to describe the overflow stream. In order to calculate the important parameters of overflow stream, we adopt a single link system with $N_y^w \leqslant w$ to replace the path $r(s,d)$ when traffic overflow to the wth spectrum layer. From the model mentioned above, each spectrum layer corresponds to a spectrum slot and traffic load only need one spectrum slot. Therefore, the average value of traffic load which overflows to the next layer $\overline{A}_y^{w+1}$can be calculated by

$$\overline{A}_y^{w+1} = A_y^w \cdot P_y^w. \quad (10)$$

However, in the multi-layer model of flexible spectrum optical networks, each layer also corresponds to one spectrum slot, and different traffic loads need different numbers of spectrum slots. For some traffic load requiring several spectrum slots, it's necessary to combine result of current spectrum layer with those of other spectrum layers. Meanwhile, whether or not to meet the spectrum consecutiveness constraint influences the results of spectrum allocation and network blocking probability. This letter discusses the computation of overflow under two different conditions, namely without spectrum consecutiveness constraint and with the constraint.

Spectrum consecutiveness constraint has not been considered firstly. Considering different kinds of services, different services between the same SD pair must be counted respectively. Because spectrum layers are independent and the spectrum slots can be randomly distributed, different services in the same spectrum layer have the same competitive ability. The average value of the kth kind of service blocked in the wth spectrum layer is $\overline{A}_y^w(k) \cdot \overline{A}_y^{w+1}/\overline{A}_y^w$. Then, the average value of traffic load which pass through the kth service in the wth spectrum layer can be calculated by

$$\overline{PASS}_y^w(k) = \overline{A}_y^w(k) - \overline{A}_y^w(k) \cdot \overline{A}_y^{w+1}/\overline{A}_y^w, \quad (11)$$

where $\overline{A}_y^w(k)$ is the average value of the kth kind of service which overflows to the next spectrum layer (w layer). Then, let us calculate the flow which overflows to the next spectrum layer ($w+1$ layer).

1. $k > w$

Assuming that the kth kind of traffic load needs k spectrum slots, the traffic load on the wth spectrum layer must be blocked and overflow to the next spectrum layer totally.

$$\overline{A}_y^{w+1}(k) = \overline{A}_y^w(k). \quad (12)$$

2. $k \leqslant w$

The kth traffic load needs not only to consider the flow passing through the spectrum layer, but also to compare with the previous spectrum layer. Then, the kth largest value can be found from the average value of flow which is calculated in w spectrum layers, the kth largest value equals to $\overline{PA}_y^w(k)$, which means that, $\overline{PA}_y^w(k)$ is the kth largest value in set $\left\{\overline{PASS}_y^1(k), \overline{PASS}_y^2(k), \cdots, \overline{PASS}_y^w(k)\right\}$. Then, the average value of the kth service overflow to the next spectrum layer can be calculated as

$$\overline{A}_y^{w+1}(k) = \overline{A}_y^w(k) - \overline{PA}_y^w(k). \quad (13)$$

Meanwhile, the average value of flow which passes through the wth spectrum layer is

$$\overline{PASS}_y^{i+1}(k) = \overline{PASS}_y^i(k) - \overline{PA}_y^w(k),$$
$$\text{when } 1 \leqslant i \leqslant w, \overline{PASS}_y^i(k) \geqslant \overline{PA}_y^w(k). \quad (14)$$

With spectrum consecutiveness constraint, the spectrum slots allocated on the lightpath must be consecutive. Services with different traffic loads between the same SD pair must be counted respectively. Because spectrum layers are independent and the spectrum slots must be consecutively distributed, different traffic load in the same spectrum layer have different competitive ability. Assuming that there are N kinds of traffic loads, and let the least common multiple of 1, 2, $\cdots$, N be LCM. The average value $\overline{PASS}_y^w(k)$ of kth service passing through the wth spectrum layer can be derived through

$$\frac{\overline{PASS}_y^w(1)}{\overline{A}_y^w(1)} \cdot \frac{1}{\text{LCM}} = \frac{\overline{PASS}_y^w(2)}{\overline{A}_y^w(2)} \cdot \frac{2}{\text{LCM}} = \cdots = \frac{\overline{PASS}_y^w(N)}{\overline{A}_y^w(N)} \cdot \frac{N}{\text{LCM}}, \quad (15)$$

$$\overline{A}_y^w - \overline{A}_y^{w+1} = \sum_{k=1}^{N} \overline{PASS}_y^w(k), \quad (16)$$

where $\overline{A}_y^w(k)$ is the average value of the kth service which overflows to the next spectrum layer (w layer). Then, let us calculate the flow which overflows to the next spectrum layer ($w+1$ layer).

