Fast calculation of wave front amplitude propagation: a tool to analyze the 3D image on a hologram
(Invited Paper)

J. -S. Chen and D. P. Chu*

University of Cambridge, Photonics and Sensors Group, Department of Engineering, 9 JJ Thomson Avenue, Cambridge, UK

**Corresponding author: dpc31@cam.ac.uk*

Received March 4, 2014; accepted April 23, 2014; posted online May 30, 2014

A simple approach to calculate the amplitude component of a wave front propagating in space from a hologram is proposed. It is able to calculate the amplitude distribution on a plane at any distance rapidly using a standard GPU. This is useful for analyzing and reconstructing the 3D image encoded on a hologram.

OCIS codes: 090.1995, 070.7345, 100.6890.
doi: 10.3788/COL201412.060021.

1. Introduction

Digital hologram calculation has been developed for decades and the efficiency of algorithms has also been improved at the same time. However, there wasn't an efficient method to calculate a single cross-section of the 3D scene encoded on a hologram. In principle one can apply Huygens-Fresnel principle to compute the wave propagation[1]. This method calculates all wave fronts propagated from every point on the hologram plane to every point on a targeted image plane, and then sums them up at each pixel of the image plane. Apparently, this method is extremely time-consuming because a digital hologram can have millions or even billions of pixels and the targeted image plane as well. Therefore the total calculation can be in the order of quadrillions, and makes it difficult to be used in practice. Researchers from Nihon University proposed a method based on fast Fourier transform (FFT) to simulate a reconstructed 3D image from a hologram[2,3]. They deal with small segments of a hologram to decide the direction and amplitude of the light ray from each segment and apply a ray model to simulate the reconstructed image. This method treats each segment as an individual component, which is a simplification of the real situation. Lights from different pixels on a hologram do interfere with each other even if they are not from the same segment. In addition, this method is based on ray tracing which does not faithfully reconstruct a wave front. These two drawbacks make it difficult to extract the cross-section image at a given distance from a hologram. As a result, to check the quality of the 3D scene encoded on a hologram one has to either perform a large number of calculations or optically reconstruct the whole 3D image and then capture it to analyze, which are both time consuming and labor intensive.

To simplify the way to analyze the 3D scene encoded on a hologram, a fast simulation method is proposed here based on the layer-based method which was developed for easy generation of 3D holograms[4]. It should be mentioned that this proposed method can only deal with the amplitude component of the wave front as propagated from a 3D hologram, but this is sufficient for most image analysis applications.

2. Layer-based method

The layer-based method is a way to generate 3D holograms using an optical Fourier transform (FT) which links the image plane and hologram plane in an f-f system, as shown in Fig. 1(a) where I is the targeted image at the image plane and $H(f)$ the hologram at the hologram plane. These two planes are located at two sides of the lens by the distance of f, which is the focal length of the lens. Computationally, the image plane and the hologram plane are each other's FT. Note that an f-f system is symmetric, so FT and FT^{-1} have the same optical meaning. This means if we place a hologram at any side of a lens at the focal length distance and provide a planar light wave to illuminate it along the lens's axis toward the lens, the wave front on the image plane at the focal length distance on the other side of the lens will be the FT of this hologram. If we could move the hologram physically forward to the lens plane without affecting the lens itself, the phase component of the wave front at the image plane would change but not the amplitude component, which is of little concern in analyzing the reconstructed image since human eyes don't detect phase. This concept is illustrated in Fig. 1(b) where ϕ is the change of phase after moving the hologram forward, $L(f)$ the phase representation of the lens with a focal length of f and $H(\mathrm{f}) \otimes L(f)$ the wave front right in front of the lens. The "$\otimes$" operator means the direct multiplication between the pair of the corresponding matrix elements.

The fact that moving the hologram along the lens axis doesn't change the amplitude profile at the image plane can be shown by Eq. (1)[5], where $I_f(u, v)$ is the complex image on the image plane, (u, v) the 2D coordinate normal to the lens axis, $h_f^{\mathrm{FT}}(u, v)$ the complex value of the Fourier transform of $H(f)$, d the distance between hologram and the lens, λ the wavelength of the light (532 nm in our experiment) and k the wavenumber defined as $2\pi/\lambda$. It is clear that changing d will only affect the phase part but not the amplitude part of the image.

