Incoherently coupled spatial soliton families in biased two-photon photorefractive crystals with both the linear and quadratic electro-optic effect

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Different from the cases discussed preciously, nonlinear changes of refractive index in the photorefractive materials are influenced by both the linear and quadratic electro-optic effect simultaneously now. Here we present the evolution equations of one-dimension incoherently coupled spatial soliton families due to two-photon effect in biased photorefractive crystals with both the linear and quadratic electro-optic effect and discuss their existence conditions and properties in detail. Our analysis indicates that these soliton families can exist in all three possible realizations: dark-dark, bright-bright and dark-bright provided that the incident beams have the same polarization, wavelength and are mutually incoherent. Finally, the stabilities of these soliton families have been discussed by means of beam propagation methods.

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Thus far, photorefractive spatial solitons based on single-photon photorefractive effect have been investigated extensively in both theory and experiments[1−23]. In 2003, Castro-Camus *et al.* presented a new model of the two-photon photorefractive effect[24]. Screening solitons, photovoltaic solitons and screening-photovoltaic solitons resulting from only the linear electro-optic effect (Pockels effect) or quadratic electro-optic effect (dc Kerr effect) in two-photon photorefractive materials and their properties have been reported in the literature already[25−28]. However, some types of two-photon photorefractive materials that have large electro-optic effect, including both the linear and quadratic electro-optic effects near the phase-transition temperature have been found, such as ferroelectric $KTa_xNb_{1-x}O_3$(KTN) crystals[29,30], $LiNbO_3$ single crystals[15,25,31] and so on. Very recently, we have demonstrated that photorefractive spatial solitons can also be supported in biased two-photon photorefractive crystals involving both the linear and quadratic electro-optic effect in steady-state regime[32]. Study on photorefractive soliton pairs and soliton families have been gradually developed[33−35].

In this letter, we present the evolution equations of the one-dimension incoherently coupled spatial soliton families in biased two-photon photorefractive materials with both the linear and quadratic electro-optic effect. In steady state our results predict that incoherently coupled dark-dark, bright-bright and dark-bright hybrid soliton families can be supported under appropriate conditions provided that the incident beams have the same polarization, the same wavelength and are mutually incoherent. These soliton families owe their existence to both the linear term and quadratic electro-optic term in the meantime in our analysis. The characteristics and properties of these soliton families will be discussed in detail. Finally, the stabilities of such solitons have been discussed by means of beam propagation methods.

Through beam splitter a optical beam is split into some sub-beams, some optical devices can be used to change the optical paths between the beam splitter and the two-photon crystal to guarantee that optical path differences of the sub-beams are greater than the coherence length, so mutually incoherent optical beams with the same wavelength and the polarization can be obtained. Let us consider N such optical beams with the same wavelength and the polarization but are mutually incoherent, propagate along z axis in biased two- photon photorefractive crystal with both the linear and quadratic electro-optic effect. We assume that the optical c axis of two-photon photorefractive crystal is oriented along the x, y and z coordinate of the system and is illuminated by the gating beam whose intensity can be considered to remain constant during propagation. The incident optical beams are linearly polarized along x and the external bias electric field is applied in the same direction. Only the one-dimensional nonlinear diffraction will be taken into account in our configuration by assuming that the variables vary much more rapidly in the x direction. In the steady-state regime, the envelopes U_i of the N beams then satisfy the following dynamical evolution equations[32]:

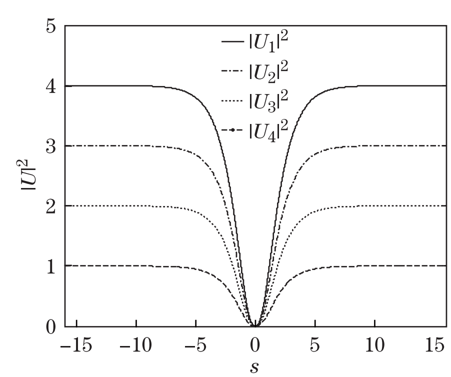

Fig. 1. Intensity profile of soliton components of dark soliton family with four components for $E_0 = -3 \times 10^3$ V/m.

