

Indoor positioning modeling by visible light communication and imaging

Jinguo Quan (权进国)^{1,2*}, Bo Bai (白勃)³, Shuang Jin (金爽)², and Yan Zhang (张岩)¹

¹Key Laboratory of Network Oriented Intelligent Computation, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China

²Division of Information Science and Technology, Shenzhen Graduate School, Tsinghua University, Shenzhen 518055, China

³Xi'an Electronic Engineering Research Institute, Xi'an 710100, China

*Corresponding author: quanjg@gmail.com

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This letter presents a model of an indoor light positioning system (LPS) based on white LEDs and a camera. The position of an LPS receiver is determined through both its relative position to LEDs according to their images captured by the camera and LEDs' absolute position information in the navigation frame, obtained through a visible light communication (VLC) link. The error performance of the proposed LPS is analyzed. The mean error and mean square error (MSE) of estimated receiver position using least squares (LS) and weighted least squares (WLS) estimators are both derived in the presence of non-uniform measurement bias and white Gaussian noise. The effects of communication data rate on the positioning accuracy are also studied through BER performance.

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Currently indoor positioning with high accuracy and reliable real-time performance is in urgent need and has become one of the most exciting features of the next generation wireless systems^[1,2]. Assisted by indoor positioning systems, various location-based services can be realized. Light positioning system (LPS) with white LEDs coupled with an inexpensive imaging sensor such as embedded in a mobile handset^[3,4] is an emerging technology. It can provide concurrent indoor positioning and illumination. The light signal emitted from a modulated white LED, carrying LED's position information, is received by an imaging sensor through a visible light communication (VLC) channel, and then the position of the imaging sensor is estimated based on the received signal's attributes such as amplitude and angle of arrival (AOA). Meanwhile, due to a large number of small size pixels, high spatial resolution is achievable, leading to a possibly low cost and high accurate LPS. Since an LPS operates in the visible light spectrum, it does not create electromagnetic interference to existing radio frequency (RF) systems, critical for RF restricted environments.

In this letter, the LPS model with a white LED and camera is developed. Performance of a camera used as both an image sensor and a communication receiver is studied, error performance resulted from corrupted LED position information are further investigated. And the numerical results are also presented.

A typical LPS with a white LED and camera is shown below. Consider a LPS with a camera and N white LEDs mounted on the ceiling whose positions in the navigation frame are known as $\mathbf{S}_i = [X_i, Y_i, Z_i]^T$, $i=1, 2, \dots, N$, shown in Fig. 1. The white LEDs' image points $\mathbf{S}'_i = [x_i, y_i]^T$ are captured on the image sensor with the center of the image sensor set as the origin of the relative coordinate system. The focal length of the image lens is fixed as

f , the distance from the lens of the camera to the image sensor plane is D , and from the lens to the floor is Z_C . Let us focus on the two-dimensional (2D) position vector $\mathbf{p}_C = [X_C, Y_C]^T$ of the camera position represented by the center of the image lens \mathbf{O} , which is the same as the position of the image sensor's center in the navigation frame. The error performance of least square (LS) and weighted least square (WLS) estimators are derived and given in the following subsections, respectively.

Assume the height Z_C of the camera (defined as the height of the image lens) is given. According to the homothetic triangle theory and Newton's law of lens imaging, we can easily obtain the position of the camera $\mathbf{p}_{C_i} = [X_{C_i}, Y_{C_i}]^T$ from pairs of white LED \mathbf{S}_i and its image \mathbf{S}'_i on the image sensor as

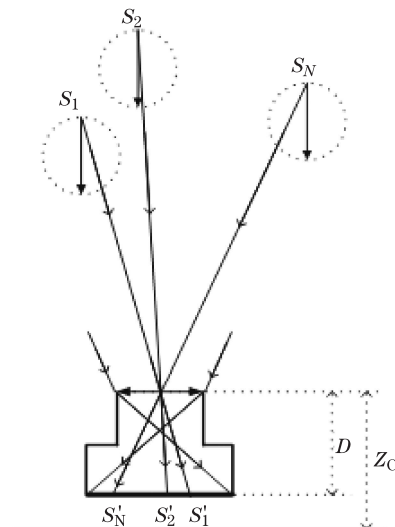


Fig. 1. LPS model with a white LED and a camera.

