Holographic display based on compressive sensing

(Invited Paper)

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We propose a method to improve the quality of the reconstructed images based on compressive sensing principles. The pseudo-inverse matrix and the total variation minimization algorithms are combined to reduce the sampling number of the computer generated hologram. Numerical simulations are performed and the results indicate that the peak signal to noise ratio is increased and the sampling ratio is decreased at the same time for holographic display.

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Holographic display can provide all the visual information of actual objects, so it is believed to be a potential technique for 3D display. However, the large amounts of the data processing of the hologram take up a lot of memory space, and this is a big problem for the real-time holographic display. In recent years, the research on compressive sensing (CS) has drawn much attention, where the reconstructed image with high quality can be achieved from the hologram at a low sampling ratio. CS has already been applied to digital holography (DH) where high-dimensional signals can be recovered from low-dimensional measurements. Clemente *et al.*[1] combined compressive holography with single-pixel optical imaging for DH. Brady *et al.*[2] proposed compressive tomographic holography, and Rivenson *et al.*[3] derived the formulas when using compressive sensing in 3D object tomography recovery. Horisaki *et al.*[4] further proposed a multidimensional imaging method by compressive Fresnel holography. These works indicate that CS can decrease the sampling ratio and improve the imaging quality. It inspires us to think over whether CS can be used in computer holographic display.

In this letter, we propose a simple method to improve the qualities of reconstructed images from a series of Fresnel hologram data based on CS principles. We also derive the theoretical formulas and perform the numerical simulations to verify the effectiveness of the method.

As is well-known, according to the Shannon-Nyquist theorem, we have to sample the signals at least twice the highest spatial frequency if we want to reconstruct the original ones. Not subjected to this limitation, CS can realize compression in the acquisition process directly and recover original signals by solving optimization problems. The acquisition model is described as

$$\mathbf{y} = \mathbf{\Phi x}, \tag{1}$$

where $\mathbf{x}$ is the input signal in the form of a column vector and is also K-sparse in some basis such as Fourier transform basis or wavelet transform basis, $\mathbf{y}$ is the measurement, and $\mathbf{\Phi}$ is the sensing matrix. If the number of data picked from $\mathbf{y}$ is M rather than the total number N ($M < N$), the signal is compressed, and the sampling ratio is M/N. Corresponding rows of the sensing matrix are kept, so its size changes into $M \times N$. Precise reconstruction of $\mathbf{x}$ can be acquired when two requirements[5] are met

$$(1-\delta_K) \parallel \mathbf{x} \parallel_2^2 \leqslant \parallel \mathbf{\Phi x} \parallel_2^2 \leqslant (1+\delta_K) \parallel \mathbf{x} \parallel_2^2, \tag{2}$$

where δ_K is a constant ranging from 0 to 1 and

$$M = O[K \log(N/K)]. \tag{3}$$

It is an ill-posed inverse problem to recover the signal from the measurement. For two-dimensional images, the results are quite satisfying by solving the following optimization problem[6]:

$$\min \mathrm{TV}(\mathbf{x}) \quad \text{s.t.} \ \mathbf{y} = \mathbf{\Phi x}, \tag{4}$$

where $\mathrm{TV}(\mathbf{x})=\sum_{m,n} \sqrt{(x_{m+1,n}-x_{m,n})^2+(x_{m,n+1}-x_{m,n})^2}$. There are also other kinds of algorithms for different conditions.

The diffraction problem can be described as Fig. 1. The objective wave $o(x_0, y_0)$ propagates in free space, and Fresnel diffraction formula[7] can be used under paraxial approximation condition

$$u(x,y) = \frac{1}{\mathrm{j}\lambda z} \iint_{\Sigma} o(x_0,y_0) \exp\left[jk\frac{(x-x_0)^2+(y-y_0)^2}{2z}\right] \mathrm{d}x_0 \mathrm{d}y_0, \tag{5}$$

where $u(x,y)$ is the complex amplitude on the hologram plane, x and y are the coordinate values of the hologram plane, x_0 and y_0 are those of the object plane, z is the distance of dissemination, and λ is the wavelength in free space. Because of the discretization in calculation, we treat the objective wave as a collection of points, as shown in Fig. 1. Then the complex amplitude of each pixel on the hologram plane can be expressed as

$$u(x_p, y_q) = \frac{1}{j\lambda z}\sum_{n=1}^{N}\sum_{m=1}^{M} o(x_{0m}, y_{0n}) \exp \left[\mathrm{j}k \frac{(x_p - x_{0m})^2 + (y_q - y_{0n})^2}{2z} \right] \Delta x_0 \Delta y_0, \quad (6)$$

where $u(x_p, y_q)$ is the complex amplitude of one pixel on the hologram plane and $M \times N$ is the resolution of the objective wave.