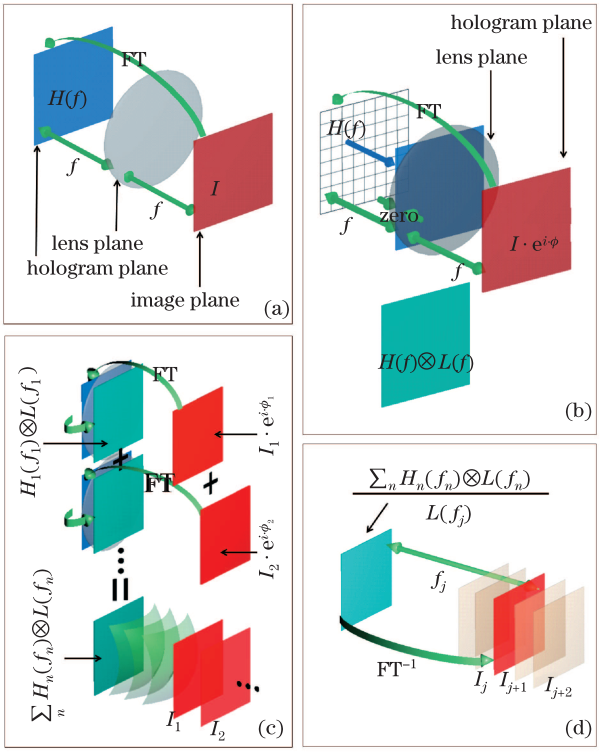

Fig. 1. Illustration of the layer-based method for hologram calculation and its reverse process as the proposed simulation method: (a) a normal FT in a f-f optics system; (b) the concept of moving hologram plane forward without changing the amplitude in image plane; (c) the concept of the layer-based method; and (d) the reverse process of the layer-based method, on which the of proposed simulation algorithm is based.

$$I_f(u,v) \sim \frac{\exp\left(j\frac{k}{2\cdot f}\left(1-\frac{d}{f}\right)(u^2+v^2)\right)}{j\cdot\lambda\cdot f} \times h_f^{\mathrm{FT}}(u,v). \quad (1)$$

Consequently, to calculate the 3D hologram of a 3D object at the lens plane, we can firstly slice the object into layers and then calculate the hologram by summing up the sub-holograms, each of which is the product of the FT of one sliced layer with a corresponding holographic lens of a focal length equal to the distance between the lens plane and the targeted image plane. Note that here "sum up" means matrix addition, and "multiply" means the $\otimes$ operation mentioned earlier. We call this type of hologram "layer-based holograms", as shown in Fig. 1(c), where n is the index of layers and ϕ the phase change of each reconstructed image planes. This method is fast and easy to implement as well as capable to adjust the resolution of accommodation cue. Its image-based nature allows it to adapt easily versatile graphic rendering techniques and extract graphics features.

3. Reverse Layer-based method

Note that the relationship between the hologram plane and the image plane here doesn't change even when we reverse the process. We can use the process in Fig. 1(d) to obtain the amplitude information at any given image plane from the hologram plane according to the procedure:

The first step is to calculate the lens pattern corresponding to the distance between the hologram plane and the targeted image plane. This lens pattern can be calculated by placing a point source at the focal point of the lens and then propagating a wave to every pixel of the hologram plane based on Huygens-Fresnel principle, as shown in Eq. (2) where f_j is the focal length of the lens, $d_j(\alpha,\ \beta)$ the distance from the point source to a given pixel, $(\alpha,\ \beta)$ the coordinate of the given pixel on the hologram plane and $L(f_j)$ the lens pattern:

$$L(f_j) = \sum_{\text{allpixles}} \frac{1}{d_j(\alpha,\beta)} \cdot e^{-i\cdot 2\pi\cdot\frac{d_j(\alpha,\beta)}{\lambda}} \quad (2)$$

The second step is to deduct the lens pattern from the 3D hologram at the lens plane and then perform an inverse FT on the remaining hologram there. Note that the deduction here means dividing each pixel of the 3D hologram to the corresponding pixel of the lens hologram. This step can be summarized by Eq. (3), where H is the 3D hologram which is equal to $\sum_n H(f_n)\otimes L(f_n)$ in this case, $L(f_n)$ the lens hologram with a focus length of f_n, n the index of the layers, $W(f_m)$ is the wave front at the image plane away from the lens plane by a distance f_m, where m is the index of the target plane.