1. $k > w$

Because the kth kind traffic load need at least k spectrum slots, the traffic load on the wth spectrum layer must be blocked and thus overflow to the next spectrum layer totally.

$$\overline{A}_y^{w+1}(k) = \overline{A}_y^{w}(k). \tag{17}$$

2. $k \leqslant w$

The kth traffic load needs not only to consider the flow passing through the spectrum layer, but also to compare with the previous $(k-1)$ spectrum layer. By choosing the minimum value from the average value of flow calculated in wth spectrum layers, the kth largest value can be found. It equals to $\overline{PA}_y^{w}(k)$,

$$\overline{PA}_y^{w}(k) = \min\left\{\overline{PASS}_y^{w}(k), \overline{PASS}_y^{w-1}(k), \cdots, \overline{PASS}_y^{w-k+1}(k)\right\}. \tag{18}$$

Then, the average value of the kth service overflow to the next spectrum layer can be calculated as

$$\overline{A}_y^{w+1}(k) = \overline{A}_y^{w}(k) - \overline{PA}_y^{w}(k). \tag{19}$$

Meanwhile, the average value of the flow which passes through w spectrum layers is

$$\overline{PASS}_y^{i+1}(k) = \overline{PASS}_y^{i}(k) - \overline{PA}_y^{w}(k), \quad \text{for } w-k+1 \leqslant i \leqslant w. \tag{20}$$

After computing the average value of kth service which overflows to the next spectrum layer under the two conditions above and assuming that there are N kinds of traffic loads, the average value of the actual stream which overflows to the next spectrum layer, i.e. $\overline{A}_y^{w+1}$ can be computed by

$$\overline{A}_y^{w+1} = \sum_{k=1}^{N} \overline{A}_y^{w+1}(k). \tag{21}$$

The variance of the actual stream which overflows to the next spectrum layer, i.e. $\overline{V}_y^{w+1}$ can be computed by Riordan equation.

$$\overline{V}_y^{w+1} = \overline{A}_y^{w+1} \cdot \left(1 - \overline{A}_y^{w+1} + \frac{\lambda y}{N_y^w \leqslant w+1+\overline{A}_y^{w+1} - \lambda y}\right), \tag{22}$$

where N_y^w is the equivalent trunk capacity of the first w spectrum layers, which can be computed by

$$\lambda y \cdot Er(\lambda y, N_y^w) = \overline{A}_y^{w+1}. \tag{23}$$

It is the general Erlang-B equation of non-integral capacity. Then, the kurtosis coefficient of the actual stream which overflows to the next spectrum layer $\overline{Z}_y^{w+1}$ is the ratio of variance and average value.

$$\overline{Z}_y^{w+1} = \overline{V}_y^{w+1} / \overline{A}_y^{w+1}. \tag{24}$$

Table 2. Algorithm A

$\overline{A}_y^1 = \lambda y$, $\overline{V}_y^1 = \lambda y$, and $\overline{Z}_y^1 = 1$, for $\forall y \in Y$, from the first spectrum layer (w= 1), calculate the blocking probability to the wth spectrum layer ($w = W$) in sequence.

1	Set w=1.
2	Assuming the initial value $A_y^w = \overline{A}_y^w$, and set the initial path blocking probability $P_y^{'} = 0$ for $\forall y$.
3	Using Eqs. (3)−(5) to calculate P_y^w.
4	In the wth spectrum layer, using Eqs. (6) and (7) to calculate A_y^w for $\forall y$.
5	If $\exists y \in Y$, $\frac{\left\lvert P_y^w - P_y^{'} \right\rvert}{P_y^w} > \varepsilon$, then
6	$P_y^{'} = P_y^w$, turn to step 3.
7	Else
8	Using Eq. (11) to calculate $\overline{PASS}_y^{w}(k)$.
9	$w = w + 1$
10	Using Eqs. (12) and (13) to calculate $\overline{A}_y^{w}(k)$.
11	Using Eq. (14) to update $\overline{PASS}_y^{w-1}(k)$.
12	Using Eqs. (21)−(24) to calculate $\overline{A}_y^{w}$, $\overline{V}_y^{w}$ and $\overline{Z}_y^{w}$.
13	If $w \leqslant W$, then
14	Turn to step 2 .
15	Else
16	Using Eqs. (8) and (9) to calculate the total blocking probability of path and network respectively
17	Return Py for $\forall y$ and P.
18	End if
19	End if

