3D diffractive lenses to overcome the 3D Abbe subwavelength diffraction limit

Igor Minin* and Oleg Minin

Siberian State Academy of Geodesy, 10 Plahotnogo, Novosibirsk, 630108, Russia

**Corresponding author: prof.minin@gmail.com*

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The innovative radiating structures as a conical millimeter wave FZP lens are proposed for subwavelength focusing. The results of FDTD simulation and experimental verification are discussed. It has been shown that in contrast to the flat diffractive optics the curvilinear 3D diffractive conical optics is capable of overcoming 3D Abbe barrier with a focal distance F greater than 2λ. The focal intensity distribution for such type of lenses is not circularly symmetric and thus the focal spot in the high numerical aperture case is no longer an Airy pattern. These results may find useful applications in optical microscopes, including "reverse-microscope", nondestructive testing, microoptics, and nanooptics.

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In optical waveband it has been demonstrated that a radially polarized field can be focused by an aplanatic lens to a spot size significantly smaller (half maximum area (HMA) $\sim 0.16\lambda^2$ and full width at half maximum (FWHM)$\sim 0.45\lambda$) than that for linear polarization (HMA$\sim 0.26\lambda^2$)[1]. Later the simulation results showed that by means of diffractive lens a focal spot with HMA equal to $0.14\lambda^2$ may be obtained[2]. A method for generating sub-wavelength (0.44λ) longitudinally polarized beam using lens axicon, which could propagate without divergence over lengths of about 2λ in free space was presented in Ref. [3].

In millimeter wave band the focusing properties of flat diffractive lenses were first investigated by means of FDTD simulation and the results of the focal fields of a phase correcting Fresnel lens examination were described for several small values of $F/D < 0.2$ and with $F \leqslant 2\lambda$[4−6]. It has been shown that a spatial resolution (focal spot diameter at half intensity equal to the FWHM) of less than $0.3-0.4\lambda$ is achievable without the immersion medium (lenses). Moreover, it has been shown that the focal intensity distribution is not circularly symmetric and thus the focal spot in the high numerical aperture (NA) case is no longer an Airy pattern. It was also shown that with the exception of one case ($F = 2\lambda$), the axial resolutions are all less than 2.0λ, which is significantly finer than hose achieved by lenses with large values of F/D. So the "Abbe barrier" was completely broken by such flat diffractive lenses with unique 3D super resolution[4−6].

Five years later investigations in the optical range[7,8] fully confirmed the results of earlier studies in the millimeter range[4−6]. It was shown that, when illuminated by a linearly polarized Gaussian beam, the binary zone plate with a focal length equal to the wavelength (λ=532 nm) forms an elliptical focal spot with a diameter at half intensity equal to FWHM$_x$=0.44λ and FWHM$_y$=0.52λ[7,8].

But subwavelength resolution beyond the Abbe barrier is possible for flat diffractive lens only with F$< \lambda$. The main aim of this work is to investigate and confirm the possibilities of subwavelength resolution focusing for 3D diffractive lens with F$> \lambda$.

It is well known that a Fresnel zone plate can be produced conformable to some curvilinear formations[9]. And curvilinear lens can be made on an arbitrary-shaped 3D surface, but the FZP-like lens with a rotational symmetry surface has better radiation characteristics and not only the phase function but also the 3D surface shape are free parameters that can be used for focusing characteristics optimization, including resolution power both for operating with quasi-monochromatic radiations and femtosecond pulses[10].

The innovative radiating structures with conical millimeter wave FZP lens and lens antenna were first proposed in 1991, then described and studied both theoretically and experimentally in Refs. [9,11]. Like the plane phase-reversal flat FZP lens, the cone-shape zone plate lens transforms in a step-wise manner where the incident plane wave into a spherical wave converged in geometrical-optic approximation to the primary focus F. This action is illustrated in Fig. 1(a). R_k are the ring zone radii, Z_k are the axial zone coordinates of a ring conical lens. Points O and F are the cone apex and primary focal points, or $|OF| = F$, where F is the focal length of lens. $< x >$ is the maximum flexure of the diffractive lens surface. For a given design with wavelength λ, cone half-opening angle α, and focal length F,

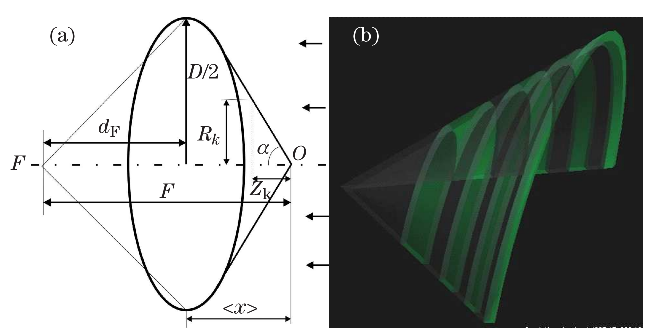

Fig. 1. Diffractive conical lens: (a) the ray-tracing technique, (b) CAD model of conical lens.