$$\mathbf{p}_{Ci} = [X_{Ci}, Y_{Ci}]^T = [X_i + \lambda_i \cdot x_{mi}, Y_i + \lambda_i \cdot y_{mi}]^T, \quad (1)$$

where $\lambda_i = \frac{Z_i - Z_C}{D}$, and $[x_{mi}, y_{mi}]^T$ is the measured position of the white LED image point $\mathbf{S}'_i = [x_i, y_i]^T$ on the image sensor. N values ($i = 1, 2, \dots, N$) obtained from N white LEDs will be identical to $\mathbf{p}_C = [X_C, Y_C]^T$ in the absence of noise. In the presence of measurement noise, we adopt the LS and WLS estimators to estimate \mathbf{p}_C from N measurement results corrupted by independent Gaussian noise $\mathbf{n}_x = [n_{x1}, n_{x2}, \dots, n_{xN}]^T$ and $\mathbf{n}_y = [n_{y1}, n_{y2}, \dots, n_{yN}]^T$ in X and Y axes, respectively, which is mainly introduced by the imaging distortion and weak incident light. Assume $\mathbf{n}_x = \boldsymbol{\mu}_x + \mathbf{v}_x$, $\mathbf{n}_y = \boldsymbol{\mu}_y + \mathbf{v}_y$, where $\boldsymbol{\mu}_x$ and $\boldsymbol{\mu}_y$ are the noise mean, v_x and v_y are zero mean Gaussian noise with variance σ_x^2 and σ_y^2 , independent across N white LEDs. Then according to Eq. (1), the squared error is

$$J = [x_m - (\mathbf{X} - X_C \cdot \mathbf{1}) \boldsymbol{\Lambda}]^T \cdot [x_m - (\mathbf{X} - X_C \cdot \mathbf{1}) \boldsymbol{\Lambda}] + [y_m - (\mathbf{Y} - Y_C \cdot \mathbf{1}) \boldsymbol{\Lambda}]^T \cdot [y_m - (\mathbf{Y} - Y_C \cdot \mathbf{1}) \boldsymbol{\Lambda}], \quad (2)$$

where $\mathbf{1}$ is an $N \times 1$ all one vector, $\boldsymbol{\Lambda} = \text{diag}\{-\frac{1}{\lambda_1}, \dots, -\frac{1}{\lambda_N}\}$ is the coefficient matrix.

Differentiating (2) with respect to X_C and Y_C , and setting the result to zero, we can obtain the LS estimated position as

$$\hat{\mathbf{p}}_C = \frac{1}{\text{tr}(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T)} (\mathbf{P} \boldsymbol{\Lambda} - [x_m, y_m]^T) \boldsymbol{\Lambda}^T \mathbf{1}, \quad (3)$$

where $\mathbf{P} = [\mathbf{X}, \mathbf{Y}]^T$ is the white LEDs' position matrix and $\text{tr}(\cdot)$ is the trace of a matrix. The estimation error vector is

$$\delta \mathbf{p}_C = \mathbf{p}_C - \hat{\mathbf{p}}_C = -\frac{1}{\text{tr}(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T)} [\delta x, \delta y]^T \boldsymbol{\Lambda}^T \mathbf{1}. \quad (4)$$

The mean error and the mean square error (MSE) can be easily derived as

$$E[\delta \mathbf{p}_C] = -\frac{1}{\text{tr}(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T)} [\boldsymbol{\mu}_x, \boldsymbol{\mu}_y]^T \boldsymbol{\Lambda}^T \mathbf{1}, \quad (5)$$

$$E[(\delta X_C)^2; (\delta Y_C)^2] = \frac{1}{\text{tr}^2(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T)} \begin{bmatrix} \mathbf{1}^T \boldsymbol{\Lambda} \cdot (\text{diag}\{\sigma_x^2\} + \boldsymbol{\mu}_x \cdot \boldsymbol{\mu}_x^T) \cdot \boldsymbol{\Lambda}^T \mathbf{1} \\ \mathbf{1}^T \boldsymbol{\Lambda} \cdot (\text{diag}\{\sigma_y^2\} + \boldsymbol{\mu}_y \cdot \boldsymbol{\mu}_y^T) \cdot \boldsymbol{\Lambda}^T \mathbf{1} \end{bmatrix}. \quad (6)$$