$$i\frac{\partial U_1}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U_1}{\partial s^2}-\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left(1+\frac{\sigma}{1+\sum\limits_{i=1}^{N}|U_i|^2}\right)U_1-\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left(1+\frac{\sigma}{1+\sum\limits_{i=1}^{N}|U_i|^2}\right)^2U_1=0,$$

$$i\frac{\partial U_2}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U_2}{\partial s^2}-\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left(1+\frac{\sigma}{1+\sum\limits_{i=1}^{N}|U_i|^2}\right)U_2-\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left(1+\frac{\sigma}{1+\sum\limits_{i=1}^{N}|U_i|^2}\right)^2U_2=0,$$

$$i\frac{\partial U_N}{\partial \xi}+\frac{1}{2}\frac{\partial^2 U_N}{\partial s^2}-\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left(1+\frac{\sigma}{1+\sum\limits_{i=1}^{N}|U_i|^2}\right)U_N-\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left(1+\frac{\sigma}{1+\sum\limits_{i=1}^{N}|U_i|^2}\right)^2U_N=0. \quad (1)$$

where $\alpha_1=(k_0x_0)^2n_e^4r_{33}E_0/2$, $\alpha_2=(k_0x_0)^2n_e^4g_{\text{eff}}\varepsilon_0^2(\varepsilon_r-1)^2E_0^2/2$, $\sigma=\gamma_1N_A/s_2I_{2d}$, $\rho=I_{2\infty}/I_{2d}$. Here $k=k_0n_e=(2\pi/\lambda_0)\,n_e$ with n_e being the unperturbed index of refraction and λ_0 the free-space wavelength. r_{33} and g_{eff} are the effective linear and quadratic electro-optic coefficient, respectively. ε_0 and ε_r are the vacuum and relative dielectric constants, respectively. $I_{2d}=\beta_2/s_2$ is dark irradiance with β_2 being the thermoionization probability constant for the transitions between the conduction band and intermediate allowed level. s_2 is photoexcitation cross section of the soliton beam, γ_1 is the recombination factor of the intermediate allowed level valence band. $I_2=I_{2d}\sum\limits_{i=1}^{N}|U_i\,(x,z)|^2$ is the total intensity of the N incident beams. $I_{2\infty}=I_2\,(x\to\pm\infty,z)$ and E_0 represent the total intensity and the value of the space charge field at $x\to\pm\infty$, respectively. x_0 is an arbitrary spatial width for scale. For simplicity, any loss effects have been neglected.

To obtain the dark soliton family, the normalized envelopes U_i is expressed as $U_i(s,\xi)=\rho^{1/2}C_i^{1/2}y(s)\exp(i\nu\xi)$, where ν represents a nonlinear shift of the propagation constant. $y\,(s)$ is a normalized odd function bounded between $0\leqslant|y\,(s)|\leqslant1$, and satisfies $y\,(0)=0$, $y\,(s\to\pm\infty)=\pm1$ and when $s\to\pm\infty$, all the derivatives of $y\,(s)$ are zeros. C_i is defined as $C_i=I_{2i\max}/I_{2\max}$ and satisfies $\sum\limits_{i=1}^{N}C_i=1$. Substituting these U_i into Eq. (1) yields

$$\ddot{y}-2\nu y-2\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left(1+\frac{\sigma}{1+\rho y^2}\right)y$$
$$-2\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left(1+\frac{\sigma}{1+\rho y^2}\right)^2y=0. \quad (2)$$