the k^{th} zone radius R_{k} and axial zone location Z_{k} can be calculated by[9]

$$\begin{cases} R_k = (F+z_k)/\text{tg}\alpha + [(F+z_k)/\text{tg}^2\alpha + z_k^2 - F^2]^{1/2} \\ Z_k = \dfrac{h\cos\alpha[(1-\cos^2\alpha/n^2)^{1/2} - \sin\alpha/n]}{(1-\cos^2\alpha/n^2)^{1/2}\text{tg}\alpha} \\ +\dfrac{hn}{(1-\cos^2\alpha/n^2)^{1/2}} - \dfrac{hn}{\sin\alpha} - F - \dfrac{k\lambda}{2}, \end{cases} \tag{1}$$

where h is the depth of the dielectric supporting layer normal to conical surface and n is dielectric refractive index. If $h \to 0$ and $\alpha \to \pi/2$, Eq. (1) becomes the expression for the radius of the classical Fresnel zone plate zones.

The phase profile was machined on the inner surface of a shallow cone, and the phase step was calculated using the expression

$$h = [0.5\lambda/(n-1)](1-\cos^2\alpha/n^2)^{1/2}. \tag{2}$$

As shown theoretically and experimentally, the longitudinal resolving power and depth definition (axial resolution Δ_z) can be controlled by choosing the flexure of the diffractive lens surface[9]. For a concave diffractive lens the limiting axial resolution in geometrical-optic approximation is

$$\Delta_z \to \lambda/2 \text{ when } R_{\text{k}}/Z_{\text{k}} \to 0 \, (\text{or} < x > \to F).$$

This specific behavior of axial resolution of diffractive optical elements on a non-flat surface makes it possible to design systems that possess much higher 3D resolution power and gain than other known classical lenses. Therefore, the main conclusion is that the longitudinal resolving power of the diffractive optical element can be controlled by choosing the flexure of the diffractive optical element surface and its spatial orientation[9]. Also the latter important effect is due to the reduced FZ lens spherical aberration as well as the reduction of the zone shadowing effect in diffractive optical elements on curvilinear surfaces[9,12]. It could be noted that by selection of diffractive optic surface and its orientation it is possible to minimize the selective types of aberrations[9].

The parameters of the diffractive lens both binary and phase correcting (Fig. 2) were selected as follows: $F/D = 1.26$, D/λ=20 (the dimension of lens diameter was limited by the computer's capabilities), F/λ=25, 90< α <20, and the lens material was polystyrene (n = 1.59). The Fresnel number FN of a lens with a lens diameter D was defined by FN=$D^2/4\lambda F$[14] and was equal to 5. For large Fresnel numbers (FN>>1), geometric optics is well suitable to derive the focal length of a lens. For low Fresnel numbers (FN<10), the focal length is shifted towards the lens due to the influence of the diffraction. In the case of binary conical FZP the lens consists of several metal screens of different annular holes normal to optical axis and situated along the conical surface[13,15]. The results of FDTD simulation of different types of conical phase reversal zone plates with different cone angles are listed in Fig. 3 (we use the commercial software Remcom XFDTD[16] with a mesh grid size of $\lambda/20$ both in x- and y-directions).

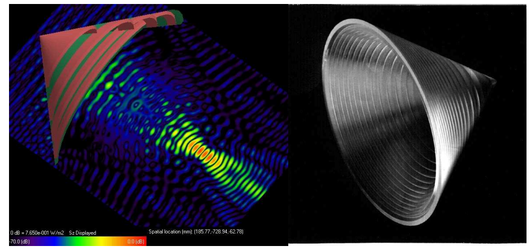

Fig. 2. FDTD simulation of conical lens focusing (left, red-air, green-dielectric) and experimental diffractive conical lens (right).

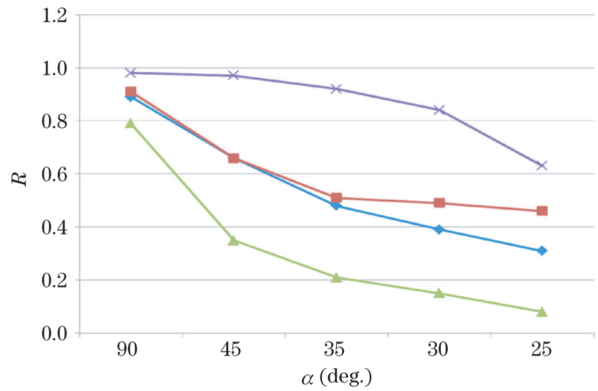

Fig. 3. FDTD simulation of the resolution power (R) of 3D conical diffractive lenses: blue – Δx, red – Δy, green – Δz, and the purple curve indicates the asymmetry of the focal spot $\Delta x/\Delta y$. The value of Δz is in the unit of classical depth of focus $8\lambda\left(\frac{F}{D}\right)^2$[14]. All other values are in the unit of Rayleigh radius[8]. The cone half-opening angle α is shown at the horizontal axis.

