Data-embedded-error-diffusion hologram

(Invited Paper)

P. W. M. Tsang[1*] and T.-C. Poon[2,3]

[1]*Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, China*

[2]*Bradley Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA 24061, USA*

[3]*Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

[*]*Corresponding author: eewmtsan@cityu.edu.hk*

Received April 7, 2014; accepted April 23, 2014; posted online May 28, 2014

This paper describes a method for converting a complex Fresnel hologram into a phase-only hologram that can be embedded with large amount of data. Briefly, each row of pixels in the hologram is scanned sequentially in a left-to-right direction. The magnitude of each visited pixel is set to a constant, and its phase is embedded with the data. Subsequently, the error is diffused to the neighborhood pixels. The phase hologram realized with such means, which is referred to as the data-embedded-error-diffusion (DEED) hologram, is capable of preserving high fidelity on the content of the hologram and the embedded data.

OCIS codes: 090.0090, 090.1995, 090.1760.

doi: 10.3788/COL201412.060017.

1. Introduction

In this paper, we propose a method to address two important issues in digital holography. The first one is the generation of phase-only hologram, a method which is required to overcome the practical limitation of existing holographic display. The second issue is, how to embed large amount of external data into a phase-only hologram. We shall demonstrate that our proposed method is capable of handling both issues. Regarding the problem on holography display, it is well known that the three-dimensional image represented by a digital hologram can be observed visually, if the hologram can be displayed with a complex electronic accessible device. However, existing devices such as a spatial light modulator (SLM) or a liquid crystal on silicon (LCoS), are only capable of reproducing either the phase, or the magnitude of a complex holograms. An effective way of overcoming this problem, is through the optical integration of a pair of devices for displaying the magnitude and phase (or the real and imaginary) components of a complex hologram[1−3]. Alternatively, a complex hologram can be converted into a double phase-only holograms[4], and displayed with a pair of phase-only devices. Despite the success of this straightforward approach, the optical setup, which requires precise alignment of two high-resolution display devices, could be rather cumbersome. In some recent attempts, the pair of components of a complex hologram is presented in non-overlapping partitions on a single device, and the wavefronts scattered from each partition are merged with a grating[5−8]. Similar to the use of a pair of devices, the optical setups of such methods are complicated, and the display area is reduced by 2 times. In view of these shortcomings, research on the generation of Phase-Only Hologram (POH) has been identified as a potential and viable solution to the above mentioned problems. The factors that sustain such long term interest are manifolds. Basically, POH can be displayed with a single, phase-only spatial light modulator. In addition, the reconstructed image is brighter, and also inherently free from the zeroth-order diffraction and the twin image. However, the favorable features of the POH do not come without a cost, as the removal of the magnitude component of a hologram could lead to severe loss on the pictorial information it represents.

Other solutions in generating POH include the adoption of iterative steps to adjust the pixel values (each leading to a phase change to the illuminating beam that falls on it) of the POH progressively, so that the reconstructed image will ultimately match with a target image[9,10]. Despite the effectiveness, the computation time is generally lengthy, especially if the target image is comprising of multiple depth planes. A complex hologram can also be directly converted into a POH by dropping the magnitude component, if random noise is added to the source image prior to the generation of the hologram. On the downside such approach, which is referred to as the One Step Phase Retrieval, requires fast display of multiple holograms frames (each representing the same source image, but added with different noise signals) to suppress the effect of the added noise[11,12]. Recently, it has also been reported that a digital complex Fresnel hologram can be swiftly converted into a POH with the use of error diffusion[13,14]. In this approach, the magnitude of each pixel in the source hologram is forced to a constant value, and the resulting error distributed to the neighboring pixels with error diffusion[15]. The reconstructed image of a POH obtained with this method when comparing from that obtained from the original hologram, is very similar.