By means of y-boundary condition, $y(s)$ and ν can be expressed as

$$s=\pm\int_y^0 d\tilde{y}\left\{-\frac{2\alpha_1\sigma}{1+\rho+\sigma}\left[(\tilde{y}^2-1)-\frac{1+\rho}{\rho}\ln\left(\frac{1+\rho\tilde{y}^2}{1+\rho}\right)\right]\right.$$
$$-\frac{2\alpha_2\sigma}{(1+\rho+\sigma)^2}\left[\frac{\sigma\rho(1-\tilde{y}^2)^2}{1+\rho y^2}-2(1+\rho)(1-\tilde{y}^2)\right.$$
$$\left.\left.-\frac{2(1+\rho)^2}{\rho}\ln\left(\frac{1+\rho\tilde{y}^2}{1+\rho}\right)\right]\right\}^{-1/2}, \quad (3)$$

$$\nu=-\alpha_1-\alpha_2. \quad (4)$$

Through numerical analysis we can get that dark soliton family due to two-photon effect is possible only when $\alpha_1<-2\alpha_2$[32]. In other words, such dark soliton families own their existence to both the linear and quadratic electro-optic term in the meantime. Then dark soliton family components can be obtained through $U_i(s,\xi)=\rho^{1/2}C_i^{1/2}y(s)\exp(i\nu\xi)$.

To illustrate our results, we consider the follow example: let $\lambda_0=532$ nm, $x_0=40$ μm, $\rho=10$, $E_0=-3\times10^3$V/m. The crystal parameters are taken to be $n_e=2.2$, $r_{33}=30\times10^{-12}$ m/V, $g_{\text{eff}}=0.17\text{m}^4/\text{C}^2$, $\varepsilon_r=10000$, $\sigma=10^4$[15, 25, 31]. Figure 1 shows the normalized intensity profiles of the dark soliton family with four components ($C_1=0.40$, $C_2=0.3$, $C_3=0.20$, $C_4=0.10$). The FWHM of these family components is found to be 167.82 μm.

The bright spatial soliton family can be analyzed in a similar way. In this case, the optical beam intensity is expected to vanish at infinity ($s\to\pm\infty$), and thus $\rho=I_{2\infty}/I_{2d}=0$. We express U_i as $U_i\,(s,\xi)=r^{1/2}C_i^{1/2}y\,(s)\exp\,(i\nu\xi)$, where $r=I_{2\max}/I_{2d}=I_2\,(0)/I_{2d}$ and $y\,(s)$ is a normalized real function bounded between $0\leqslant y\,(s)\leqslant1$. For the bright spatial soliton family, we require that $y\,(0)=1$, $\dot{y}\,(0)=0$, $y\,(s\to\pm\infty)=0$. The parameters C_i, ν are as the same as the dark soliton family. Substituting this expression of U_i into Eq. (1), we readily get

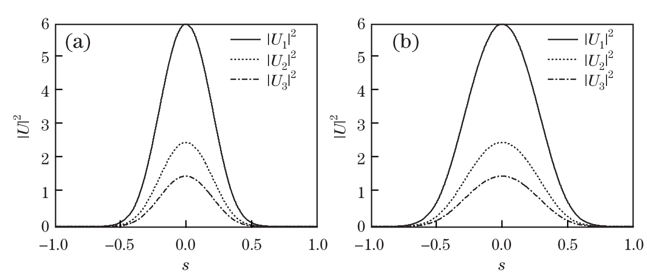

Fig. 2. Intensity profile of soliton components of bright soliton family with three components for (a)$E_0=1\times10^5$V/m and (b) $E_0=-1\times10^5$V /m.

$$\ddot{y} - 2\nu y - \frac{2\alpha_1}{1+\sigma}\left(1+\frac{\sigma}{1+ry^2}\right)y - \frac{2\alpha_2}{(1+\sigma)^2}\left(1+\frac{\sigma}{1+ry^2}\right)^2 y = 0. \tag{5}$$

