One step hologram calculation for multi-plane objects based on nonuniform sampling

(Invited Paper)

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Received February 21, 2014; accepted April 4, 2013; posted online May 20, 2014

The nonuniform sampling method in hologram plane is proposed to reconstruct objects on multi-plane simultaneously. The hologram is nonuniformly sampled by decomposing it into several parts with various sampling rates. The hologram is calculated based on the nonuniform fast Fourier transform (NUFFT) algorithm. In the experiment, we load this nonuniformly sampled hologram on phases-only spatial light modulator (SLM), and by illumination with collimated light objects with different sampling rates are reconstructed at different distant planes simultaneously. Both of the numerically simulation and optical experiments are performed to demonstrate the feasibility of our method. The experiment also shows that our proposed nonuniform sampled hologram for multi-plane objects is calculated by only one step, better than conventional method that needs several steps of calculation proportional to the numbers of object planes.

OCIS codes: 090.1760, 090.2870, 090.0090.

doi: 10.3788/COL201412.060020.

Holographic display technology has the capability to reconstruct all the information of an object. However, the three-dimensional (3D) object always contains too much information and it is time consuming to calculate the computer generated hologram. To reduce the computational burden, one approach is decomposing the object into several layers or slices, which corresponded to different planes[1]. And therefore, the calculation is converted from three dimensions to two dimensions which dramatically reduce the information in the calculation. Conventional method to calculate the computer generated hologram for multi-plane object is based on the scalar diffraction calculation such as the Fresnel diffraction and Angular spectrum method[2−5]. During the calculation, the light filed in each object plane is numerically propagated to the hologram plane respectively by the diffraction calculation. The hologram is then obtained by adding all of the complex distribution propagated from each object plane. So the diffraction will be calculated several times according to the number of the object planes.

During the diffraction calculation, the fast Fourier transform (FFT) is normally used to accelerate the calculation. It is well known that if the sampling rate of the hologram is changed, the image will be reconstructed in another distant plane. This principle has been widely used in the digital holography microscopy for image reproduction of object at different distances from CCD[6−8]. In this letter, we use this principle to calculate a hologram for objects on multi-plane by nonuniformly sampling the hologram plane.

Let us first consider the Fresnel diffraction of a hologram reconstruction in one dimension which is expressed as

$$f(x)=\int h(u)\cdot\exp\left[\frac{i\pi u^2}{\lambda z}+\frac{i\pi x^2}{\lambda z}-\frac{i2\pi ux}{\lambda z}\right]\mathrm{d}u, \quad (1)$$

where $h(u)$ and $f(x)$ represents the hologram and the image respectively, λ is the wavelength, and z is the distance of diffraction. According to Ref. [6], if the sampling rate in the hologram is changed from Δu to $\Delta u'=\alpha\cdot\Delta u$, then u is changed to $u'=\alpha\cdot u$. Consequently, Eq. (1) is rewritten as

$$\begin{aligned}f(x)&=\int h(u)\cdot\exp\left[\frac{\mathrm{i}\pi\alpha^2u^2}{\lambda\alpha^2z}+\frac{\mathrm{i}\pi\alpha^2x^2}{\lambda\alpha^2z}-\frac{\mathrm{i}2\pi\alpha^2xu}{\lambda\alpha^2z}\right]\mathrm{d}u\\&=\int h(u')\cdot\exp\left[\frac{\mathrm{i}\pi u'^2}{\lambda z'}+\frac{\mathrm{i}\pi x'^2}{\lambda z'}-\frac{\mathrm{i}2\pi u'x'}{\lambda z'}\right]\mathrm{d}u, \quad (2)\end{aligned}$$

where $z'=\alpha^2z$, $x'=\alpha\cdot x$. It is clear from Eq. (2) that if the sampling rate in hologram changes from Δu to $\alpha\cdot\Delta u$, then the image will be reconstructed in a different distance $z'=\alpha^2z$ with a scale of α in size.