The LS estimator can be improved by adopting a WLS criterion when replacing the squared error J with

$$J = [x_m - (\mathbf{X} - X_C \cdot \mathbf{1}) \boldsymbol{\Lambda}]^T \mathbf{W}_X [x_m - (\mathbf{X} - X_C \cdot \mathbf{1}) \boldsymbol{\Lambda}] + [y_m - (\mathbf{Y} - Y_C \cdot \mathbf{1}) \boldsymbol{\Lambda}]^T \mathbf{W}_Y [y_m - (\mathbf{Y} - Y_C \cdot \mathbf{1}) \boldsymbol{\Lambda}], \quad (7)$$

where $\mathbf{W} = \boldsymbol{\Phi}^{-1}$ is the symmetric weighting matrix applicable to X and Y coordinates, $\boldsymbol{\Phi}$ is the noise correlation matrix

$$\boldsymbol{\Phi} = \boldsymbol{\mu} \boldsymbol{\mu}^T + \text{diag}\{\sigma_1^2, \dots, \sigma_N^2\}. \quad (8)$$

Through a similar procedure, we can obtain the WLS estimated position, the WLS estimation errors, the mean

error and the MSE as

$$\hat{\mathbf{p}}_C = \frac{1}{\|\boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T\|} (\mathbf{P} \boldsymbol{\Lambda} - [x_m, y_m]^T) \mathbf{W} \boldsymbol{\Lambda}^T \mathbf{1}, \quad (9)$$

$$\delta \mathbf{p}_C = \mathbf{p}_C - \hat{\mathbf{p}}_C = -\frac{1}{\|\boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T\|} [\delta x, \delta y]^T \mathbf{W} \boldsymbol{\Lambda}^T \mathbf{1}, \quad (10)$$

$$E[\delta \mathbf{p}_C] = -\frac{1}{\|\boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T\|} [\boldsymbol{\mu}_x, \boldsymbol{\mu}_y]^T \mathbf{W} \boldsymbol{\Lambda}^T \mathbf{1}, \quad (11)$$

$$E[(\delta X_C)^2; (\delta Y_C)^2] = \frac{1}{\|\boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T\|^2} \begin{bmatrix} \mathbf{1}^T \boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T \cdot (\text{diag}\{\sigma_x^2\} + \boldsymbol{\mu}_x \cdot \boldsymbol{\mu}_x^T) \cdot \mathbf{W} \boldsymbol{\Lambda}^T \mathbf{1} \\ \mathbf{1}^T \boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T \cdot (\text{diag}\{\sigma_y^2\} + \boldsymbol{\mu}_y \cdot \boldsymbol{\mu}_y^T) \cdot \mathbf{W} \boldsymbol{\Lambda}^T \mathbf{1} \end{bmatrix}, \quad (12)$$

where $\|\boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T\| = \mathbf{1}^T \boldsymbol{\Lambda} \mathbf{W} \boldsymbol{\Lambda}^T \mathbf{1}$. Assume all the white LED image points have identical noise mean μ_0 and standard deviation σ_0 in both X and Y axes, the weighting matrix \mathbf{W} can be simplified from matrix $\boldsymbol{\Phi}$ to

$$\mathbf{W} = \mu'_0 \mathbf{1} \cdot \mathbf{1}^T + \sigma'^2_0 \mathbf{I}, \quad (13)$$

where $\mu'_0 = -\frac{1}{\sigma_0^2} \cdot \frac{\mu_0^2}{N \cdot \mu_0^2 + \sigma_0^2}$, $\sigma'_0 = \frac{1}{\sigma_0}$, and \mathbf{I} is a $N \times N$ identity matrix. The difference between LS estimated coordinates by Eq. (3) and WLS by Eq. (9) is

$$\hat{\mathbf{p}}_{\text{LS}} - \hat{\mathbf{p}}_{\text{WLS}} = \frac{\mu'_0 (\mathbf{P} \boldsymbol{\Lambda} - [x_m, y_m]^T) \text{tr}(\boldsymbol{\Lambda})}{[\mu'_0 \text{tr}^2(\boldsymbol{\Lambda}) + \sigma'^2_0 \text{tr}(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T)]} \cdot [\text{tr}(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T) \cdot [\mathbf{1} \cdot \text{tr}(\boldsymbol{\Lambda} \boldsymbol{\Lambda}^T) - \boldsymbol{\Lambda}^T \mathbf{1} \cdot \text{tr}(\boldsymbol{\Lambda})]]. \quad (14)$$