In this letter, we have adopted the error diffusion framework for converting a complex digital hologram into a phase-only hologram, as well as embedding large amount of external data into the hologram. Briefly, each row of pixels in a complex, digital Fresnel hologram is processed sequentially from the top to the bottom row. The pixels in each row, in turn, are scanned from a left-to-right direction. The magnitude of each visited pixel is forced to a constant value, leaving behind the phase value of the pixel. Next, the least M digits of the phase value are replaced by the data to be embedded. The error arising from with the dropping of the magnitude, together with the perpetuation of the inserted data, will be diffused to the neighborhood pixels. The hologram realized with our proposed method is referred to as the data-embedded-error-diffusion (DEED) hologram. Practical application based on this method will be demonstrated with a case study demonstrating the concealment of an intensity image into a DEED hologram. We shall show that with a suitable choice of M, both the embedded intensity image and the reconstructed image of the hologram, can be preserved with favorable quality and high fidelity.

2. Data-embedded-error-diffusion (DE-ED) phase-only hologram

In this section, we shall describe our proposed method for converting a complex Fresnel hologram to a data-embedded-phase-only DEED hologram based on error diffusion. To begin with, consider a digital Fresnel hologram $H(u, v)$ that is generated from a three-dimensional object scene as given in Eq. (1).

$$H(u,v) = \sum_{k=0}^{N-1} a_k \exp\left(\frac{i2\pi r_k(u,v)}{\lambda}\right), \quad (1)$$

where u and v are the horizontal and the vertical coordinates, respectively. N is the total number of object points, λ is the wavelength of the optical beam. a_k is the intensity of the kth point in the scene, and $r_k(u, v)$ is the distance between the kth object point to a location (u, v) on the hologram. It can be inferred from Eq. (1) that each pixel of $H(u, v)$ is a complex quantity that can be expressed in terms of its magnitude and phase components, i.e.

$$H(u,v) = |H(u,v)| \angle (H(u,v)), \quad (2)$$

where $\angle[\cdot]$ represents the phase of the complex quantity within the bracket. Without loss of generality, we assume that the magnitude of $H(u,v)$ is normalized between $[0,1]$, with the maximum value '1' representing a transparent pixel that only imposes phase changes on the light passing through it. The data to be embedded is assumed to be a two dimensional array of data, denoted by $I(u,v)$. Each data point of $\angle[H(u,v)]$ and $I(u,v)$ are represented as a P-bits binary number. Next, the bit-string of $I(u,v)$ is shifted to the right hand side by $(P-M)$ bits, resulting in a bit-string of the revised image $D(u,v)$ given by

$$D(u,v) = I(u,v) >> (P-M), \quad (3)$$

where $R >> S$ denotes shifting a binary number R by S bits to the right hand side. The hologram is processed from the top to the bottom row. In each row, the pixels are sequentially visited along the left-to-right directions. To convert the hologram into a phase-only image, each visited is converted to a phase-only value by setting the magnitude to '1', while retaining the phase component intact. To embedded the external data, the least M significant digits of the bit-string of the phase quantity of the hologram pixel are replaced with the bit-string of $D(u,v)$. The revised bit-string of the phase only hologram $H_\mathrm{p}(u, v)$, which contains the embedded data, is given by

$$|H_\mathrm{p}(u,v)| = 1,$$
$$\angle|H_\mathrm{p}(u,v)| = \{\angle[H(u,v)] \wedge B_M\} \vee D(u,v), \quad (4)$$

where B_M is a P-bits binary number with the least M significant bits set to zero, and the rest set to '1'. $\wedge$ and $\vee$ are the logical 'AND' and the 'OR' operators, respectively.

