Factors affecting the spectrum of an electromagnetic light wave on scattering from a semisoft boundary medium

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The spectrum of an electromagnetic light wave on scattering from a semisoft boundary medium is discussed within the accuracy of the first-order Born approximation. It is shown that spectral shifts and spectral switches are affected both by the polarization of the incident light wave and by the characters of the scattering medium. Moreover, numerical results show that the direction at which the spectral switch occurs is governed by the characters of the scattering medium, whereas the magnitude of the spectral switch is affected by the polarization of the incident light wave.

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The far-zone scattered field of light waves on scattering from a medium is a topic of considerable importance due to potential applications in areas such as medical diagnosis. Since the discovery made by Wolf that the spectrum of light may change as it is scattered from a medium[1], numerous papers were published on the scattering of light waves from various scatterers[2–9]. In discussion of light wave scattering, the far-zone scattered spectrum attracts much attention[10–13]. It is found that the distribution of the scattered spectrum is closely related to the characters of the scattering medium. This phenomenon may provide a method for the determination of the structure characters from the measurements of the scattered spectrum. Recently, a new model, that is, the semisoft boundary model, was presented[14]. It is found that when a scalar light wave is scattered from a semisoft boundary medium, not only spectral shifts but also spectral switches can be produced in the far-zone scattered field[15]. On the other hand, the weak scattering theory was generalized from scalar light waves to vector light waves in recent years[16,17]. It is shown that the polarization of the incident light wave is also a critical factor affecting the spectral coherence of the far-zone scattered field[18]. Then one may wonder whether there is any influence of the polarization on the spectrum of an electromagnetic light wave on scattering. In this letter, we will generalize the vector scattering theory from soft boundary medium to semisoft boundary one, and discuss factors which play roles in the distribution of the far-zone scattered spectrum.

As shown in Fig. 1, consider that an electromagnetic plane light wave, with a propagation direction specified by a real unit vector $\mathbf{s}_0$, is incident on a scattering medium. The properties of the incident field at a pair of points $\mathbf{r}_1'$ and $\mathbf{r}_2'$ are characterized by the so-called cross-spectral density matrix, which is defined as[19]

$$\begin{aligned}\ddot{\mathbf{W}}^{(\mathrm{in})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0,\omega) &\equiv \left[W_{ij}^{(\mathrm{in})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0,\omega)\right] \\ &= \left[\left\langle E_i^*(\mathbf{r}_1',\mathbf{s}_0,\omega)E_j(\mathbf{r}_2',\mathbf{s}_0,\omega)\right\rangle\right], \\ &(i = x,y;\ j = x,y). \end{aligned} \tag{1}$$

Here the angular brackets denote the ensemble average, the asterisk denotes the complex conjugate, and E_x and E_y are the two mutually orthogonal components at frequency ω of the electric field perpendicular to the propagation direction (i.e., the z direction), with a form of

$$E_i(\mathbf{r}',\mathbf{s}_0,\omega) = a_i(\omega)\exp(ik\mathbf{s}_0\cdot\mathbf{r}'),\ \ (i = x,y), \tag{2}$$

where $a_i(\omega)$ is a random function and $k = \omega/c$ with c being the speed of light in vacuum. Assume that the two Cartesian coordinate components of the field are independent of each other. On substituting Eq. (2) into Eq. (1), one can readily find

$$\ddot{\mathbf{W}}^{(\mathrm{in})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0,\omega) = \begin{bmatrix} W_{xx}^{(\mathrm{in})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0,\omega) & 0 \\ 0 & W_{yy}^{(\mathrm{in})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0,\omega)\end{bmatrix}, \tag{3}$$

$$W_{ii}^{(\mathrm{in})}(\mathbf{r}_1',\mathbf{r}_2',\mathbf{s}_0,\omega) = S_i(\omega)\exp[ik\mathbf{s}_0\cdot(\mathbf{r}_2'-\mathbf{r}_1')], \tag{4}$$

$$S_i(\omega) = \left\langle a_i^*(\omega)a_i(\omega)\right\rangle,\ \ (i = x,y). \tag{5}$$