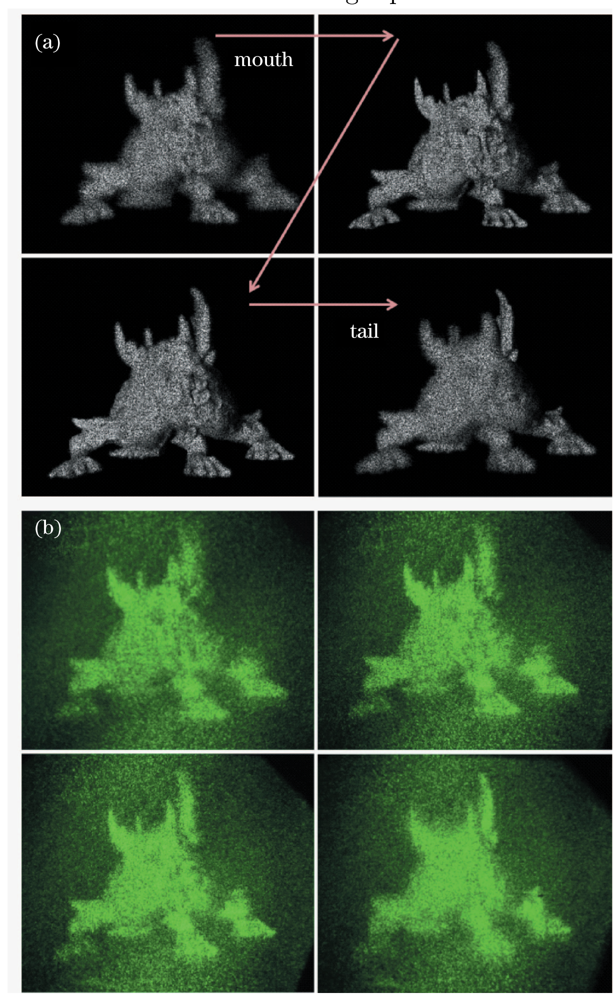

Fig. 2. (a) Amplitude images constructed from the simulated wave front at different distances from a layer-based 3D hologram at the lens plane using the proposed method; (b) physical amplitude image formed on a scatter screen placed at the corresponding distances from the same hologram.

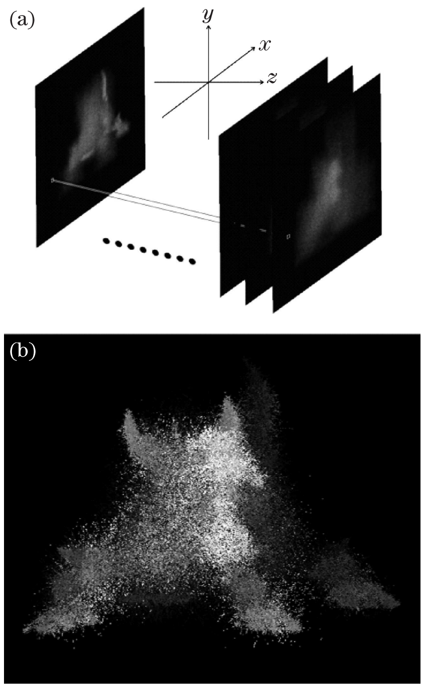

Fig. 3. (a) An illustration of multiple image planes reconstructed from the calculated wave front (amplitude only). The transparent bar connecting the hologram plane and different image planes indicates a same x-y coordinate. (b) The depth map of the encoded 3D image as reconstructed for the same hologram used in Fig. 2.

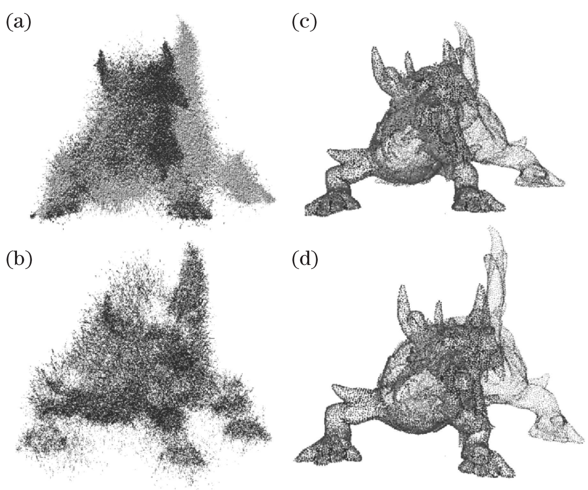

Fig. 4. 3D data points calculated for two different viewing angles. (a) and (b) are the results calculated from the reconstructed depth map; (c) and (d) are the source data.

$$W(f_m) = FT^{-1}\Big[\frac{H}{L(f_m)}\Big], \qquad (3)$$

It is worth mentioning that the H used in Eq. (3) is not necessarily to be a layer-based 3D hologram constructed from $\sum_n H(f_n) \otimes L(f_n)$. Instead it can be a 3D hologram generated by any method, such as a point-based or polygon-based method, and the advantage of rapid calculation by using this reverse layer-based procedure to analyze the encoded 3D images is the same for all holograms.