When all the white LED lights are with the same height Z_0 , the coefficient matrix becomes $\boldsymbol{\Lambda} = -\frac{D}{Z_0 - Z_C} \mathbf{I}$, and Eq. (14) reduces to $\hat{\mathbf{p}}_{\text{LS}} - \hat{\mathbf{p}}_{\text{WLS}} = 0$, which means the LS estimates have the same performance as the WLS in this special case.

With the advantage of the CMOS based camera, we can easily get access to the specific region of interest, the pixels capturing the white LED image, and obtain the incident light power of the white LED on the pixels^[5,6]. The system noise that limits the performance of indoor VLC is mainly introduced by the channel and the camera. One typical noise source is the ambient light noise, and the image sensor noise of the camera^[7] usually includes the dark current noise, quantum shot noise, thermal noise, amplifier noise, MOS device noise, ADC noise, and the fixed pattern noise (FPN). The ambient light noise can be easily removed using an electrical filter. Assume interference from other white LEDs and reflected light would not take a dominant role. Therefore we only consider quantum shot noise, introduced by incident light signal, ambient light noise, dark current noise, and thermal noise in calculating the system's SNR, while all other camera noises are neglected. The system quantum shot noise variance is given by

$$\sigma_{\text{shot}}^2 = 2e \cdot \left[R_D(\lambda) \cdot P_r + \frac{R_D(\lambda) \cdot P_b \cdot I_2}{\text{Num}} + \frac{I_d \cdot I_2}{\text{Num}} \right] \cdot B, \quad (15)$$

where e is the electron charge, $R_D(\lambda) = \eta(\lambda) e \lambda / hc$ is the camera responsibility with respect to light wavelength λ , I_d is the dark current noise, $I_2 = 0.562$ is the noise bandwidth factor, B is the channel bandwidth, Num is the total pixel number of the image sensor, P_b is the total received ambient light noise.

The equivalent system noise is a Gaussian noise with a

total variance that is the sum of contributions from shot noise and thermal noise^[5]

$$\sigma_{\text{total}}^2 = \sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2. \quad (16)$$

The system SNR can be expressed as

$$\text{SNR} = \frac{(R_D \cdot P_r)^2}{\sigma_{\text{total}}^2} = \frac{(R_D \cdot P_r)^2}{\sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2}. \quad (17)$$

The communication BER is given by $\text{BER} = Q(\sqrt{\text{SNR}})$ for on-off keying (OOK) modulation scheme, where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-y^2/2} dy.$$

With the interference of the VLC channel noise or camera noise, LED position information may be misinterpreted randomly at the image sensor. So when the position information of only one LED is incorrectly interpreted, the mean error and the MSE are

$$\mu_{\text{in}} = \sum_{i=1}^N \alpha_i \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij} \cdot (X_j - X_i), \quad (18)$$

$$\sigma_{\text{in}}^2 = \sum_{i=1}^N \alpha_i \sum_{\substack{j=1 \\ j \neq i}}^N \beta_{ij} \cdot (X_j - X_i)^2, \quad (19)$$

where α_i is the probability that the position information transmission of the i th white LED, among all N LEDs, will have an error, and β_{ij} is the probability for the position information of the i th white LED to be misinterpreted as the one for the j th white LED. To simplify the computation, set the probabilities $\alpha_i = 1/N$ and $\beta_{ij} = 1/(N-1)$, and considering all possible cases with incorrectly decoded LED position information, from LED 1 to LED N , the total mean error and MSE are

$$\mu_{\text{total}} = \sum_{k=1}^N k \cdot \mu_{\text{in}} \cdot C_N^k \cdot (1 - P_e)^{N-k} \cdot P_e^k = 0, \quad (20)$$

$$\sigma_{\text{total}}^2 = \sum_{k=1}^N k \sigma_{\text{in}}^2 C_N^k (1 - P_e)^{N-k} P_e^k = \frac{1}{N(N-1)} \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N (X_j - X_i)^2 \sum_{k=1}^N k C_N^k (1 - P_e)^{N-k} P_e^k, \quad (21)$$

where P_e is the decoding error probability of a VLC link from a white LED to the receiver, namely the BER.