Equation (5) represents the spectrum of the incident field along the ith direction. In this case, the spectrum of the incident field can be expressed as

$$S^{(i)}(\omega) = S_x(\omega) + S_y(\omega). \tag{6}$$

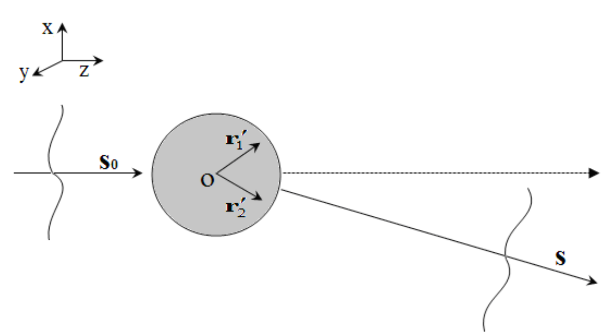

Fig. 1. Illustration of the notations.

Assume that the medium is a weak scatterer so that the scattering can be analyzed within the accuracy of the first-order Born approximation[20]. The far-zone scattered field then can be expressed as[17]

$$\mathbf{E}^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\int_D F(\mathbf{r}',\omega)\left\{\mathbf{E}^{(i)}(\mathbf{r}',\mathbf{s}_0,\omega)-[\mathbf{s}\cdot\mathbf{E}^{(i)}(\mathbf{r}',\mathbf{s}_0,\omega)]\mathbf{s}\right\}\times G(r\mathbf{s},\mathbf{r}',\omega)\mathrm{d}^3r', \tag{7}$$

where $\mathbf{s}=(s_x,s_y,s_z)$ denotes the direction of the scattering path, $\mathbf{E}^{(i)}(\mathbf{r}',\mathbf{s}_0,\omega)$ is the incident field, $F(\mathbf{r}',\omega)$ is the scattering potential, and $G(r\mathbf{s},\mathbf{r}',\omega)$ is the free-space Green's function, which can be approximated by[20]

$$G(r\mathbf{s},\mathbf{r}',\omega)\sim\frac{\exp(\mathrm{i}kr)}{r}\exp(-\mathrm{i}k\mathbf{s}\cdot\mathbf{r}'). \tag{8}$$

The three Cartesian coordinate components can be obtained from Eq. (7) as

$$E_x^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\int_D F(\mathbf{r}',\omega)G(r\mathbf{s},\mathbf{r}',\omega)\{(1-s_x^2)E_x(\mathbf{r}',\mathbf{s}_0,\omega)-s_xs_yE_y(\mathbf{r}',\mathbf{s}_0,\omega)\}\mathrm{d}^3r', \tag{9a}$$

$$E_y^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\int_D F(\mathbf{r}',\omega)G(r\mathbf{s},\mathbf{r}',\omega)\{-s_xs_yE_x(\mathbf{r}',\mathbf{s}_0,\omega)+(1-s_y^2)E_y(\mathbf{r}',\mathbf{s}_0,\omega)\}\mathrm{d}^3r', \tag{9b}$$

$$E_z^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\int_D F(\mathbf{r}',\omega)G(r\mathbf{s},\mathbf{r}',\omega)\{-s_xs_zE_x(\mathbf{r}',\mathbf{s}_0,\omega)-s_ys_zE_y(\mathbf{r}',\mathbf{s}_0,\omega)\}\mathrm{d}^3r'. \tag{9c}$$

The properties of the scattered field at a pair of points specified by position vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$ can also be characterized by a cross-spectral density matrix, which is defined as[19]

$$\ddot{\mathbf{W}}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2,\mathbf{s}_0,\omega)\equiv\left[W_{ij}^{(s)}(r\mathbf{s}_1,r\mathbf{s}_2,\mathbf{s}_0,\omega)\right]=\left[\left\langle E_i^{(s)*}(r\mathbf{s}_1,\mathbf{s}_0,\omega)E_j^{(s)}(r\mathbf{s}_2,\mathbf{s}_0,\omega)\right\rangle\right],\quad(i=x,y,z;\ j=x,y,z). \tag{10}$$

The scattered spectrum, which can be obtained from the cross-spectral density matrix, is defined as[17]

$$S^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\mathrm{Tr}\ \ddot{\mathbf{W}}^{(s)}(r\mathbf{s},r\mathbf{s},\mathbf{s}_0,\omega), \tag{11}$$

where Tr denotes the trace of the matrix.