Since the main calculation is matrix multiplication, the calculation process can be described as: firstly, we arrange the $n \times n$ object into a $N \times 1$ vector where $N = n \times n$, and then arrange the factors $\exp\left[\mathrm{j}k\frac{(x_p-x_{0m})^2+(y_q-y_{0n})^2}{2z}\right]\Delta x_0 \Delta y_0$ into a matrix, and finally their product is calculated as illustrated in Fig. 2.

For clarity, Eq. (6) is changed into the form of matrix multiplication

$$\mathbf{u} = C\mathbf{P}\mathbf{o} = \mathbf{\Phi}\mathbf{o}, \quad (7)$$

where $\mathbf{u}$ is the complex amplitude distribution, C is the complex constant, and $\mathbf{P}$ is the matrix determined by

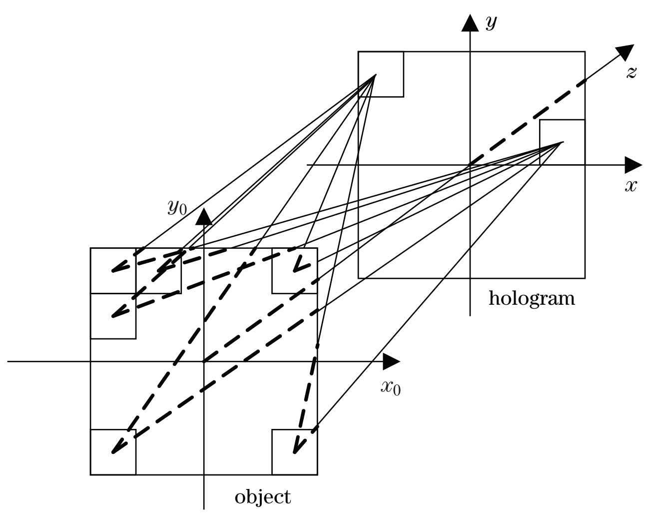

Fig. 1. Fresnel diffraction model.

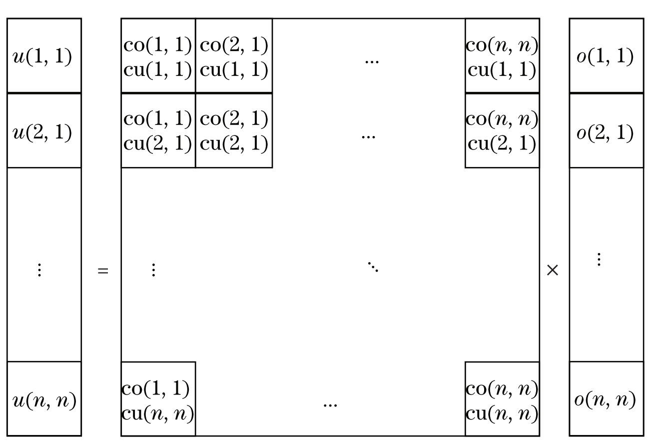

Fig. 2. Calculation process, $\mathrm{co}(m, n)$ are coordinate values of the object plane and $\mathrm{cu}(m, n)$ are those of the hologram plane.

the position factors. We can see that Fresnel diffraction process is a standard acquisition model of CS, so compression can be achieved.