In this section, we evaluate the performance of both the LS and WLS estimators for the LPS with white LEDs and camera, and examine the accuracy of the derived analytical results. We adopt the mean error (bias), root mean square error (RMSE) and the variance of the positioning algorithms as performance metrics^[8]

$$\text{Bias} = \sqrt{\{E[\delta X_C]\}^2 + \{E[\delta Y_C]\}^2}, \quad (22)$$

$$\text{RMSE} = \sqrt{E[(\delta X_C)^2] + E[(\delta Y_C)^2]}, \quad (23)$$

$$\text{VAR} = \sqrt{\text{var}^2(X_C) + \text{var}^2(Y_C)}, \quad (24)$$

and examine the effects of different parameters on estimator performance, in particular measurement noise bias μ and additive Gaussian noise standard deviation σ . Also the positioning error resulting from VLC link is evaluated with respect to the communication data rate. All the numerical results are the sample averages of 10^4 independent realizations. The distribution of white LEDs and camera (in meters) is shown in Fig. 2. To simplify the simulation, we assume every LED image has the same standard deviation in both X and Y axes, and Table 1 gives the related parameters.

Set the noise mean for each LED image as 10% of its distance to the image sensor origin. Figures 3 and 4 show the bias and RMSE performance of the LS and WLS estimators with known and estimated LED position against the white noise standard deviation σ , respectively. The standard deviation takes values from 0 to 25 mm with a step size 1 mm, and the communication data rate in Fig. 4 is 500 kbps. The variance performance of LS and WLS estimators for LPS with estimated LED positions against the white noise standard deviation σ is compared in Fig. 5. Not surprisingly, the variance performance of LS estimator with measurement bias is worse than the WLS estimator, and there exists a big gap between the CRLB^[9] for either the LS or the WLS estimator with

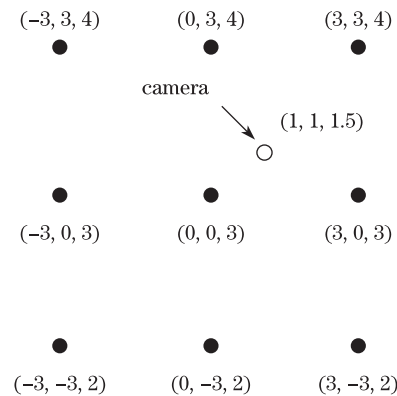


Fig. 2. Distribution of white LEDs and a camera.

Table 1. Related Parameters

Symbol	Quantity	Value
P_s	White LED Light Power	20 mW
$\Phi_{1/2}$	Semi-angle at Half Power	60 deg.
P_b	Average Ambient Light Power	60 μ W
R_D	Camera Responsivity	0.21 A/W
T_K	Absolute Temperature	295 K
G	Open Loop Voltage Gain	10
g_m	FET Transconductance	30 mS
Γ	FET Channle Noise Factor	1.5
I_2	Noise Bandwidth Factor I_2	0.562
I_3	Noise Bandwidth Factor I_3	0.0868
H	Fixed Capacitance	112 pF/cm ²
I_d	Dark Current	20 nA
D	Image Distance	25 mm
Num	Total Pixel Numbers	1024×1024

measurement bias. Performance against VLC system BERs, resulted from different communication data rate from 800 kbps to 1.4 Mbps with a step of 50 kbps, is given in Fig. 6. The mean of imaging noise is 10% to the image sensor origin, and deviation σ is fixed as 10 mm. It shows that the RMSE error dramatically increases when the system BER grows, and the RMSE performance of WLS estimator is firstly much better than the LS estimator.

In conclusion, this letter presents an indoor LPS model using white LEDs and a camera, and corresponding LS and WLS position estimation algorithms. The mean error and MSE of the LS and WLS estimators are both derived, which show a better performance of the WLS estimator with estimated LED positions. The effects of VLC error on the estimator performance and relevant communication parameters like data rate are also studied. The analytical and simulation results match well.