The method for design of the hologram for multi-plane objects is described in Fig. 1. The 3D object composes four letters (A, B, C, and D) which are located at four different distances. The two-dimensional (2D) image of this 3D object is located in the image plane. Let us assume that the size of the 2D image is $4N$. The origin of the coordinate system is in the center of this image and each letter is just located in different quadrants (see Fig. 1). According to the position of four letters, we design the hologram as follow. The hologram which has the same size $4N$ is first decomposed into four parts equally and each part (size $N\times N$) is responsible for the reconstruction of one letter. The sampling rates in each part are not the same, as shown in Fig. 1. From the first quadrant to the fourth quadrant, the sampling rate in sequence is $\Delta u_2=\alpha_2\Delta u$, $\Delta u_1=\alpha_1\Delta u$, $\Delta u_3=\alpha_3\Delta u$, and $\Delta u_4=\alpha_4\Delta u$, here Δu is the sampling rate of the spatial light modulator (SLM) which will be introduced below. So in this way the hologram is composited of four parts and each part has the same resolution ($N\times N$) but different samplings. Then the light is propagated from the

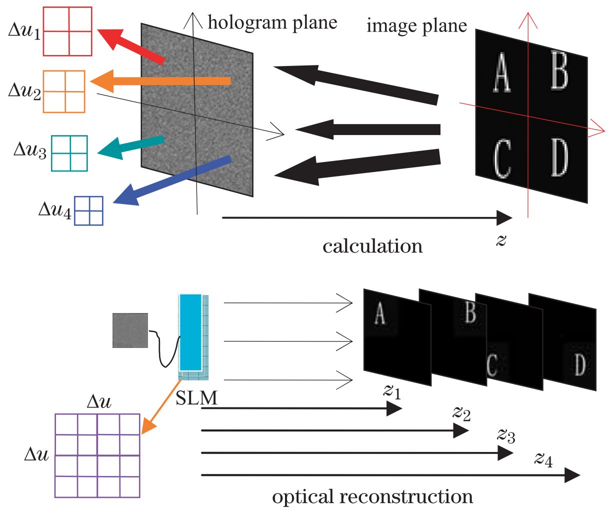

Fig. 1. (Color online) Schematic diagram of the calculation and reconstruction of nonuniform sampled hologram for multi-plane objects.

2D image to this nonuniform sampled hologram by using the diffraction theory. So in this way the information in the hologram for reconstructing the letters is encoded according to the quadrant positions, for example, the first quadrant of the hologram mainly contributes to the reconstruction of letter "B" but has less contributions to other letters, so as the conclusion for other three quadrants of the hologram. It should be noted that to realize one step calculation we combine four letters into one image, and the term "one step" means the diffraction calculation is only between two planes, rather than from each object plane to the hologram plane in conventional calculation method.

In the optical reconstruction process as shown in Fig. 1, the hologram is loaded into the SLM device which has the uniform sampling rate Δu. So the sampling rate of the hologram is changed in the reconstruction process compared with that in the calculation. According to the principle described in Eq. (2), the change of sampling rate from Δu_1 to Δu will give rise to the movement of reconstruction position of letter "A" from the distance z to z_1 as shown in Fig. 1. So as for the reconstruction of letter "B", "C", and "D" respectively. Based on the change of the sampling rate in four parts, the hologram reconstructs four images on four different planes with only one step diffraction calculation.

In the following, we will explain the details of hologram calculation. The FFT algorithm is normally used to accelerate the calculation of hologram. However, the FFT based algorithm (Eq. (1)) requires uniform sampling in both of the image plane x and the hologram plane u. In our method, the image plane is uniformly sampled, but the hologram is divided into four parts and each part has its own sampling rate (Δu_1, Δu_2, Δu_3 and Δu_4), so the hologram is nonuniformly sampled, in which the FFT based algorithm cannot be used. And thus, we use nonuniform fast Fourier transform (NUFFT) algorithm to calculate the nonuniform sampled Fresnel diffraction. NUFFT is a latest widely used method which permits one to calculate the discrete Fourier transform between nonuniform sampled points. Several algorithms of NUFFT calculation have been proposed in the past years[9−11]. The whole calculation of the Fresnel diffraction from the image plane to the nonuniform sampled hologram plane can be simply written as

$$\begin{aligned} h(u) &= \exp\left(\frac{i\pi u^2}{\lambda z}\right)\cdot \text{NUFFT}\left\{f(x)\cdot\exp\left(\frac{i\pi x^2}{\lambda z}\right)\right\} \\ &= \text{NUFRE}\left\{f(x)\right\}, \end{aligned} \tag{3}$$