The main calculation load is to calculate holographic lenses of specific focal lengths, which is not computationally heavy. Even if the number of the reconstructed image planes is in the order of thousands, the total computation load is still much less than that of the traditional reconstruction method based on Huygens-Fresnel principle. In addition, FFT is also computationally efficient. Both of them can be done very fast by parallel calculation on a graphic processing unit (GPU). We can simulate a hologram with a resolution of 1,024 x 768 propagating to an image plane of the same resolution in less than 100 milliseconds on our computer, making it possible to reconstruct 1,000 image planes for analysis within 2 minutes.

The hardware of the computer which we use is of standard consumer grade. GPU is GTX 460SE, with a 650-MHz graphic clock, 288 cores, 1-GB memory and 108.8-GB/sec memory bandwidth. CPU is Intel i3 560 at 3.33GHz. The programming interface is MATLAB R2011b with an open source library GPUmat V.028 which allows MATLAB to run parallel calculation on a Nvidia GPU.

4. Image results

In the following, we show the reconstruction results using a layer-based hologram, as shown in Fig. 2(a). At the same time, we load this hologram onto a binary SLM on with 13.68-μm pixel pitch and locate a scatter at different distances from the hologram to capture both the amplitude of the replayed wave front and the associated accommodation cue, as shown in Fig. 2(b). Both results clearly show convincible 3D image reconstruction at images planes of different distances and the corresponding changes of the accommodation cue. The good agreement between the calculation and experiment supports the effectiveness of the proposed approach.

Such an approach can be very useful in image analysis. For example, by calculating the wave front propagating from a hologram at a number of image planes as shown in Fig. 3(a), we can obtain the amplitude distribution of the propagating wave in space. For each coordinate on the x-y plane. the amplitude values at different positions along the z axis can be used to locate the amplitude maximum and obtain the z coordinate for a focused point. By applying this procedure to all the pixels, we can construct the depth map of the 3D image, as shown in Fig. 3(b). Based on such a depth map, a 3D image can be calculated for any given viewing angle, as shown in Fig. 4.

It is noted that there is noticeable noise in the holographic images reconstructed through either calculation or optical replay. This is partly due to the spatial and phase quantization of the hologram and partly due to the wave propagation property, which causes the energy from the non-focused points interferes with that at the focused points. Therefore, the actual measured value on a voxel point in space from the reconstructed wave front is not as clear as its graphics 3D raw data. It is possible to reduce such noises but this is not to be discussed in this letter.

5. Conclusion

In conclusion, a simple approach of rapid calculation of a 2D image layer of the 3D object recorded on a digital hologram was proposed. It can be used to simulate, check and analyze the image quality and properties from any computer generated holograms.

6. Acknowledgement

JSC and DPC would like to thank the UK Engineering and Physical Sciences Research Council (EPSRC) for the support through the Platform Grant in Liquid Crystal Photonics. JSC also want to thank Taiwan Education Ministry for their funding to his PhD study in Cambridge.

References

1. J. W. Goodman, *Introduction to Fourier Optics*, 3rd ed. (Roberts & Company, 2005) pp. 63-91.
2. H. Yoshikawa, T. Yamaguchi, and H. Fujita, *In Adaptive Optics: Analysis and Methods/Computational Optical Sensing and Imaging/Information Photonics/Signal Recovery and Synthesis Topical Meetings on CD-ROM, OSA Technical Digest (CD)* (Optical Society of America, 2007).
3. T. Yasuda, M. Kitamura, M. Watanabe, M. Tsumuta, T. Yamaguchi, and H. Yoshikawa, Proc. SPIE **7233,** 72330H (2009).
4. J. -S. Chen, Q. Smithwick, and D. Chu, Proc. SPIE **8648,** 864824 (2013).
5. J. W. Goodman, *Introduction to Fourier Optics*, 3rd ed. (Roberts and Company, 2005) pp. 103-108.