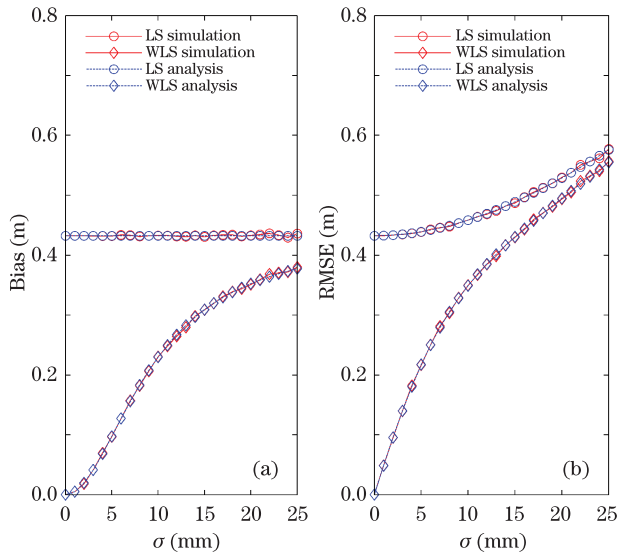


Fig. 3. Performance of LPS with known LED positions. (a) Bias; (b) RMSE

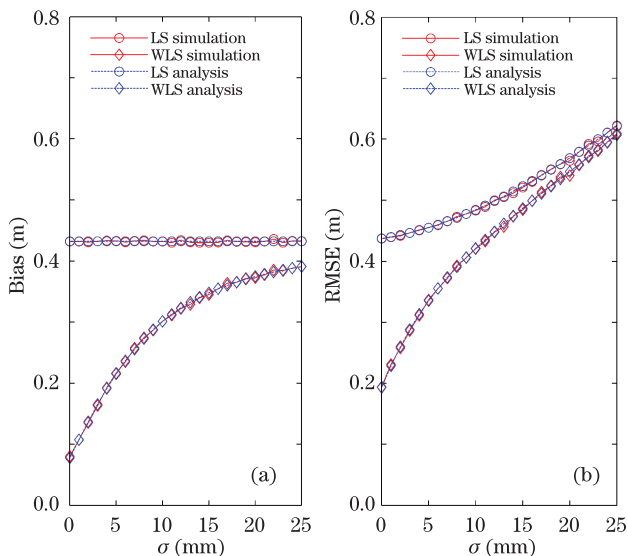


Fig. 4. Performance of LPS with estimated LED positions. (a) Bias; (b) RMSE

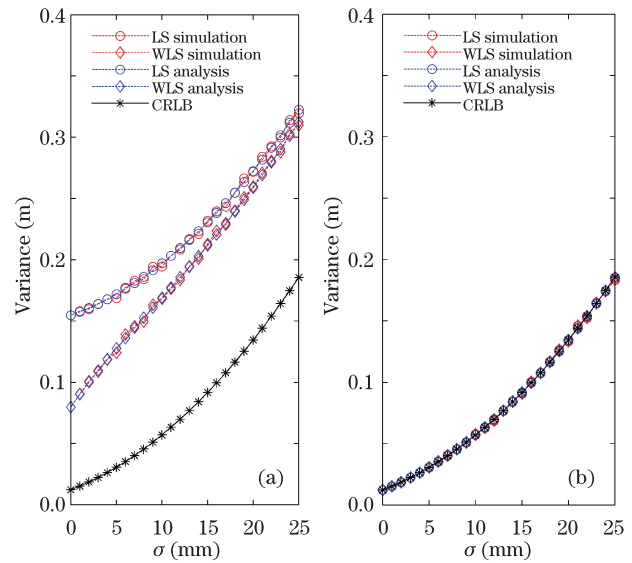


Fig. 5. Variance performance with estimated LED positions. (a) With measurement bias; (b) without measurement bias

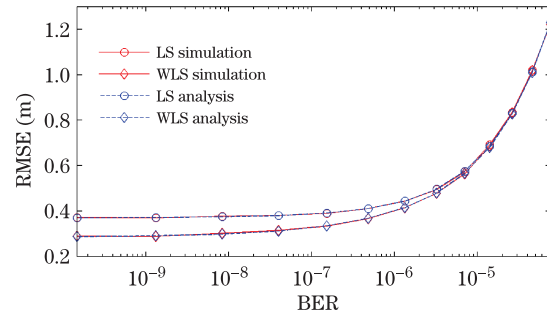


Fig. 6. RMSE performance of LPS versus system BERs.

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