where NUFFT$\{\cdot\}$ denotes the calculation of NUFFT transform[12−14], and the NUFRE$\{\cdot\}$ is the symbol of the nonuniform sampled Fresnel diffraction. According to the Refs. [11,14], the NUFRE can be defined into two types. The first type is the calculation from nonuniform sampled plane to the uniform sampled plane, which is defined as type 1, and the formula can be written as

$$\begin{aligned} f(x) &= \text{NUFRE1}\left\{h(u)\right\} \\ &= \exp\left(\frac{i\pi x^2}{\lambda z}\right)\cdot \text{NUFFT1}\left\{h(u)\cdot\exp\left(\frac{i\pi u^2}{\lambda z}\right)\right\}, \end{aligned} \tag{4}$$

where NUFFT1 is the type 1 of NUFFT. Here, the plane u is nonuniform sampled and plane x is uniform sampled. The second type (defined as type 2) is the calculation from uniform sampled plane to the nonuniform sampled plane, likewise, the expression can be written as

$$\begin{aligned} h(u) &= \text{NUFRE2}\left\{f(x)\right\} \\ &= \exp\left(\frac{i\pi u^2}{\lambda z}\right)\cdot \text{NUFFT2}\left\{f(x)\cdot\exp\left(\frac{i\pi x^2}{\lambda z}\right)\right\}, \end{aligned} \tag{5}$$

where NUFFT2 is the type 2 of NUFFT. The plane x is uniform sampled and plane u is nonuniform sampled. In our situation the calculation from image plane to the hologram plane is type 2 (Eq. (5)). Here $f(x)$ can be considered as the light distribution of the whole image composed of four letters and $h(u)$ can be considered as the light distribution of the whole hologram composed of four parts with different samplings in Fig. 1. It should be noted that the NUFFT operation in Eq. (5) takes the crucial function for realizing the calculation from the image to the nonuniform sampled hologram in only one step. And when the calculated hologram is loaded into the SLM which has a uniform sampling rate, the sampling rates in each part is changed to uniform, and thus will cause the hologram reconstruct the four objects (letters) at different planes according to Eq. (2).

We first present the computer simulation results to demonstrate our proposed method of the nonuniform sampled hologram for objects on multi-plane. A 3D object is composed of four Chinese characters which are located at different planes. Figure 2(a) is a 2D image with four Chinese characters that is used to calculate the nonuniform sampled hologram. This 2D image has the distance z = 500 mm from the hologram plane, and the sampling of this image is uniform. Correspondingly, the hologram is divided into four parts where different sampling rates are imposed as shown in Fig. 2(b). The sampling rate from the first to the fourth parts of the hologram are $\Delta u_1 = 1.3\Delta u$,

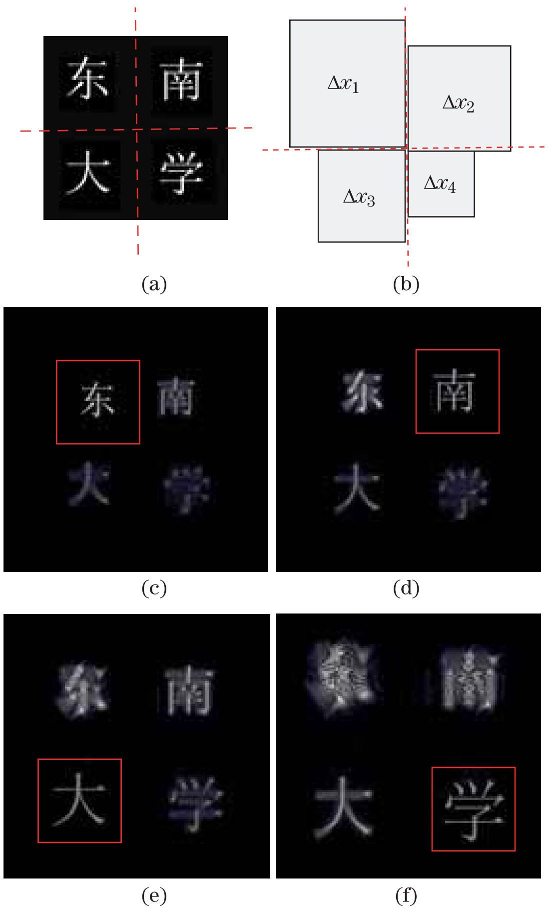

Fig. 2. (Color online) Numerical simulation results of the reconstruction of the nonuniform sampled hologram on different planes. (a) is the target image composed of four Chinese characters, (b) shows the sampling method of the hologram, (c)–(f) are the reconstructions on four different planes of different orders with each character clearly focused on each plane.