The scattering potential of a semisoft boundary scatterer can be expressed as[15]

$$F(\mathbf{r}',\omega)=(\omega/c)^2\eta(\mathbf{r}',\omega), \tag{12}$$

$$\eta(\mathbf{r}',\omega)=A\sum_{m=1}^{M}(-1)^{m-1}C_M^m\exp(-m\beta_M\frac{\mathbf{r}'^2}{2\sigma^2}). \tag{13}$$

Equation (13) is the dielectric susceptibility of the scatterer with

$$C_M^m=\frac{M!}{m!(M-m)!}, \tag{14a}$$

$$\beta_M=-\ln[1-(1-e^{-1})^{\frac{1}{M}}]. \tag{14b}$$

It is shown from Eq. (13) that the parameter M plays a critical role in the boundary of the scatterer. In a special case (i.e., $M=1$), the scatterer reduces to a soft boundary medium. Otherwise, the scatterer is a semisoft boundary one.

On substituting Eqs. (2), (8), and (12), first into Eq. (9), and then into Eq. (11), and after some calculations, one can find the far-zone scattered spectrum as

$$S^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\frac{(1-s_x^2)S_x(\omega)+(1-s_y^2)S_y(\omega)}{r^2}\left(\frac{\omega}{c}\right)^4\left|\tilde{\eta}(\mathbf{K},\omega)\right|^2, \tag{15}$$

$$\tilde{\eta}(\mathbf{K},\omega)=\int\eta(\mathbf{r}',\omega)\exp(-\mathrm{i}\mathbf{K}\cdot\mathbf{r}')\mathrm{d}^3r'. \tag{16}$$

Equation (16) is the three-dimensional Fourier transform of the dielectric susceptibility with

$$\mathbf{K}=k(\mathbf{s}-\mathbf{s}_0). \tag{17}$$

On substituting Eq. (13) into Eq. (16), and manipulating the three-dimensional Fourier transform, one finds

$$\tilde{\eta}(\mathbf{K},\omega)=A\sum_{m=1}^{M}(-1)^{m-1}C_M^m\frac{(2\pi)^{3/2}\sigma^3}{(m\beta_M)^{3/2}}\exp(-\frac{\sigma^2}{2m\beta_M}\mathbf{K}^2). \tag{18}$$

It follows from Eqs. (15) and (18) that the scattered field can be expressed as

$$S^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\frac{(1-s_x^2)S_x(\omega)+(1-s_y^2)S_y(\omega)}{r^2}A^2\left(\frac{\omega}{c}\right)^4\times\left|\sum_{m=1}^{M}(-1)^{m-1}C_M^m\frac{(2\pi)^{3/2}\sigma^3}{(m\beta_M)^{3/2}}\exp\left[-\frac{\sigma^2}{2m\beta_M}\mathbf{K}^2\right]\right|^2. \tag{19}$$

For the simplicity of following discussion, let us consider the scattered spectrum in a special plane, that is, the Y–Z plane. In this case, the unit vector of the scattering direction should be expressed as

$$\mathbf{s}=(0,\ s_y,\ s_z). \tag{20}$$

On substituting Eq. (20) together with Eq. (17) into Eq. (19), we obtain the far-zone scattered spectrum as

$$S^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)=\frac{S_x(\omega)+\cos^2\theta S_y(\omega)}{r^2}A^2\left(\frac{\omega}{c}\right)^4\times\left|\sum_{m=1}^{M}(-1)^{m-1}C_M^m\frac{(2\pi)^{3/2}\sigma^3}{(m\beta_M)^{3/2}}\exp\left[-\frac{\sigma^2k^2(1-\cos\theta)}{m\beta_M}\right]\right|^2, \tag{21}$$

where θ is the angle made by $\mathbf{s}$ and $\mathbf{s}_0$. As shown in Eq. (21), the scattered spectrum is, in general, different from the incident one and changes with the directions of the scattering path. In the following, an example is illustrated to shown the effects of the characters of the scattering medium and polarization of the incident light wave on the distributions of the far-zone scattered spectrum. For the following discussion, let us introduce the relative spectral shift of the far-zone scattered field, which is defined as[21]

$$\frac{\delta\omega}{\omega_0}=\frac{\omega_m-\omega_0}{\omega_0}, \tag{22}$$

where ω_0 is the central frequency of the incident light wave and ω_m is the frequency at which the scattered spectral density $S^{(s)}(r\mathbf{s},\mathbf{s}_0,\omega)$ takes its maximum value.