By integrating Eq. (5) twice yields

$$s = \pm\int_y^1 d\tilde{y}\left\{\frac{2\alpha_1\sigma}{r(1+\sigma)}\left[\ln(1+r\tilde{y}^2)-\tilde{y}^2\ln(1+r)\right] + \frac{2\alpha_2\sigma}{(1+\sigma)^2}^2\left\{\frac{2}{r}\left[\ln(1+r\tilde{y}^2)-\tilde{y}^2\ln(1+r)\right] + \frac{\sigma r\tilde{y}^2(1-\tilde{y}^2)}{(1+r)(1+r\tilde{y}^2)}\right\}\right\}^{-1/2}, \tag{6}$$

$$\nu = -\frac{\alpha_1}{1+\sigma}\left[1+\frac{\sigma}{r}\ln(1+r)\right] - \frac{\alpha_2}{(1+\sigma)^2}\cdot\left[1+\frac{2\sigma}{r}\ln(1+r)+\frac{\sigma^2}{1+r}\right]. \tag{7}$$

Through numerical calculation we can conclude that Eq. (6) can support bright solitons only when $\alpha_1 > -\alpha_2/6$[32]. It is easy to see that the sign of $\alpha_1$and $\alpha_2$also can be different by altering the polarity of the external bias electric field as long as $\alpha_1 > -\alpha_2/6$, which is quite different from the cases presented previously. The bright soliton components can be obtained through $U_i(s,\xi) = r^{1/2}C_i^{1/2}y(s)\exp(i\nu\xi)$.

Figure 2 depicts the normalized intensity profiles of bright soliton family with three components ($C_1 = 0.60$, $C_2 = 0.25$, $C_3 = 0.15$) for $E_0 = 1\times10^5$, -1×10^5 V/m, $r = 10$. The FWHM of these bright soliton components are found to be 18.22, 24.72 μm, respectively. From Fig. 2, it is found that the width of these family components for $E_0 = -1\times10^5$ V/m ($\alpha_1 < 0$, $\alpha_2 > 0$) is also different from that of $E_0 = 1\times10^5$ V/m ($\alpha_1 > 0$, $\alpha_2 > 0$). In other words, in the case of the external biased field with equal in magnitude but opposite in polarity the photorefractive effect is weakened even counteracted by the interaction between the linear and quadratic electric-optic effect. So the width of these family components can also be adjusted by altering the polarity besides changing the strength of the external biased field.

Let us consider M bright and N dark beams that propagate collinearly in such photorefractive crystals. The total intensity of M+N mutually incoherent optical beams can be written as $I_2 = I_{2d}(\sum_{i=1}^{M}|U_i(x,z)|^2 + \sum_{j=1}^{N}|V_j(x,z)|^2)$, where $U_1, U_2,\cdots, U_M$and $V_1, V_2,\cdots, V_N$ are the normalization envelopes of the M bright and N dark beams, respectively, and satisfy the following dynamical evolution equations:

$$\mathrm{i}\frac{\partial U_p}{\partial\xi} + \frac{1}{2}\frac{\partial^2 U_p}{\partial s^2} - \alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\cdot\left(1+\frac{\sigma}{1+\sum_{i=1}^{M}|U_i|^2+\sum_{j=1}^{N}|V_j|^2}\right)U_p - \alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\cdot\left(1+\frac{\sigma}{1+\sum_{i=1}^{M}|U_i|^2+\sum_{j=1}^{N}|V_j|^2}\right)^2 U_p = 0, \tag{8a}$$

$$\mathrm{i}\frac{\partial V_q}{\partial\xi} + \frac{1}{2}\frac{\partial^2 V_q}{\partial s^2} - \alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\cdot\left(1+\frac{\sigma}{1+\sum_{i=1}^{M}|U_i|^2+\sum_{j=1}^{N}|V_j|^2}\right)V_q - \alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\cdot\left(1+\frac{\sigma}{1+\sum_{i=1}^{M}|U_i|^2+\sum_{j=1}^{N}|V_j|^2}\right)^2 V_q = 0. \tag{8b}$$

Here, $p\in(1,2,\cdots,M)$ and $q\in(1,2,\cdots,N)$.