Table 3. Algorithm B

	$\overline{A}_y^1 = \lambda y$, $\overline{V}_y^1 = \lambda y$, and $\overline{Z}_y^1 = 1$, for $\forall y \in Y$, from the first spectrum layer (w=1), calculate the blocking probability to the wth spectrum layer ($w = W$) in sequence.
1	Set w= 1.
2	Assuming the initial value $A_y^w = \overline{A}_y^w$, and set the initial path blocking probability $P_y^{'} = 0$ for $\forall y$.
3	Using the Eqs. (3)−(5) to calculate P_y^w.
4	In the wth spectrum layer, using Eqs. (6) and (7) to calculate A_y^w for $\forall y$.
5	If $\exists y \in Y$, $\frac{\left\lvert P_y^w - P_y^{'} \right\rvert}{P_y^w} > \varepsilon$, then
6	$P_y^{'} = P_y^w$, turn to step 3.
7	Else
8	Using Eqs. (15) and (16) to calculate $\overline{PASS}_y^w(k)$.
9	$w = w + 1$
10	Using Eqs. (17)−(19) to calculate $\overline{A}_y^w(k)$.
11	Using Eq. (20) to update $\overline{PASS}_y^{w-1}(k)$.
12	Using Eqs. (21)−(24) to calculate $\overline{A}_y^w$, $\overline{V}_y^w$ and $\overline{Z}_y^w$.
13	If $w \leq W$, then
14	Turn to step 2 .
15	Rlse
16	Using Eqs. (8) and (9) to calculate the total blocking probability of path and network respectively
17	Return Py for $\forall y$ and P.
18	End if
19	End if

In previous part, we propose the layered graph model for flexible spectrum optical networks and analyze the overflow under two conditions of without spectrum consecutiveness constraint and with the constraint. If the difference of path blocking probability computed by two times iteratively is less than ε, the current spectrum layer can be considered as a stable one and the algorithm will enter the next spectrum layer iterative computation until the convergence of the models.

The steps for the layered graph model without spectrum consecutiveness constraint is illustrated by algorithm A, as in Table 2.

The main steps for the layered graph model in flexible spectrum optical network under with spectrum consecutiveness constraint is illustrated by algorithm B, as in Table 3.

The blocking probability of flexible spectrum optical networks with and without spectrum consecutiveness constraint can be computed and obtained by the layered graph model. Meanwhile, the simulation results have been given based on an optical network testbed, which is built on a single-core virtual machine with 1 GB RAM running Linux on an IBM X3650 server.

We adopt NSFNET topology with 14 nodes and 21 links as shown in Fig. 1. Assuming that there are three kinds of services, service A requires 1 spectrum slot, service B requires 2 spectrum slots, and service C requires 3 spectrum slots. The ratio of the number of these three kinds of services is assumed to be 1:1:1, which will not affect the results. In the simulation, the traffic load and link capacity are assumed to be average, which means that the traffic loads among all the SD node pairs are same and the number of spectrum slot on each link is also same. The simulation result is obtained after running 10^6 services and in the simulation, $\varepsilon = 10^{-2}$.

We verify the result of the layered graph model for flexible spectrum optical networks without spectrum consecutiveness.

The traffic load between each SD nodes pair follows Poisson distribution with arrival rate of 1. Then, making a comprehensive statistics of the three kinds of services, we plot the blocking probability of the entire network as shown in Fig. 2. We can observe that with the link capacity increasing, blocking probability will decrease. Also we can see that blocking probability deduced from layered graph model matches well with that of the simulation. However, the blocking probability of analytical is lower than the simulation results which are the actual values and the upper bound. The reason is that the overflow from the up layer to the lower layer does not follow the exact Poisson distribution strictly. More precisely speaking, the offload used on the lower layer is smaller than the actual value.

Figure 3 gives the blocking probability under difference traffic loads (Erlangs) without spectrum consecutiveness constraint, which are calculated by formula derivation and simulation as well. In our simulation, each link has 24 spectrum slots. From Fig. 3, we can get that with the traffic load increasing, blocking probability will increase and the difference of blocking probability obtained by our model and the simulation will decrease. Separate statistics of three different services as well as comprehensive statistics of the total network prove that blocking probability deduced from our layered graph model compares well with that from simulation, especially under high traffic load. The difference between the analytical and simulation results is mainly caused by the inaccurate Poisson distribution of the overflow, which will also be motivation for more accurate model.

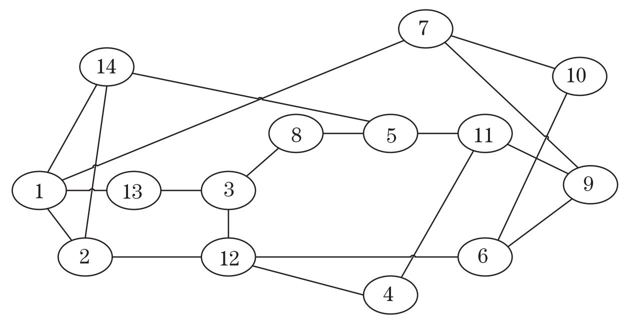

Fig. 1. NSFNET topology with 14 nodes.