If $\delta\omega/\omega_0 < 0$ (i.e., $\omega_m < \omega_0$), the scattered spectrum is red-shifted; if $\delta\omega/\omega_0 > 0$ (i.e., $\omega_m > \omega_0$), the scattered spectrum is blue-shifted.

As an example, let us assume that the spectrum of the incident field has a distribution of Gaussian profile, which can be expressed as[1]

$$S_i(\omega) = B_i \exp\left[\frac{-(\omega-\omega_0)^2}{2\Gamma_i^2}\right], \ (i = x,y), \tag{23}$$

where B_i is a constant, Γ_i is the spectral width of the field along the ith direction, and ω_0 is the central frequency. On substituting Eq. (23) into Eq. (21), one can find the far-zone scattered spectrum of an electromagnetic light wave on scattering from a semisoft boundary medium. In the following, we present some numerical results to show the effects of the characters of the scatterer and the polarization of the incident light on the scattered spectrum of the far-zone scattered field.

Firstly, let us consider the effect of the characters of the scatterer on the changes of the far-zone scattered spectrum. In Fig. 2, the relative spectral shifts with different boundary conditions are presented. It is shown that when the scatterer is a soft boundary medium (i.e., $M = 1$), with the increasing scattering angle, the spectrum is first blue-shifted, and then red-shifted. However, for scatterer with semisoft boundary (i.e., $M = 2$ or $M = 10$), with the increasing scattering angle, the spectrum experiences a rapidly changes from red-shift to blue-shift, that is, a spectral switch (for a detailed definition of the spectral switch, please refer Ref. [21]). Moreover, it can also be seen from Fig. 2 that the direction at which the spectral shift occurs is affected by the boundary of the scatterer. In Fig. 3, we study the influence of the effective width of the dielectric susceptibility on the relative spectral shifts of the far-zone scattered field. As shown in Fig. 3, if the scattering angle is small, the influence of the effective width of the dielectric susceptibility on relative spectral shifts is not obvious, and the spectrum is blue-shifted. However, when the scattering angle continues to increase, the influence of the effective width of the dielectric susceptibility on the relative spectral shift becomes obvious. Figure 3 also shows that the effective width of the dielectric susceptibility is a critical factor on the direction at which the spectral switch occurs.

As shown in Eq. (21), the polarization of the incident light wave is also a critical factor on the distribution of the far-zone scattered spectrum. Figure 4 shows the relative spectral shift of the far-zone scattered field for incident light waves with different polarizations. As shown in Fig. 4, when the scattering angle is small, the scattered spectrum is blue-shifted, and the spectral shift is affected by the polarization of the incident light wave. With the increasing scattering angle, spectral switch can be produced in the scattered spectrum. Moreover, numerical results show that the direction at which the spectral switch occurs is almost invariant with the changes of the polarization of the incident light, whereas the degree of the spectral switch (i.e., the magnitude of the transition from red-shift to blue-shift) is affected by the polarization of the incident light wave.

In conclusion, we discuss the far-zone scattered spectrum of an electromagnetic light wave on weak scattering from a semisoft boundary medium. It is shown that the far-zone scattered spectrum is affected by characters of the scatterer and the polarization of the incident light wave. Moreover, the direction at which the spectral switch occurs is governed by the characters of the scatterer, whereas the magnitude of the spectral