To obtain the dark-bright hybrid soliton family solutions of Eq. (8), the normalized envelopes U_i and V_j are expressed as $U_i(s,\xi) = r^{1/2}C_i^{1/2}f(s)\exp(i\mu\xi)$ and $V_j(s,\xi) = \rho^{1/2}D_j^{1/2}g(s)\exp(i\nu\xi)$, where $\sum_{i=1}^{M}C_i = 1$, $\sum_{j=1}^{N}D_j = 1$. r and ρ are the radio of the peak value of the total intensity of M bright and N dark optical beams to the dark-irradiance I_{2d}, respectively. $f(s)$ corresponds to a bright envelope bounded between $0\leqslant y(s)\leqslant 1$ and satisfies that $f(0) = 1$, $\dot{f}(0) = 0$, $f(s\to\pm\infty) = 0$; $g(s)$ denotes a dark normalized field profile also bounded between $0\leqslant|g(s)|\leqslant 1$ and $g(0) = 0$, $g(s\to\pm\infty) = 1$. Substitution these form of U_i and V_j into Eq. (8) lead to the following equations:

$$\frac{\partial^2 f}{\partial s^2} = 2\left[\mu+\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left(1+\frac{\sigma}{1+rf^2+\rho g^2}\right)+\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left(1+\frac{\sigma}{1+rf^2+\rho g^2}\right)^2\right]f, \tag{9a}$$

$$\frac{\partial^2 g}{\partial s^2} = 2\left[\nu+\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left(1+\frac{\sigma}{1+rf^2+\rho g^2}\right)+\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left(1+\frac{\sigma}{1+rf^2+\rho g^2}\right)^2\right]g. \tag{9b}$$

We now look for particular solutions which also satisfy the condition $f^2+g^2 = 1$. In this limit, Eq. (9) take the forms of

$$\frac{\partial^2 f}{\partial s^2} = 2\left\{\mu+\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left[1+\frac{\sigma}{(1+\rho)(1+\delta f^2)}\right]+\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left[1+\frac{\sigma}{(1+\rho)(1+\delta f^2)}\right]^2\right\}f, \tag{10a}$$

$$\frac{\partial^2 g}{\partial s^2}=2\left\{\nu+\alpha_1\left(\frac{1+\rho}{1+\rho+\sigma}\right)\left\{1+\frac{\sigma}{(1+\rho)\left[1+\delta\left(1-g^2\right)\right]}\right\}+\alpha_2\left(\frac{1+\rho}{1+\rho+\sigma}\right)^2\left\{1+\frac{\sigma}{(1+\rho)\left[1+\delta\left(1-g^2\right)\right]}\right\}^2\right\}g, \quad (10b)$$

where the parameter $\delta$is defined as $\delta=(r-\rho)/(1+\rho)$. From Eq. (10), the values of $\mu$and νcan be readily obtained by employing the $f-g$ boundary conditions and are given by

$$\mu=-\frac{\alpha_1}{1+\rho+\sigma}\left[1+\rho+\frac{\sigma}{\delta}\ln\left(1+\delta\right)\right]-\frac{\alpha_2}{\left(1+\rho+\sigma\right)^2}\left[\left(1+\rho\right)^2+\frac{2\sigma\left(1+\rho\right)}{\delta}\ln\left(1+\delta\right)+\frac{\sigma^2}{1+\delta}\right], \quad (11)$$

$$\nu=-\alpha_1-\alpha_2. \quad (12)$$

Equation (10) can be solved approximately provided that $|\delta|<<1$, that is to say, the peak values of the total intensity of M bright beams are approximately equal to that of the N dark ones. In this case, μ can be approximately rewritten as

$$\mu\doteq\alpha_1\left[\sigma\delta/2\left(1+\rho+\sigma\right)-1\right] +\alpha_2\left[\sigma\delta/(1+\rho+\sigma)-1\right], \quad (13)$$

and thus Eq. (10) can be reduced to

$$\frac{\partial^2 f}{\partial s^2}=\frac{\sigma\delta}{1+\rho+\sigma}\left(\alpha_1+2\alpha_2\right)f-\frac{2\sigma\delta}{1+\rho+\sigma}\left(\alpha_1+2\alpha_2\right)f^3, \quad (14a)$$

$$\frac{\partial^2 g}{\partial s^2}=-\frac{2\sigma\delta}{1+\rho+\sigma}\left(\alpha_1+2\alpha_2\right)g+\frac{2\sigma\delta}{1+\rho+\sigma}\left(\alpha_1+2\alpha_2\right)g^3. \quad (14b)$$