$\Delta u_2 = 1.1\Delta u$, $\Delta u_3 = 0.9\Delta u$, and $\Delta u_4 = 0.7\Delta u$ (Δu is the sampling rate of the SLM). So the hologram looks scaled in different parts as shown in Fig. 2(b). The hologram is calculated by the type 2 nonuniform sampled Fresnel diffraction (Eq. (5)) in one step.

Then the reconstruction process of the complex hologram is simulated by using the angular spectrum method, and the sampling rate of the SLM is Δu. Due to the principle in Eq. (2), the change of the sampling rate will result in the reconstruction of four characters in different planes. The relationship between the new distance and the distance z can be calculated as shown in Eq. (2). For the first character, the sampling rate changes from $1.3\Delta u$ to Δu, the reconstruction distance is changed from z to $(1/1.3)^2 z$. Followed by this rule, The reconstruction distance on four different planes of each Chinese character is $z_1 = z/(1.3)^2 = 296$ mm, $z_2 = z/(1.1)^2 = 413$ mm, $z_3 = z/(0.9)^2 = 617$ mm, and $z_4 = z/(0.7)^2 = 1020$ mm respectively. The reconstruction result by computer simulation is shown in Figs. 2(c)–(f). From Figs. 2(c)–(f) we can see that each Chinese character is clearly in focus on its plane while the other characters are out of focus, which confirms that the nonuniform sampled hologram can reconstruct objects on multiply planes.

In the optical reconstruction experiment, the nonuniform sampled computer generated hologram is loaded into the SLM. Because the SLM we used is a phase type SLM, so it is necessary to encode the complex hologram into a phase-only hologram (kinoform). We used the Gerchberg-Saxton (GS) algorithm to optimize the hologram[15]. The iteration starts by multiplying an initial random phase to the image. The diffraction from uniform sampled image to nonuniform sampled hologram is calculated by NUFRE2, then we extract only the phase component and impose unity amplitude constraint, the inverse diffraction from hologram to image is calculated by NUFRE1, and the amplitude of the image is imposed while remaining the phase component. By repeating this iterative loop with a few steps, the hologram can be optimized to be used for the phase-only SLM.

Figure 3 shows the optical reconstruction of the optimized phase-only hologram. Figure 3(a) is the optical setup for reconstruction. In order to remove the zero-order noise, the phase hologram is added with an additional phase factor of a divergence lens therefore all of the reconstructed image planes are away from the focal plane of the lens. And a filter is placed at the focal plane to remove the zero-order noise as shown in Fig. 3(a). The SLM we used is holoeye Pluto with 1920×1080 resolutions and 8-μm sampling rate. The reconstructed images are captured directly by the CMOS of a camera at different distances. Figures 3(b)–(e) are the reconstructed images of four Chinese characters at the distance of $z_1 = 296$ mm, $z_2 = 413$ mm, $z_3 = 617$ mm, and $z_4 = 1\,020$ mm, respectively. Each character is clearly focused at its planes while the other characters are out of