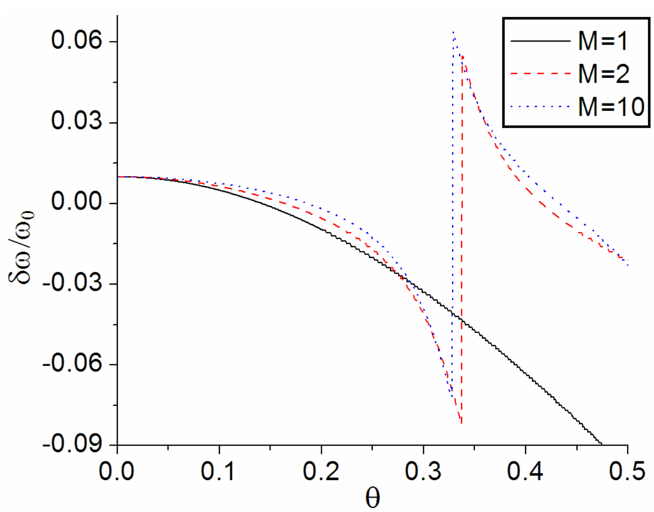

Fig. 2. Relative spectral shifts of the scattered field for scatterers with different boundary values (i.e., M). The parameters for calculation are: $A = 1$, $B_x = 1$, $B_y = 1$, $\omega_0 = 3\times10^{15}$ s^{-1}, $\Gamma_x = 0.05\omega_0$, $\Gamma_y = 0.075\omega_0$, $c = 3\times10^8$ m/s, $k_0 = \omega_0/c$, and $k_0\sigma = 10$.

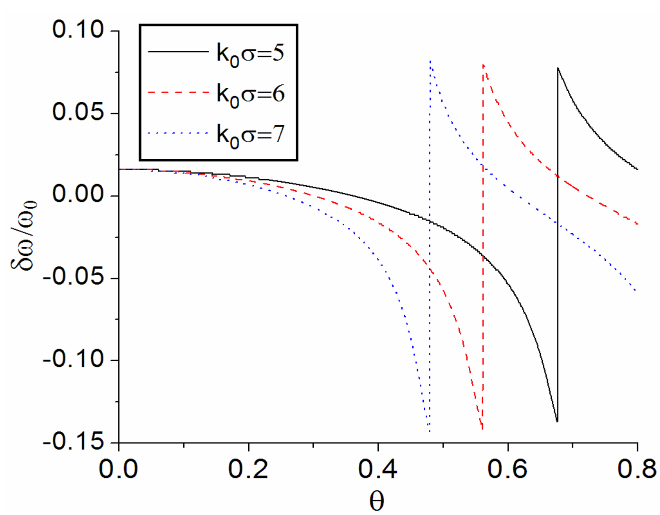

Fig. 3. Relative spectral shifts of the scattered field for scatterers with different effective widths of the dielectric susceptibility (i.e., $k_0\sigma$). The parameters for calculation are: $A = 1$, $B_x = 1$, $B_y = 1$, $\omega_0 = 3\times10^{15}$ s^{-1}, $\Gamma_x = 0.05\omega_0$, $\Gamma_y = 0.1\omega_0$, $c = 3\times10^8$ m/s, $k_0 = \omega_0/c$, and $M = 5$.

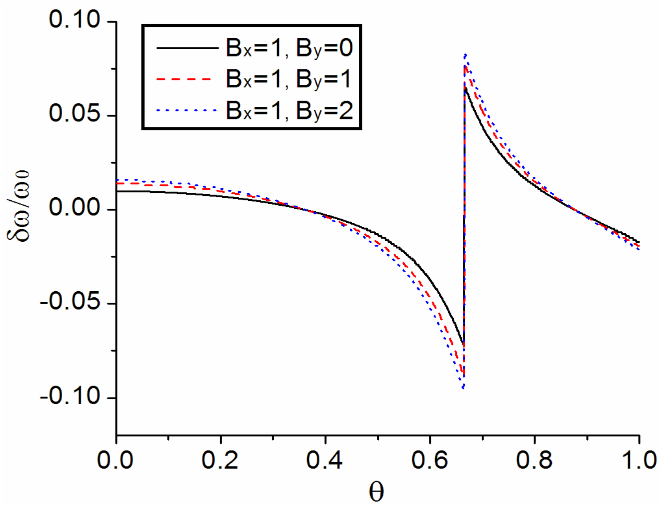

Fig. 4. Relative spectral shifts of the scattered field for incident light waves with different polarizations (i.e., B_x and B_y). The parameters for calculation are: $A = 1$, $\omega_0 = 3\times10^{15}$ s^{-1}, $\Gamma_x = 0.05\omega_0$, $\Gamma_y = 0.075\omega_0$, $c = 3\times10^8$ m/s, $M = 5$, $k_0 = \omega_0/c$, $k_0\sigma = 10$.

switch is affected by the polarization of the incident light wave.

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