The results of simulations in the units of wavelength are shown in Table 1 to compare with data from Fig. 3. In approximation the focal spot is ellipse with the lengths of $\Delta x/\lambda$ and $\Delta y/\lambda$ in Table 1. The square of focal spot S, the "value" of 3D focal spot V in FWHM in the axial direction, and lenses Gain are also shown in Table 1.

Additional investigations and comparison with simulation of simple model of metal rings binary conical FZP[13,15], which consists of several metal screens of different annular holes situated along the conical surface and perpendicular to optical axis, showed that the resolution power of phase reversal conical diffractive lens was about 5−7% better than that of the amplitude binary lens. Moreover, the FZP lens is superior in axial resolution compared to the classical millimeter wave lens (plane-spherical or plane-hyperbolic lens). Also the results obtained by FDTD simulations and simple approximate algorithm[13] have shown a good agreement. It is also surprising that simple model of diffractive lens based on several flat metallic annular rings placed along conical surface and normal to optical axis[13,15] gives the results similar to the classical dielectric conical diffractive lens.

In experimental verification we used a diffractive optical element fabricated on a conical surface (Fig. 2(b)). It was manufactured of a numerically controlled lathe, using optical-grade polystyrene with the following optical constants: diffractive index n=1.59 and absorption coefficient $k \sim 10^{-3}$. The nominal radiation wavelength

Table 1. Focusing Characteristics of Dielectric Diffractive Conical Lenses

α (deg.)	90	45	35	30	25
$\Delta x/\lambda$	1.31	0.92	0.67	0.55	0.42
$\Delta y/\lambda$	1.32	0.91	0.70	0.68	0.67
$\Delta z/\lambda$	9.2	3.65	2.47	1.7	0.90
$\Delta F/\lambda$	23	13		6.4	2.45
S/λ^2	1.36	0.65	0.37	0.29	0.22
V/λ^3	12.49	2.4	0.91	0.5	0.2
G, dB	11.8	21.1	21.7	21.9	22.4

was λ_0=4.6 mm and the lens factor F/D was 1. The initial lens aperture D/λ_0=44[9] was limited to a value of D/λ_0= 20 by absorption materials to compare with simulations and cone angle α=35°.

The structure of focal spot was visualized by the method of movable probe. The diffraction field was mechanically scanned by an open end of a waveguide used as receiver connected to a detector head. D-407 type diode was attached to the tapered dielectric rod waveguide (DRW) antenna to measure the received power and recorded on a selective nanovoltmeter. DRW antenna as a probe was built of a 7λ long tapered dielectric rod and a conventional metallic waveguide with a junction or matching part between the rod and waveguide[17]. Additional details about the experimental setup and experimental method can be found in Ref. [9]. The root-mean-square deviation of the focal spot intensity from the calculated value was 6%.

The experimental results show that

♦ half-width (at half-height) of field intensity distribution along the optical axis for a "conical" diffractive optical element, with parameters as shown above, is twice as narrow as that of an equivalent zone plate (when radiation is incident on the side of the apex of the diffractive optics) and less than $\Delta_z < 2\lambda$;

♦ when radiation is incident on the side of the base of the diffractive optical element, the width of field intensity distribution along the optical axis is approximately 2.5 times wider than that for the equivalent zone plate[9];

♦ the resolution power of the conical lens is about 0.7 of wavelength with full cone angle of 70° in the first case.

It could be noted that the distance from the base of the cone to the focal point $\Delta F/\lambda$ is always $\Delta F > 2\lambda$ (see Table 1). Therefore, the longitudinal resolving power (axial resolution) of the diffractive optical element can be controlled by choosing the flexure of the diffractive optical element surface and its spatial orientation and could be less than Abbe barrier. So the "Abbe barrier" was completely broken by such diffractive lenses with unique 3D super resolution.

In conclusion, in contrast to the flat diffractive optics the curvilinear 3D diffractive conical optics is capable of overcoming 3D Abbe barrier with a focal distance F greater than 2λ. The focal intensity distribution for conical diffractive lens (as for phase reversal flat FZP lens[1,2]) is also not circularly symmetric and thus the focal spot in the high NA case is no longer an Airy pattern.

These results may find useful applications in optical microscopes, including "reverse-microscope", nondestructive testing, microoptics, nanooptics, to manipulate the 3D focused field distribution flexibly by use of diffractive optical elements to some applications[18] and so on.

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