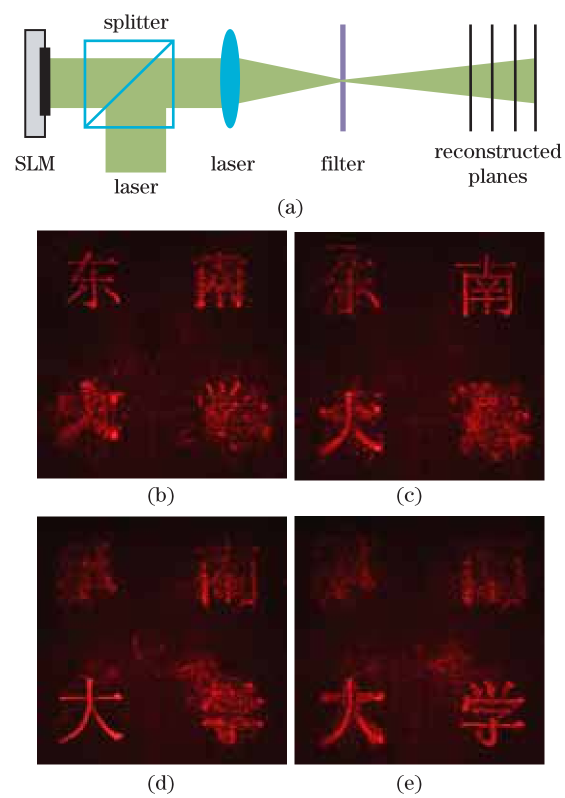

Fig. 3. (Color online) Optical reconstruction of the nonuniform sampled phase-only hologram. (a) is the optical setup for reconstruction, (b)-(e) are the reconstructions of the four Chinese characters on four different planes with each one focused respectively. (Media).

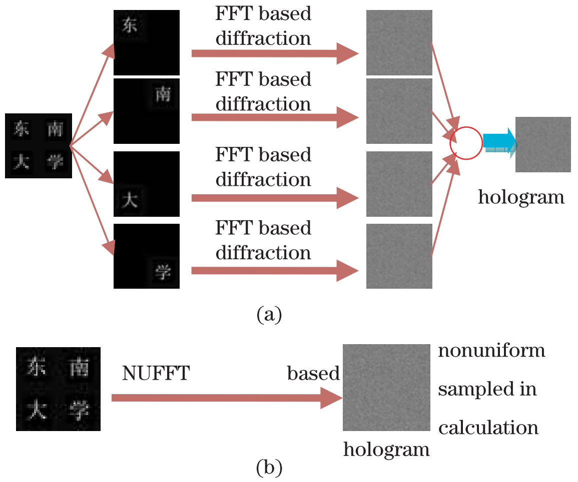

Fig. 4. (Color online) Comparison of the calculation of hologram for multi-plane reconstructions between conventional method and our nonuniform sampled method. (a) hologram calculated in conventional method, and (b) hologram calculated using nonuniform sampled method.

focus. We also record this reconstruction by moving the camera back and forth between the first focused plane and the fourth focused plane, and the whole process is vividly shown in Media.

Figure 4 shows the comparison between conventional method and our proposed method of hologram calculation for objects on multiply planes. In Fig. 4(a), the conventional method starts from the decomposition of the 3D objects into several sub-images and then each sub-image is propagated to the hologram plane by the calculation of FFT based diffraction, finally the hologram for multi-plane display is obtained by adding all of the holograms from sub-images. So if the numbers of the object planes are N, it is necessary to calculate the FFT based diffraction for N times. In Fig. 4(b), by nonuniformly sampling the hologram, we can directly calculate the diffraction from the 3D object to the hologram based on the NUFFT algorithm, and the hologram has the same function for reconstructing at different planes. It is also known that the nonuniform sampled hologram has the advantage of optimizing the numbers of sampling points and eliminating the redundant information properly[16].

It is worth mentioning that while the change of the sampling rate result in the reconstruction of object on multi-planes, the size of the image will also be changed with the different reconstruction distances, which will introduce axial distortion in the reconstruction. In order to remove this, one effective method is that the image plane is also nonuniform sampled and the sampling way should be consistent with the hologram in order to compensate the size scaling in the reconstruction. In the future we will continue our work to solve this problem.

In conclusion, we propose a method to calculate the computer generated hologram of objects on multiple planes by only one step. The hologram is nonuniformly sampled and the NUFFT algorithm is used to calculate diffraction between nonuniformly sampled planes. Both of the numerical simulation and optical experiment confirm the practicability of our method. This work has potential applications in the implementation of displaying 3D object which can be treated or decomposed into several closely planes.

This work was supported by the National "973" Program of China (Nos. 2013CB328803 and 2013CB328804), the National "863" Program of China (Nos. 2012AA03A302 and 2013AA013904), and the Aeronautical Science Foundation of China (No. 20125169021).

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