Equations (3) and (4) can be illustrated with an example illustrated in Figs. 1(a)–1(c), based on $M = 5$, and $P = 8$. Figure 1(a) shows a pixel $\angle[H(u,v)]$ on the phase-only-hologram, a pixel $I(u,v)$ of the image to be embedded into the hologram, and the binary number B_M with $M = 5$. $a_i|_{0\le i<8}$ and $b_i|_{0\le i<8}$ represent the binary bits of the hologram pixel and the image pixel, respectively. The result of applying Eq. (3) to shift the bits in the pixel $I(u,v)$ by $(P-M) = 3$ bits to the right, resulting in a revised pixel value $D(u,v)$, is shown in Fig. 1(b). The figure also shows the result of $\angle[H(u,v)] \wedge B_M$, which is formed by setting the least M significant bits of $\angle[H(u,v)]$ to zero. Fig. 1(c) depicts the result of applying Eq. (4) to combine $D(u,v)$ and $[\angle H(u,v)] \wedge B_M$ with the "OR" operation to obtain the phase term of the hologram pixel $\angle[H_\mathrm{p}(u,v)]$. From Fig. 1(b), it is noted that the bit-string of $D(u,v)$ and the bit-string of $\angle[H(u,v)] \wedge B_M$ are orthogonal to each other, so that they can be separated in the future.

The process in Eq. (4) results in an error on the value of the hologram pixel, which can be expressed as

$$E(u_j, v_j) = H(u_j, v_j) - H_\mathrm{p}(u_j, v_j). \quad (5)$$

The error $E(u_j, v_j)$ is diffused to its adjacent neighbor at the right, as well as the 3 pixels immediately below it. Adopting the syntax of the C programming language, we have

$$H(u_j, v_j+1) = H(u_j, v_j+1) + w_1 E(u_j, v_j), \quad (6)$$
$$H(u_j+1, v_j-1) = H(u_j+1, v_j-1) + w_2 E(u_j, v_j), \quad (7)$$
$$H(u_j+1, v_j) = H(u_j+1, v_j) + w_3 E(u_j, v_j), \quad (8)$$
$$H(u_j+1, v_j+1) = H(u_j+1, v_j+1) + w_4 E(u_j, v_j). \quad (9)$$

The terms w_0 to w_3 are constant coefficients, with $w_0 = 7/16$, $w_1 = 3/16$, $w_2 = 5/16$, and $w_3 = 1/16$ as suggested in Ref. [15]. To retrieve the embedded image from the DEED hologram, the least M significant bits of the each pixel in the hologram are extracted to recover the image

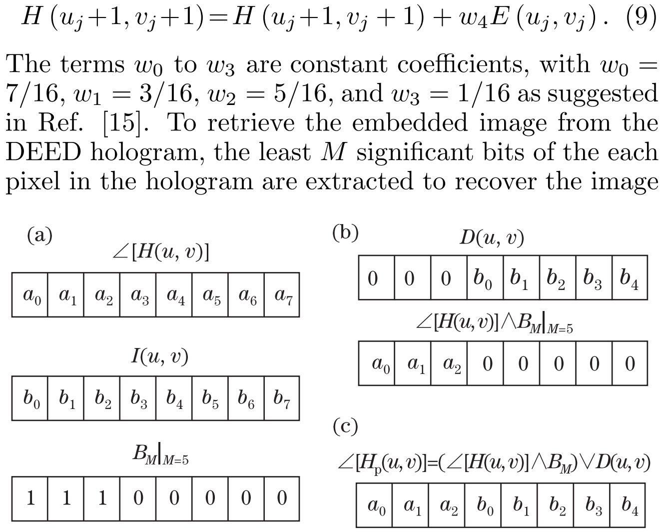


Fig. 1. (a) The bit-strings representing $\angle[H(u,v)]$, $I(u,v)$, and B_M based on $M = 5$, and $P = 8$. (b) $D(u,v)$ and $[\angle H(u,v)] \wedge B_M$. (c) $\angle[H_\mathrm{p}(u,v)]$.