After sampling, M samples are picked out from the measurement to form $\mathbf{u}_{\text{new}}$ and then a new sensing matrix $\mathbf{\Phi}_{\text{new}}$ with the size of $M \times N$ is generated, so the pseudo-inverse matrix can be achieved. The definition of pseudo-inverse matrix is[8]

$$\mathbf{AXA} = \mathbf{A}, \quad \mathbf{XAX} = \mathbf{X}, \quad (8)$$

where $\mathbf{X}$ is the pseudo-inverse matrix of $\mathbf{A}$. The singular value decomposition (SVD) of $\mathbf{\Phi}_{\text{new}}$ is given as

$$\mathbf{\Phi}_{\text{new}} = \mathbf{USV^H} = \mathbf{U}(\mathbf{\Delta 0})\mathbf{V^H}, \quad (9)$$

where $\mathbf{U}$ is a $M \times M$ matrix, $\mathbf{\Delta}$ is a $M \times M$ diagonal matrix containing singular values, the sizes of $\mathbf{S}$ and $\mathbf{V}$ are $M \times N$ and $N \times N$, respectively. $\mathbf{\Phi}_{\text{new}}^{-1}$ can be written by

$$\mathbf{\Phi}_{\text{new}}^{-1} = \mathbf{V}\begin{pmatrix}\mathbf{\Delta}^{-1} \\ \mathbf{0}\end{pmatrix}\mathbf{U^H}, \quad (10)$$

and the multiplication of $\mathbf{\Phi}_{\text{new}}^{-1}$ and $\mathbf{\Phi}_{\text{new}}$ can be obtained by

$$\mathbf{\Phi}_{\text{new}}^{-1}\mathbf{\Phi}_{\text{new}} = \mathbf{V}\begin{pmatrix}\mathbf{\Delta}^{-1} \\ \mathbf{0}\end{pmatrix}\mathbf{U^H U}(\mathbf{\Delta 0})\mathbf{V^H}$$

$$= \mathbf{V}\begin{pmatrix}\mathbf{\Delta}^{-1} \\ \mathbf{0}\end{pmatrix}(\mathbf{\Delta 0})\mathbf{V^H} = \mathbf{V}\begin{pmatrix}\mathbf{E}_{M\times M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{pmatrix}\mathbf{V^H}. \quad (11)$$

The reconstruction of the hologram by pseudo-inverse matrix can be further expressed as

$$\mathbf{\Phi}_{\text{new}}^{-1}\mathbf{u}_{\text{new}} = \mathbf{\Phi}_{\text{new}}^{-1}\mathbf{\Phi}_{\text{new}}\mathbf{o} = \mathbf{V}\begin{pmatrix}\mathbf{E}_{M\times M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{pmatrix}\mathbf{V^H o}$$

$$= \mathbf{V}\left[\mathbf{E}_{N\times N} - \begin{pmatrix}\mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{(N-M)\times(N-M)}\end{pmatrix}\right]\mathbf{V^H o}$$

$$= \mathbf{V}\mathbf{E}_{N\times N}\mathbf{V^H o} - \mathbf{V}\begin{pmatrix}\mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{(N-M)\times(N-M)}\end{pmatrix}\mathbf{V^H}$$

$$\mathbf{o} = \mathbf{o} + \mathbf{Ko}. \quad (12)$$

It is clear that the multiplication of $\mathbf{\Phi}_{\text{new}}^{-1}$ and $\mathbf{u}_{\text{new}}$ is a mix of original image and noise, and the $M \times 1$ measurement reproduces $N \times 1$ reconstructed image. If $\mathbf{\Phi}_{\text{new}}^{-1}\mathbf{u}_{\text{new}}$ and $\mathbf{\Phi}_{\text{new}}^{-1}\mathbf{\Phi}_{\text{new}}$ are viewed as the measurement and the sensing matrix, respectively, more accurate reconstruction can be obtained by the total variation minimization algorithms. We use peak signal to noise ratio (PSNR) to indicate the reconstruction effect, and it is defined as

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{\mathrm{MAX}}{\mathrm{MSE}}\right), \quad (13)$$

$$\mathrm{MSE} = \frac{1}{MN}\sum_{n=1}^{N}\sum_{m=1}^{M} \| I(m,n) - K(m,n) \|^2, \quad (14)$$

where MAX is the maximum intensity of the original image.

In order to demonstrate the validity of this method, numerical simulations are performed. Four 128 × 128 original images, Lena, cameraman, woman, and ship, are selected as the objective images, as shown in Figs. 3(a)−(d). The distance between the object plane and the hologram plane is 20 mm and the wavelength is 671 nm in free space. We randomly pick out $5\% - 100\%$ data

Fig. 3. Original images: (a) Lena, (b) cameraman, (c) woman, and (d) ship.