It can be directly shown that Eq. (14) exhibit closed form solutions of the form

$$f\left(s\right)=\mathrm{sech}\left\{\left[\sigma\delta\left(\alpha_1+2\alpha_2\right)/\left(1+\rho+\sigma\right)\right]^{1/2}s\right\}, \quad (15a)$$

$$g\left(s\right)=\tanh\left\{\left[\sigma\delta\left(\alpha_1+2\alpha_2\right)/\left(1+\rho+\sigma\right)\right]^{1/2}s\right\}. \quad (15b)$$

The normalized envelopes of dark-bright soliton components can be expressed as

$$U_i\left(s,\xi\right)=r^{1/2}C_i^{1/2}\mathrm{sech}\left\{\left[\delta\sigma\left(\alpha_1+2\alpha_2\right)/(1+\rho+\sigma)\right]^{1/2}s\right\} \times\exp\left\{i\left\{\alpha_1\left[\sigma\delta/2(1+\rho+\sigma)-1\right] +\alpha_2\left[\sigma\delta/(1+\rho+\sigma)-1\right]\right\}\xi\right\}, \quad (16a)$$

$$V_j\left(s,\xi\right)=\rho^{1/2}D_j^{1/2}\tanh\left\{\left[\delta\sigma\left(\alpha_1+2\alpha_2\right)/(1+\rho+\sigma)\right]^{1/2}s\right\} \times\exp\left[\mathrm{i}\left(-\alpha_1-\alpha_2\right)\xi\right]. \quad (16b)$$

Equation (15) show that these solutions are possible only when $\delta\left(\alpha_1+2\alpha_2\right)>0$.

To illustrate our results, Fig. 3 shows the normalized intensity profiles of the dark-bright hybrid spatial soliton family in which the peak value of the total intensity of the bright beams is slightly larger than that of the dark ones. The intensity FWHM of these family components is found to be 80.14 and 89.73 μm, respectively.

It is noteworthy to note that these incoherently coupled spatial soliton families, which stem from two-photon effect in biased photorefractive crystal with both the linear and quadratic electro-optic effect, can be simplified to incoherently coupled spatial soliton pairs when they contain only two components. Similarly, when the incident optical beams contain only one component, Eq. (1) are equivalent to that of the single soliton case already discussed previously[32]. Moreover, incoherently coupled dark-bright hybrid soliton families can be supported in our configuration when the peak values of the total intensity of bright beams and that of the dark ones are approximately equal.

Finally, we investigate the stabilities of these bright and dark soliton families by means of beam propagation

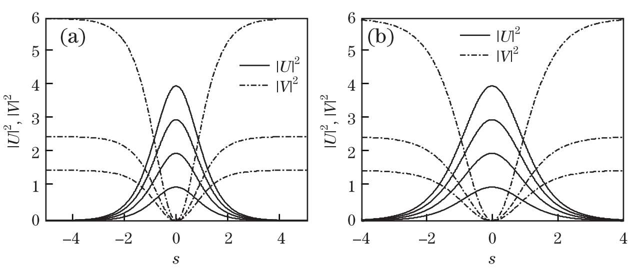

Fig. 3. Intensity profile of soliton components of dark-bright hybrid soliton family with four bright components ($C_1=0.40$, $C_2=0.3$, $C_3=0.20$, $C_4=0.10$) and three dark components ($D_1=0.60$, $D_2=0.25$, $D_3=0.15$) for (a) $E_0=1\times10^5$ V/m and (b) $E_0=-1\times10^5$ V/m for $\delta=0.01$.