$D(u,v)$. The bit-string of each pixel in $D(u,v)$ is then shifted by $(P-M)$ bits to the left hand side to recover an approximation of the original embedded image, denoted by $\tilde{I}(u,v)$. Mathematically, the bit-string of a pixel of $\tilde{I}(u,v)$ is given by

$$\tilde{I}(u,v) = \{\angle[H_\mathrm{p}(u,v)] \wedge C_M\} << (P-M), \quad (10)$$

where $R << S$ denotes shifting a binary number R by S bits to the left hand side, and C_M is a P-bits binary number with the least M bits equal to 1, and the rest of the bits set to 0. Note that C_M and B_M are orthogonal so that we could extract the information later. The operation in Eq. (10) can be illustrated with Figs. 2(a) and 2(b). Continuing with the description in the previous example depicted in Figs. 1(a)–1(c), the bit-string of the hologram pixel in $\angle[H_\mathrm{p}(u,v)]$, the binary bit-string C_M for $M=5$, and the result of applying the "AND" operator on the two bit-strings (i.e. $\angle[H_\mathrm{p}(u,v)] \wedge C_M$), are shown in Fig. 2(a). Subsequently, the bit-string $\angle[H_\mathrm{p}(u,v)] \wedge C_M$ is shifted by $(P-M)=(8-5)=3$ bits to the left hand side, resulting in the bit-string of $\tilde{I}(u,v)$ as shown in Fig. 2(b).

The reconstructed images of the DEED hologram is obtained in two stages. Firstly, a low pass filter is applied to $\angle\lfloor H_\mathrm{p}(u,v)\rfloor$, so that the "noise" that are introduced in the error diffusion and the data embedding process can be reduced. Subsequently, the filtered phase hologram can be reconstructed optically with a phase-only display device, or through numerical means.

3. Results and evaluation

In this section, we shall apply numerical simulation to evaluate the performance of our proposed method. To start with, Eq. (1) is applied to generate a digital complex Fresnel hologram $H(u,v)$ for the source image "Lenna" shown in Fig. 3(a), based on the optical settings listed in Table 1.

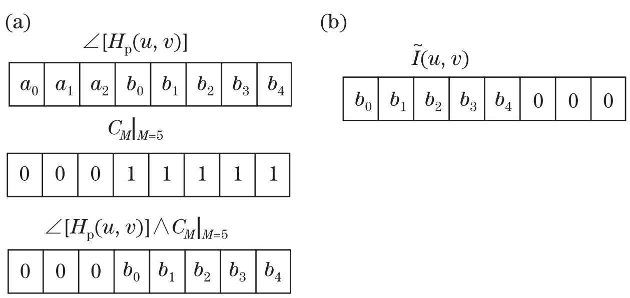


Fig. 2. (a) Bit-strings of $\angle[H_\mathrm{p}(u,v)]$ and C_M $(M=5)$, and the result of applying the "AND" operation to these two bit-strings. (b) Bit-string of $\tilde{I}(u,v)$ obtained by shifting the bit-string of $\angle[H_\mathrm{p}(u,v)] \wedge C_M$ by $(P-M)=3$ bits to the left hand side.

Table 1. Optical Settings for Generating the Complex Fresnel Hologram

Size of Hologram	2048×2048
Size of Source Image	1024×1024
Pixel Size	7×7 (μm)
Wavelength of Optical Beam (λ)	650 nm

The source image is parallel to, and at a distance of 0.3 m from the hologram. The data to be embedded is a two dimensional, 2048×2048 image "Peppers" shown in Fig. 3(b). The size of the source image is smaller than the hologram, so that there are sufficient diffraction fringes to cover the edge of the source image. To begin with, the image represented by the Fresnel hologram $H(u,v)$ is reconstructed at the focal plane, and taken to be the reference image. As the latter is practically identical to the original image "Lenna", it is not shown here. Subsequently, we have applied our proposed method to generate the DEED hologram $H_\mathrm{p}(u,v)$ from the complex Fresnel hologram, and with the image "Peppers" embedded. The pixels in the source image, the embedded image, and the DEED hologram are each represented with an 8 bits binary number (i.e., $P=8$). Three different values of M, ranging from 4 to 6, are employed to explore its effect on the DEED hologram and the embedded image. The reconstructed images of $H_\mathrm{p}(u,v)$ and the embedded images, corresponding to the three values of M, are shown in Figs. 4(a)–4(c) and Figs. 5(a)-5(c), respectively. We observed that the quality of the reconstructed images is slightly degraded with increasing values of M, while the other way round is noted for the embedded images retrieved from the DEED holograms.