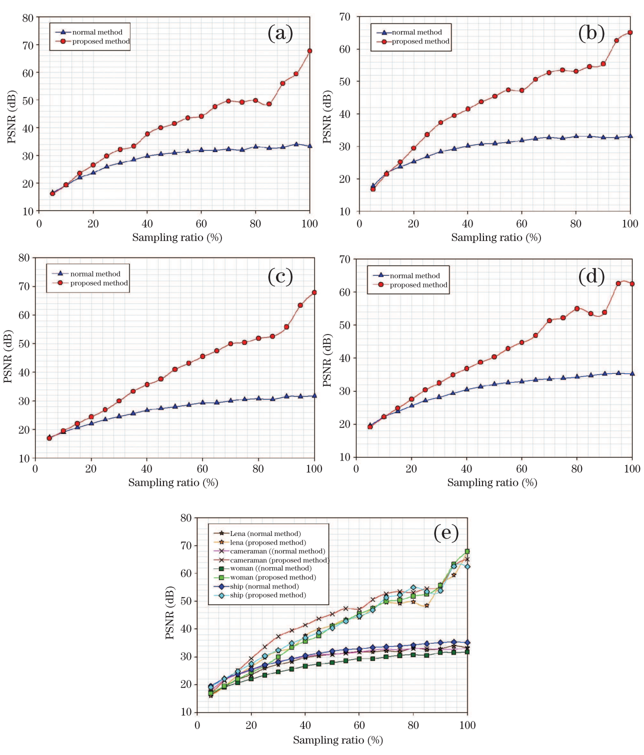

Fig. 4. PSNR of the reconstructed image as a function of the sampling ratio, (a) Lena, (b) cameraman, (c) woman, and (d) ship, where the red line with dots represents the results by our proposed method and the blue line with triangles represents those by normal method; (e) is the comparison of (a)−(d), where ☆, ×, □, and ◇ represent the results of Lena, cameraman, woman, and ship, respectively.

Fig. 5. (a)−(d) are the reconstructed images by the normal method at the sampling ratios of 15%, 25%, 35%, and 45%, respectively; (e)−(h) are the reconstructed images by our proposed method at the same sampling ratios.

from the measurements with the step size of 5%. For a good comparison, we employ the normal method that applies reconstruction algorithms to process the sampled hologram data directly, instead of the product of $\boldsymbol{\Phi}_{\mathrm{new}}^{-1}$ and $\mathbf{u}_{\mathrm{new}}$ as the proposed method. We calculate the PSNRs of the reconstructed images derived from our proposed method and the normal method, and plot the PSNR as a function of the sampling ratio, as shown in Figs. 4(a)−(d), where the red line with dots represents the results by our proposed method and the blue line with triangles represents those of the normal method. All these lines are synthesized in Fig. 4(e).

To clearly observe the reconstructed images, we exhibit the reproduced images of the cameraman as an example, as shown in Fig. 5, where (a)−(d) are the reconstructed images by the normal method at the sampling ratios of 15%, 25%, 35%, and 45%, respectively, and the PSNRs are 23.7209 dB, 26.9127 dB, 29.2285 dB, and 30.7313 dB, respectively; (e)−(h) are the reconstructed images by our proposed method at the same sampling ratios, and the PSNRs are 25.1771 dB, 33.6203 dB, 39.5331 dB, and 43.7967 dB, respectively.

It can be seen from Figs. 4(a)−(e), since the effect of noise becomes smaller, the PSNR grows as the sampling ratio increases as expected. When the sampling ratio exceeds 30%, the PSNR of the reconstructed image by the proposed method is much higher than that of the normal method, indicating that the quality is better. The line represents the PSNR of the reconstructed image by the normal method begins to flatten to about 30 dB when the sampling ratio is higher than 40%; while for our proposed method, it still grows to more than 60 dB. As shown in Fig. 5, it is obvious that the reconstruction effect of the proposed method is much better than that of the normal method, and when the sampling ratio exceeds 25%, the reconstructed images are in close proximity to the original ones.

In brief, we propose a simple method to improve the quality of reconstructed image at a low sampling ratio based on CS principles, using the pseudo-inverse matrix and the optimization algorithms. The numerical results demonstrate that the PSNRs of the reconstructed images generated by the proposed method are much higher than those of the normal method when the sampling ratio surpasses 30%. This means that better image quality can be obtained with less hologram data. It is believed to be a promising approach for reducing the data amounts and it is beneficial to 3D information transmission via Internet for 3D real-time holographic display in the future.

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