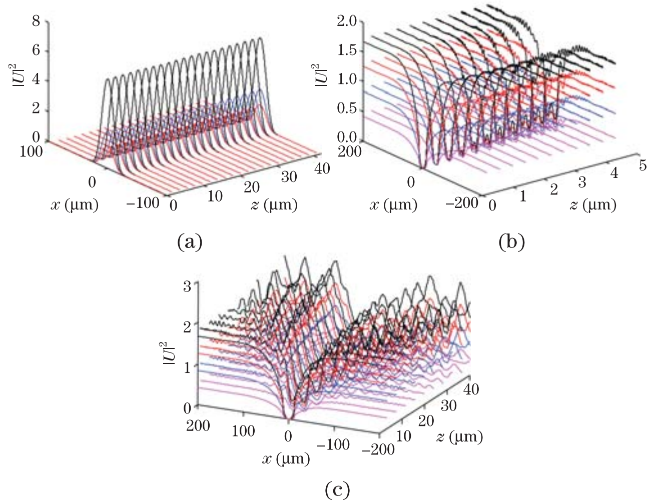

Fig. 4. (a) Stable propagation of the bright soliton family with three components for $r=10$ and $E_0=1\times10^5$ V/m; (b) Approximate stable propagation of the dark soliton family in a relative short distance and (c) unstable propagation of the dark soliton family during the longer propagation with four components for $E_0=-1\times10^5$ V/m, $\rho=4$.

methods. The solitary wave solutions obtained from Eqs. (3) and (6) are used as the input beam profiles and solve Eq. (1) numerically. As expected, our results confirm that the bright soliton family can remain invariant during propagation process, which is shown in Fig. 4(a). Thus, bright soliton family resulting from both the linear and quadratic electric-optic effect is stable with distance. The propagation of the dark soliton family is depicted in Figs. 4(b) and 4(c). It is shown that the input beams will diffract during the longer propagation process (>8 mm), which can be mainly attributed to the truncation in the process of numerical calculation. However, the propagation of dark soliton family can be regarded as an approximate stable process in comparatively short distance (∼5 mm) in the propagation direction. Moreover, after a simple analysis we can get that dark-bright hybrid soliton family is also approximate stable in comparatively short distance (∼5 mm) in the propagation direction and will be unstable during the longer propagation process (>8 mm) because of the dark soliton components in the hybrid soliton family.

In conclusion, we have presented theoretically the evolution equations of one-dimension incoherently coupled spatial soliton families in biased two-photon photorefractive material including both the linear and quadratic electro-optic effects. Under strong external bias conditions, our analysis indicates that dark-dark, bright-bright and dark-bright hybrid incoherently coupled soliton families can exist in the steady-state regime. It has been shown that bright soliton family is possible only when $\alpha_1 > -\alpha_2/6$, whereas the dark branch requires $\alpha_1 < -2\alpha_2$, and dark-bright hybrid soliton family is possible only when $\delta\,(\alpha_1 + 2\alpha_2) > 0$ is satisfied. Distinguished from the studies in the past, these incoherently coupled soliton families own their existence to both the linear term and quadratic electro-optic term in the meantime in our analysis. PR effect can be enhanced or weakened even counteracted by the interaction between the linear and quadratic electric-optic effect, so existence conditions of these spatial soliton families are more complex for a given two-photon PR crystal. For example, bright spatial soliton family can exist when $\alpha_1 < 0,\ \alpha_2 > 0$ so long as $\alpha_1 > -\alpha_2/6$ is satisfied, which is quite different from the cases discussed previously. Finally, the stabilities of such solitons have been discussed by means of beam propagation methods. It is found that bright spatial soliton family is stable, whereas those dark soliton family and dark-bright hybrid soliton family can be regarded as an approximate stable in comparatively short distance (∼5 mm) in the propagation direction whereas the input beams will diffract and unstable during the longer propagation process(>8 mm).

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