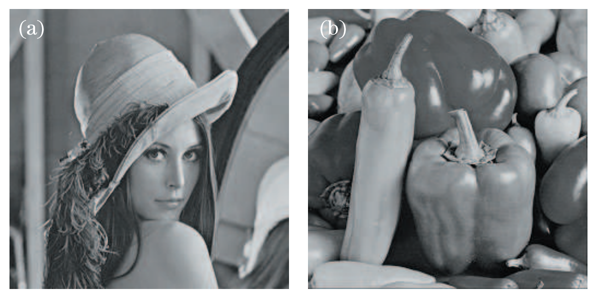


Fig. 3. (a) Source image "Lenna", (b) Embedded image "Peppers".

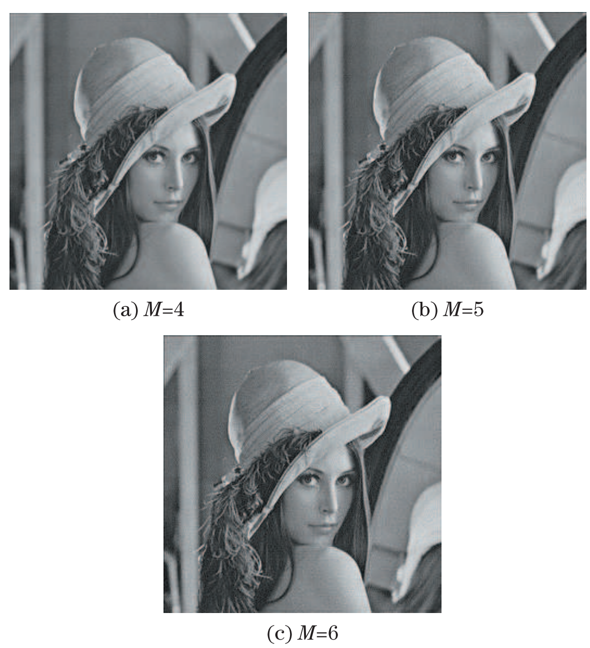
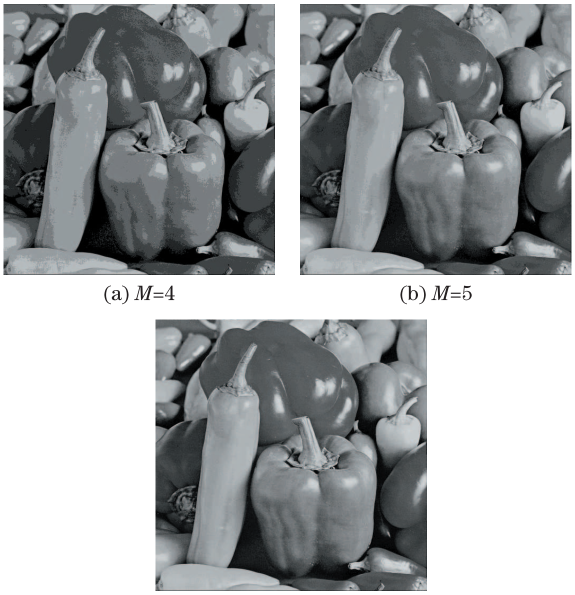


Fig. 4. (a)–(c) Reconstructed images from the DEED holograms representing the source image "Lenna", and embedded with the image "Peppers", based on different values of M.



(a) M=4 (b) M=5 (c) M=6

Fig. 5. (a)–(c) Embedded images retrieved from the DEED holograms representing the source image "Lenna", and embedded with the image "Peppers", based on different values of M.