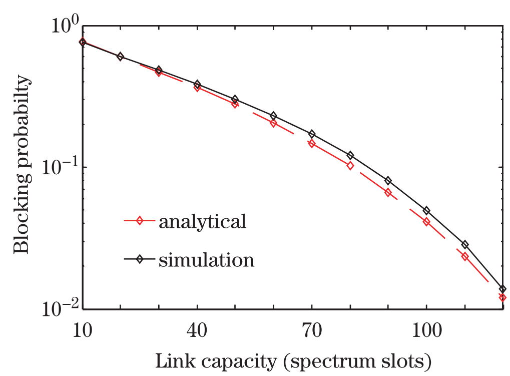

Fig. 2. Blocking probability with link capacity.

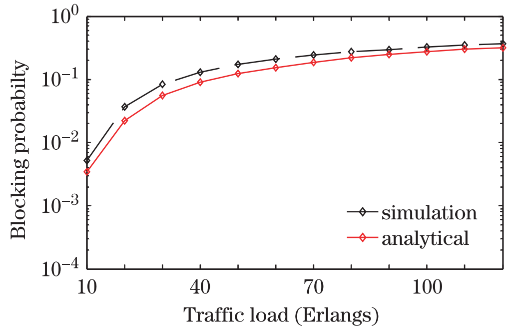

Fig. 3. Blocking probability with traffic load.

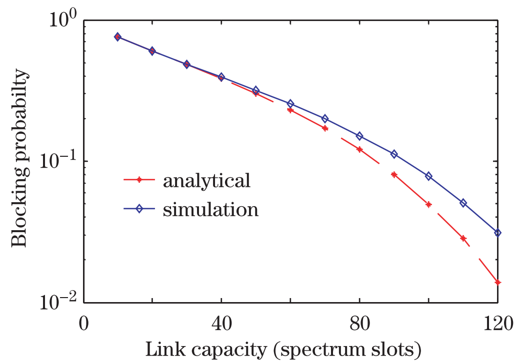

Fig. 4. Blocking probability with link capacity.

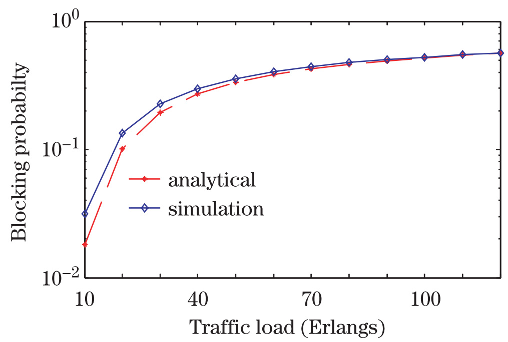

Fig. 5. Blocking probability with traffic load.

Then we verify the result with spectrum consecutiveness constraint for flexible spectrum optical network by layered graph model. The traffic load between each SD nodes pair follows Poisson distribution with arrival rate 1. Then, making a comprehensive statistics of three services, the curve of network blocking probability with link capacity is plotted in Fig. 4, which depicts that with the link capacity increasing, blocking probability will decrease.

In our simulation, each link has 24 spectrum slots. Then, making a comprehensive statistics of the three traffic loads, we plot the blocking probability curve of the entire network with traffic load as Fig. 5, which shows that with the traffic load increasing, blocking probability will increase and the difference between blocking probability obtained by our model and simulation respectively will decrease.

In conclusion, an analytical methodology is proposed for blocking probability computation for flexible spectrum optical networks. Two blocking probability analysis models are derived under consecutive and inconsecutive spectrum constraints. Overflow method is used when more than one spectrum slots are required. Numerical results show that the blocking probability of our proposed model can match the simulation results in different link capacity and traffic loads. More works about more accurate blocking probability models based on these works in the letter will be conducted in the future, and how to make the overflow from one layer to another follow Poisson distribution will be the focal point.

This work was supported by the National "863" Program of China (No. 2012AA011301), the National "973" Program of China (No. 2010CB328204), the National Natural Science Foundation of China (Nos. 61271189 and 61201154), the Research Fund for the Doctoral Program of Higher Education of China (No. 20120005120019), the Beijing Youth Elite Project for Universities, the Fundamental Research Funds for the Central Universities (No. 2013RC1201), and the Fund of State Key Laboratory of Information Photonics and Optical Communications (BUPT).

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