Table 2. Quantitative Comparison between the Fidelity of the Reconstructed Images of the DEED Holograms, and the Embedded Images Retrieved from the DEED Holograms (with Reference to the Ones Obtained from the Reference Image and the Original Image "Peppers", respectively), for Different Values of *M*

	$M=4$	$M=5$	$M=6$
Reconstructed Image from DEED Hologram (dB)	34.24	32.03	30.15
Retrieved Embedded Image from the DEED Hologram (dB)	28.86	35.27	40.47

A quantitative evaluation on the quality of the reconstructed images of the DEED holograms (in terms of peak-signal-to-noise-ratio (PSNR)) shown in Figs. 4(a)-4(c), with reference to the reconstructed image obtained from a phase hologram without the embedded image (i.e. $M = 0$) , is shown in the 1^{st} row of Table 2. The fidelity of the embedded data (image) retrieved from $H_{\mathrm{p}}(u, v)$ for different values of M is shown in the second row of the Table. We observe that the quantitative values of the fidelity for both the reconstructed images and the embedded images are in line with the results shown in Figs. 4(a)–4(c) and 5(a)–5(c).

4. Conclusion

A fast method for converting a digital, complex Fresnel hologram into a phase-only hologram that is embedded with external data is reported in this paper. The hologram that is realized with this method is known as the data-embedded-error-diffusion (DEED) hologram. In brief, the pixels in the complex hologram are scanned in a row by row manner. The magnitude of each visited pixel is forced to a constant value, while the least M significant digits of the phase quantity are replaced with the data to be embedded. Subsequently, the error resulted from the dropping of the magnitude term and the incorporation of the external data of each pixel is diffused to the neighborhood pixels. We have tested our proposed method by converting a 2048×2048 complex hologram into a DEED hologram, which is embedded with an intensity image of identical size as that of the hologram. The evaluation reveals that both the reconstructed image and the embedded image are preserved with favorable fidelity with M ranging between 4 to 6.

This work was supported by the Chinese Academy of Sciences Visiting Professorships for Senior International Scientists Program (No. 2010T2G17) and the High-End Foreign Experts Recruitment Program, China (No. GDJ20130491009).

References

1. M. Makowski, A. Siemion, I. Ducin, K. Kakarenko, M. Sypek, A. Siemion, J. Suszek, D. Wojnowski, Z. Jaroszewicz, and A. Kolodziejczyk, Chin. Opt. Lett. **9,** 120008 (2011).
2. M.-L. Hsieh, M.-L. Chen, and C.-J. Cheng, Opt. Eng. **46,** 070501 (2007).
3. R. Tudela, E. Martín-Badosa, I. Labastida, S. Vallmitjana, I. Juvells, and A. Carnicer, J. Opt. A **5,** S189 (2003).
4. C. K. Hsueh and A. A. Sawchuk, Appl. Opt. **17,** 3874 (1978).
5. X. Li, Y. Wang, J. Liu, J. Jia, Y. Pan, and J. Xie, *Dig. Holo. and 3D Img., OSA Technical Digest* DTh2A. 3 (2013).
6. S. Reichelt, R. Häussler, G. Fütterer, N. Leister, H. Kato, N. Usukura, and Y. Kanbayashi, Opt. Lett. **37,** 1955 (2012).
7. H. Song, G. Sung, S. Choi, K. Won, H. Lee, and H. Kim, Opt. Express **20,** 29844 (2012).
8. J.-P. Liu, W. Y. Hsieh, T.-C. Poon, and P. W. M. Tsang, Appl. Opt. **50,** H128 (2011).
9. R. W. Gerchberg and W. O. Saxton, Optik **35,** 237 (1972).
10. J. Yeom, J. Hong, J.-H. Jung, K. Hong, J.-H. Park, and B. Lee, Chin. Opt. Lett. **9,** 120009 (2011).
11. E. Buckley, *Proc. SID Symp.* 1074 (2008).
12. A. J. Cable, E. Buckley, P. Marsh, N. A. Lawrence, T. D. Wilkinson, and W. A. Crossland, SID International Symposium Digest of Technical Papers **35,** 1431 (2004).
13. P. W. M. Tsang and T.-C. Poon, Opt. Express **21,** 23680 (2013).
14. P. W. M. Tsang, A. S. M. Jiao, and T.-C. Poon, Opt. Express **22,** 5060 (2014).
15. R. W. Floyd and L. Steinberg, Proc SID **17,** 75 (1976).